SEMINAR EXERCISES IN PROBABILITY THEORY 732A63

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(EXERCISES BY PER SIDÉN)
14 SEPTEMBER 2020

2. Conditioning

Exercise 2.0.1 (self study)
Prove Theorem 2.1, p. 34 in Gut's book, in the discrete distribution case.
Exercise 2.0.2 (self study)
Prove Theorem 2.2(b), p. 36 in Gut's book, in the continuous distribution case.
Exercise 2.1 (2.8 in Gut's book)
The density function of the two-dimensional random variable (X, Y) is

$$f_{X,Y}(x,y) = \begin{cases} \frac{x^2}{2 \cdot y^3} \cdot e^{-\frac{x}{y}} & , 0 < x < \infty, 0 < y < 1 \\ 0 & , \text{otherwise.} \end{cases}$$

(a) Determine the distribution of Y.
(b) Find the conditional distribution of X given that $Y = y$.
(c) Use the results from (a) and (b) to compute $E(X)$ and $Var(X)$.

Exercise 2.2 (2.11 in Gut's book)
Suppose X and Y have a joint density function given by

$$f_{X,Y}(x,y) = \begin{cases} c \cdot x^2 y & , 0 < y < x < 1 \\ 0 & , \text{otherwise.} \end{cases}$$

Compute c, the marginal densities, $E(X)$, $E(Y)$, and the conditional expectations $E(Y|X = x)$ and $E(X|Y = y)$.

Exercise 2.3 (2.33 in Gut's book)
Suppose that the random variable X is uniformly distributed symmetrically around zero, but in such a way that the parameter is uniform on $(0, 1)$; that is, suppose that

$$X|A = a \sim U(-a, a) \text{ with } A \sim U(0, 1).$$

Find the distribution of X, $E(X)$ and $Var(X)$.

Exercise 2.4 (2.35 in Gut's book)
Let X and Y be jointly distributed random variables such that

$$Y|X = x \sim Bin\left(n, x\right) \text{ with } X \sim U\left(0, 1\right).$$

Compute $E\left(Y\right)$, $Var\left(Y\right)$ and $Cov\left(X, Y\right)$ (without using what is known from Section 4 about the distribution of Y).

Exercise 2.5* (2.23 in Gut's book)
The joint density function of X and Y is given by

$$f_{X,Y}\left(x, y\right) = \begin{cases} c \cdot xy & , x, y > 0,\, 4x^2 + y^2 \leq 1 \\ 0 & , \text{otherwise.} \end{cases}$$

Compute c, the marginal densities, and the conditional expectations $E\left(Y|X = x\right)$ and $E\left(X|Y = y\right)$.

Exercise 2.6* (2.30 in Gut's book)
Show that a suitable power of a Weibull-distributed random variable whose parameter is gamma-distributed is Pareto-distributed. More precisely, show that if

$$X|A = a \sim W\left(\frac{1}{a}, \frac{1}{b}\right) \text{ with } A \sim \Gamma\left(p, \theta\right),$$

then X^b has a (translated) Pareto distribution.