SEMINAR EXERCISES IN PROBABILITY THEORY 732A63

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1. INTRODUCTION AND MULTIVARIATE RANDOM VARIABLES

Exercise 1.1
Let X and Y have the joint density

$$f_{X,Y}(x,y)=\begin{cases}\frac{1}{\sqrt{2\pi}}\exp\left[-\left(\frac{x^2}{8}+2y\right)\right] & ,\ -\infty<x<\infty,\ 0<y<\infty\\ 0 & ,\ \text{otherwise.}\end{cases}$$

(a) Determine the marginal distributions $f_X(x)$ and $f_Y(y)$. Do X and Y belong to some known distributions?

(b) Compute $E\left[(X+1)Y^2\right]$.

Exercise 1.2
Suppose that X and Y are random variables with joint density

$$f_{X,Y}(x,y)=\begin{cases}\frac{2}{3}(2x+y) & ,\ \begin{cases}0<x<1\\ 0<y<1\end{cases}\\ 0 & ,\ \text{otherwise.}\end{cases}$$

(a) Compute the probability density function and cumulative distribution function of X.

(b) Compute $P(2X+Y<1)$.

Exercise 1.3 (1.1 in Gut's book)
Show that if $X\sim C(0,1)$, then so is $1/X$.

Exercise 1.4 (1.8 in Gut's book)
Show that if X and Y are independent $N(0,1)$-distributed random variables,
then $X/Y\sim C(0,1)$.

Exercise 1.5* (1.16 in Gut's book)
A certain chemistry problem involves the numerical study of a lognormal random variable X. Suppose that the software package used requires the input of $E(Y)$ and $Var(Y)$ into the computer (where Y is normal and such that $X=e^Y$), but that one knows only the values of $E(X)$ and $Var(X)$. Find expressions for the former mean and variance in terms of the latter.

Exercise 1.6* (1.43a in Gut's book)

Let X and Y be independent random variables. Determine the distribution of $(X - Y) / (X + Y)$ if $X \sim Exp(1)$ and $Y \sim Exp(1)$.