SEMINAR EXERCISES IN PROBABILITY THEORY 732A63

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(EXERCISES BY PER SIDÉN)
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1. INTRODUCTION AND MULTIVARIATE RANDOM VARIABLES

Exercise 1.0.1 (self study)
Using the presented transformation technique derive the densities for the functions of random variables on slides 20 and 21 of Per's slides.
Exercise 1.0.2 (self study)
Show equivalency between the representation of the bivariate normal distribution on slide 25 of Per's slides and the representation presented during the lecture

$$f_{X,Y}(x,y) = \frac{1}{2\pi|\mathbf{\Sigma}|}\exp(-\frac{1}{2}((x\ y)^T-\mu)^T\mathbf{\Sigma}^{-1}((x\ y)^T-\mu)).$$

Exercise 1.1
Let X and Y have the joint density

$$f_{X,Y}(x,y) = \begin{cases} \frac{1}{\sqrt{2\pi}}\exp\left[-\left(\frac{x^2}{8}+2y\right)\right] & , -\infty < x < \infty,\ 0 < y < \infty \\ 0 & , \text{otherwise.} \end{cases}$$

(a) Determine the marginal distributions $f_X(x)$ and $f_Y(y)$. Do X and Y belong to some known distributions?
(b) Compute $E\left[(X+1)Y^2\right]$.

Exercise 1.2
Suppose that X and Y are random variables with joint density

$$f_{X,Y}(x,y) = \begin{cases} \frac{2}{3}(2x+y) & , \begin{cases} 0 < x < 1 \\ 0 < y < 1 \end{cases} \\ 0 & , \text{otherwise.} \end{cases}$$

(a) Compute the probability density function and cumulative distribution function of X.
(b) Compute $P(2X+Y<1)$.

Exercise 1.3 (1.1 in Gut's book)
Show that if $X \sim C(0,1)$, then so is $1/X$.

Exercise 1.4 (1.8 in Gut's book)
Show that if X and Y are independent $N(0,1)$-distributed random variables, then $X/Y \sim C(0,1)$.

Exercise 1.5* (1.16 in Gut's book)
A certain chemistry problem involves the numerical study of a lognormal random variable X. Suppose that the software package used requires the input of $E(Y)$ and $Var(Y)$ into the computer (where Y is normal and such that $X = e^Y$), but that one knows only the values of $E(X)$ and $Var(X)$. Find expressions for the former mean and variance in terms of the latter.

Exercise 1.6* (1.43a in Gut's book)
Let X and Y be independent random variables. Determine the distribution of $(X-Y)/(X+Y)$ if $X \sim Exp(1)$ and $Y \sim Exp(1)$.