SEMINAR EXERCISES IN PROBABILITY THEORY 732A63

PER SIDÉN

2. Conditioning

Exercise 2.1 (2.8 in Gut's book)
The density function of the two-dimensional random variable (X, Y) is

$$f_{X,Y}(x, y) = \begin{cases} \frac{x^2}{2 \cdot y^3} \cdot e^{-\frac{x}{y}} & , 0 < x < \infty, 0 < y < 1 \\ 0 & , \text{otherwise.} \end{cases}$$

(a) Determine the distribution of Y.
(b) Find the conditional distribution of X given that $Y = y$.
(c) Use the results from (a) and (b) to compute $E(X)$ and $Var(X)$.

Exercise 2.2 (2.11 in Gut's book)
Suppose X and Y have a joint density function given by

$$f_{X,Y}(x, y) = \begin{cases} c \cdot x^2 y & , 0 < y < x < 1 \\ 0 & , \text{otherwise.} \end{cases}$$

Compute c, the marginal densities, $E(X)$, $E(Y)$, and the conditional expectations $E(Y|X = x)$ and $E(X|Y = y)$.

Exercise 2.3 (2.33 in Gut's book)
Suppose that the random variable X is uniformly distributed symmetrically around zero, but in such a way that the parameter is uniform on $(0, 1)$; that is, suppose that

$$X|A = a \sim U(-a, a) \text{ with } A \sim U(0, 1).$$

Find the distribution of X, $E(X)$ and $Var(X)$.

Exercise 2.4 (2.35 in Gut's book)
Let X and Y be jointly distributed random variables such that

$$Y|X = x \sim Bin(n, x) \text{ with } X \sim U(0, 1).$$

Compute $E(Y)$, $Var(Y)$ and $Cov(X, Y)$ (without using what is known from Section 4 about the distribution of Y).

Exercise 2.5* (2.23 in Gut's book)
The joint density function of X and Y is given by

$$f_{X,Y}(x,y) = \begin{cases} c \cdot xy & , x,y > 0,\ 4x^2 + y^2 \leq 1 \\ 0 & , \text{otherwise.} \end{cases}$$

Compute c, the marginal densities, and the conditional expectations $E(Y|X=x)$ and $E(X|Y=y)$.

Exercise 2.6* (2.30 in Gut's book)
Show that a suitable power of a Weibull-distributed random variable whose parameter is gamma-distributed is Pareto-distributed. More precisely, show that if

$$X|A = a \sim W\left(\frac{1}{a}, \frac{1}{b}\right) \text{ with } A \sim \Gamma(p, \theta),$$

then X^b has a (translated) Pareto distribution.