LINKÖPINGS UNIVERSITET
Department of Computer and
Information Science
Division of Statistics and Machine learning/ANd

732A66 Decision theory
Fall semester 2017
Assignment 1

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Below are four tasks that you shall try to solve. All questions put should be answered. Prepare your solutions in a nice format that can be easily read. You are allowed to help each other but there must be individual submissions (that are not just copies of one submission).

Your solutions should be submitted at latest on **Monday 23 October 2016**.

1. Assume a person is visiting his General Practitioner (GP) for some health problem. The patient shows a symptom that from the GP:s point-of-view could be the consequence of each of three different diseases. This symptom appears with disease A_1 in 25 % of all cases, with disease A_2 in 15 % of all cases, and with disease A_3 in 5 % of all cases. One can further approximately assume that a person cannot have more than one of these diseases at the same time. The symptom can also appear for other reasons. When none of the three mentioned diseases are present the probability is approximately 0.5 % that a person shows this symptom.

   a) What diagnosis should the GP give if she uses the principles of inference to the best explanation?
   b) Assume now that the prevalence of the diseases A_1, A_2 and A_3 are 0.1 %, 0.5 % and 1 % respectively. By prevalence is here meant the proportion of the relevant population (those people from which the current person belong) that has this disease at this specific point of time (point prevalence). What are the conditional probabilities of the person having respectively the diseases A_1, A_2 and A_3? What is your opinion about diagnosis according to principles of inference to the best explanation in this case?

2. *This is Exercise 17 in Chapter 4 of "Winkler: An Introduction to Bayesian inference and decision, 2nd ed."*
   In sampling from a Bernoulli process, the posterior distribution is the same whether one samples with *n* fixed (binomial sampling) or with *r* fixed (Pascal sampling). Explain why this is true. Suppose that a statistician merely samples until he is tired and decides to go home. Would the posterior distribution still be the same (that is, is the stopping rule noninformative)?

3. *This is Exercise 37 in Chapter 3 of "Winkler: An Introduction to Bayesian inference and decision, 2nd ed."*
   A bank official is concerned about the rate at which the bank's tellers provide service for their customers. He feels that all of the tellers work at about the same speed, which is either 30, 40 or 50 customers per hour. Furthermore, 40 customers per hour is twice as likely as each of the two other values, which are assumed to be equally likely. In order to obtain more information, the official observes all five tellers for a two-hour period, noting that 380 customers are served during that period. Use this new information to revise the official's probability distribution of the rate at which the tellers provide service.

4. Show that the two-parameter beta distribution belongs to the exponential class of distribution, i.e. its probability density function can be written on the form

$$f\left(\boldsymbol{x}\middle|\boldsymbol{\theta}\right)=e^{\sum_{j=1}^{k}A_j(\boldsymbol{\theta})B_j(\boldsymbol{x})+C(\boldsymbol{x})+D(\boldsymbol{\theta})}$$

(see further the presentation of meeting 5)

Now, find the form of a conjugate prior distribution for the two parameters of a beta distribution. You need only to specify it up to a proportionality constant that needs not to be calculated. Using a sample of 5 observations having this beta distribution, how is the prior distribution updated to a posterior distribution?

Assume you assign a prior distribution that is as non-informative as possible, still being a proper distribution. How will the posterior distribution develop when you increase the sample size from 5 point to 100 points? From 100 points to 1000 points?