LINKÖPINGS UNIVERSITET
Department of Computer and
Information Science
Division of Statistics and Machine learning/ANd

Assignment 2

Below are three tasks that you shall try to solve. All questions put should be answered. Prepare your solutions in a nice format that can be easily read. You can help each other but there must be individual submissions (that are not just copies of one submission).

Your solutions should be submitted at latest on **Monday 11 December 2017**.

1. *This is essentially Exercises 12 and 13 in Chapter 5 of "Winkler: An Introduction to Bayesian inference and decision, 2nd ed."*
   One nonprobabilistic decision-making criterion involves the consideration of a weighted average of the highest and lowest payoffs for each action. The weights, which must sum to 1, can be thought of as an optimism-pessimism index. The action with the highest weighted average of the highest and lowest payoffs is the action chosen by this criterion.

   a) Comment on this decision-making criterion and use it for payoff table (i) below with the highest payoff in each row receiving a weight of 0.4 and the lowest payoff receiving a weight of 0.6

Payoff table (i)

Action	State of the world				
	A	*B*	*C*	*D*	*E*
1	–50	80	20	100	0
2	30	40	70	20	50
3	10	30	–30	10	40
4	–10	–50	–70	–20	200

   b) Use the decision-making criterion described above for payoff table (ii) below, with the highest payoff in each row receiving a weight of 0.8 and the lowest payoff receiving a weight of 0.2. For payoff table (ii) the *ER* criterion would also involve a weighted average of the two payoffs in each row. Compare the criterion described above with the *ER* criterion.

Payoff table (ii)

Action	State of the world	
	I	*II*
1	10	4
2	7	9

2. *This is essentially Exercises 35ab, 38 and 39 in Chapter 5 of "Winkler: An Introduction to Bayesian inference and decision, 2nd ed."*
Suppose that a person's utility function for *total assets* (*not* changes in assets) is

$$U(A) = 200A - A^2 \text{ for } 0 \le A \le 100$$

where *A* represents total assets in thousands of dollars.

a) Graph this utility function. How could you classify this person with regard to their attitude towards risk?
b) If the person's total assets are currently \$10 000, should they take a bet in which they will win \$10 000 with probability 0.6 and loose \$10 000with probability 0.4?
c) A function that is often used to measure the degree of risk aversion in a given utility function is the Pratt-Arrow risk-aversion function. This function is of the form $r(A) = -U''(A)/U'(A)$, where $U(A)$ represents the utility function for total assets, and where the primes denote differentiation (first and second derivatives, i.e. $U'(A) = \frac{dU}{dA}$ and $U''(A) = \frac{d^2U}{dA^2}$. [*Primes have otherwise related to prior and posterior function in the textbook*]. Find $r(A)$ for the utility function given above.
d) Find the Pratt-Arrow risk-aversion functions for $0 < A < 100$ for the following utility functions, where *A* represents total assets in thousands of dollars
(i) $U(A) = 1 - e^{-0.05A}$
(ii) $U(A) = \log A = \ln A$
Graph these risk-aversion functions and the risk-aversion function in subtask c) above and compare them in terms of how the risk aversion changes as *A* increases.

3. *This is essentially Exercise 46 in Chapter 5 of "Winkler: An Introduction to Bayesian inference and decision, 2nd ed."*
In the slides to Meeting 8 (12 October) (which you find under Work plan and material on the course web) Exercise 36 and Exercise 45a in Chapter 5 of the textbook are solved. For each of the gambles in Exercise 45, show the risk premium graphically when the utility function for monetary payoffs is $U(R) = 40000 - (200 - R)^2$ for $-200 \le R \le 200$.