



Description logics

Description logics

- A family of knowledge representation languages
- Uses in different application areas (e.g., software management, configuration management, natural language processing, clinical information systems, information retrieval)
- Key technology for Ontologies and the Semantic Web

Ontologies, Description Logics and OWL terminology

Ontologies

DL

OWL

concept

concept

class

relation

role (binary)

property

axiom

axiom

axiom

instance

individual

individual

Outline

- DL languages
 - syntax and semantics
- DL reasoning services
 - algorithms, complexity
- DL systems
- DLs for the web



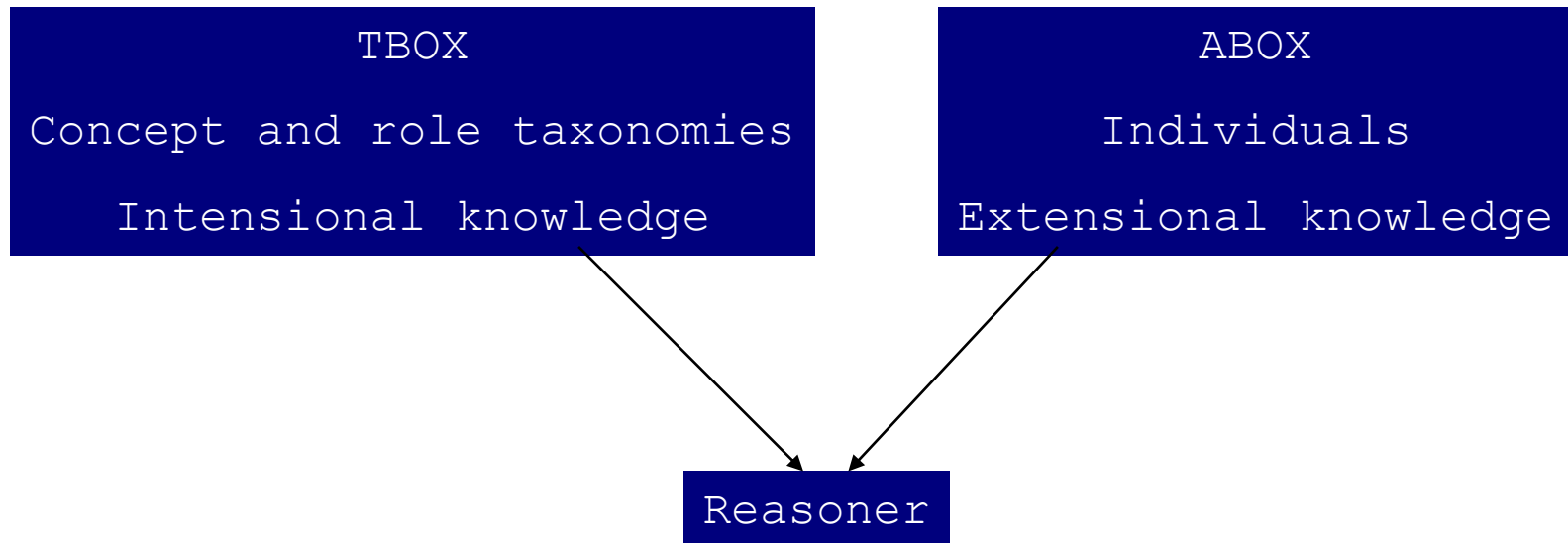
Example

Teams have at least two members, while large teams have at least 10 members. Sports teams are teams which have only athletes as members. A football team is a team which has at least 11 members and all the members are football players. Football players are athletes. Real Madrid is a football team that has Eden Hazard as a member.



DL SYNTAX

Tbox and Abox



\mathcal{AL}

R atomic role, A atomic concept

$C, D \rightarrow A$ | (atomic concept)

T | (universal concept, top) *owl:thing*

\perp | (bottom concept) *owl:nothing*

$\neg A$ | (atomic negation) *owl:complementOf*

$C \cap D$ | (conjunction) *owl:intersectionOf*

$\forall R.C$ | (value restriction) *owl:allValuesFrom*

$\exists R.T$ (limited existential quantification)

owl:someValuesFrom

$\mathcal{AL}[X]$

C $\neg C$ (concept negation) *owl:complementOf*

\mathcal{U} $C \cup D$ (disjunction) *owl:unionOf*

\mathcal{E} $\exists R.C$ (existential quantification)
owl:someValuesFrom

\mathcal{N} $\geq n R, \leq n R$ (number restriction)
owl:maxCardinality, owl:minCardinality

\mathcal{Q} $\geq n R.C, \leq n R.C$ (qualified number restriction)
owl:maxQualifiedCardinality, owl:minQualifiedCardinality

Concepts and relations

Team ← Concept/class
(Team)

\neg Team
(**not** Team)

Team $\cap \geq 10$ hasMember
(Team **and at least** 10 members)

Team $\cap \leq 10$ hasMember
(Team **and at most** 10 members)

Relation/role/property

Concepts and relations

Team $\cap \forall$ hasMember.Football-player

(Team **and all** members **are** football players)

Team $\cap \exists$ hasMember.Football-player

(Team **and there is a** member **that is a** football player)

$\mathcal{AL}[\mathcal{X}]$

\mathcal{R} $R \cap S$ (role conjunction)

\mathcal{I} R^- (inverse roles)

\mathcal{H} (role hierarchies)

\mathcal{F} $u_1 = u_2, u_1 \neq u_2$ (feature (dis)agreements)

$S[X]$

S \mathcal{ALC} + transitive roles

$S\mathcal{HIQ}$ \mathcal{ALC} + transitive roles
 + role hierarchies
 + inverse roles
 + number restrictions

Tbox - Terminological axioms

- $C = D$ ($R = S$)

owl:equivalentClass / owl:equivalentProperty

- $C \subseteq D$ ($R \subseteq S$)

rdfs:subClassOf / rdfs:subPropertyOf

Football-player \subseteq Athlete

(**Every** football player **is** an athlete)

- (disjoint $C D$)

owl:disjointWith

Tbox

- An equality whose left-hand side is an atomic concept is a definition.
- A finite set of definitions T is a Tbox (or terminology) if no symbolic name is defined more than once.

Example

Team $\subseteq \geq 2$ hasMember

Large-Team = Team $\cap \geq 10$ hasMember

Sports-team = Team $\cap \forall$ hasMember.Athlete

Football-Team = Team $\cap \geq 11$ hasMember

$\cap \forall$ hasMember.Football-player

Football-player \subseteq Athlete

DL as sublanguage of FOPL

Team(this)

\wedge

$(\exists x_1, \dots, x_{11}:$

hasMember(this, x1) \wedge ... \wedge hasMember(this, x11)

$\wedge x_1 \neq x_2 \wedge \dots \wedge x_{10} \neq x_{11})$

\wedge

$(\forall x: \text{hasMember}(\text{this}, x) \rightarrow \text{Football-player}(x))$

Abox

- Assertions about individuals:
 - $C(a)$
 - $R(a,b)$

Example

Football-Team(Real_Madrid)
hasMember(Real_Madrid, Eden_Hazard)

Knowledge base

A knowledge base is a tuple $\langle T, A \rangle$
where T is a Tbox and A is an Abox.

Example

Team $\subseteq \geq 2$ hasMember

Large-Team = Team $\cap \geq 10$ hasMember

Sports-team = Team $\cap \forall$ hasMember.Athlete

Football-Team = Team $\cap \geq 11$ hasMember

$\cap \forall$ hasMember.Football-player

Football-player \subseteq Athlete

Football-Team(Real_Madrid)

hasMember(Real_Madrid, Eden_Hazard)

Example - OWL

<Declaration> <ObjectProperty IRI="#hasmember"/> </Declaration>

<Declaration> <Class IRI="#football-player"/> </Declaration>

<Declaration> <Class IRI="#athlete"/> </Declaration>

<Declaration> <Class IRI="#team"/> </Declaration>

<Declaration> <Class IRI="#large-team"/> </Declaration>

<Declaration> <Class IRI="#sports-team"/> </Declaration>

<Declaration> <Class IRI="#football-team"/> </Declaration>

<Declaration> <NamedIndividual IRI="#Real_Madrid"/> </Declaration>

<Declaration> <NamedIndividual IRI="#Eden_Hazard"/> </Declaration>

Example - OWL

Large-Team = Team \cap ≥ 10 hasMember

```
<EquivalentClasses>
  <Class IRI="#large-team"/>
  <ObjectIntersectionOf>
    <Class IRI="#team"/>
    <ObjectMinCardinality cardinality="10">
      <ObjectProperty IRI="#hasmember"/>
    </ObjectMinCardinality>
  </ObjectIntersectionOf>
</EquivalentClasses>
```

Example - OWL

Football-Team = Team \cap ≥ 11 hasMember \cap \forall hasMember.Football-player

```
<EquivalentClasses>
  <Class IRI="#football-team"/>
  <ObjectIntersectionOf>
    <Class IRI="#team"/>
    <ObjectAllValuesFrom>
      <ObjectProperty IRI="#hasmember"/>
      <Class IRI="#football-player"/>
    </ObjectAllValuesFrom>
    <ObjectMinCardinality cardinality="11">
      <ObjectProperty IRI="#hasmember"/>
    </ObjectMinCardinality>
  </ObjectIntersectionOf>
</EquivalentClasses>
```




DL SEMANTICS

\mathcal{AL} (Semantics)

An interpretation \mathcal{I} consists of a non-empty set $\Delta^{\mathcal{I}}$ (the domain of the interpretation) and an interpretation function $\cdot^{\mathcal{I}}$ which assigns to every atomic concept A a set $A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$ and to every atomic role R a binary relation $R^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$.

The interpretation function is extended to concept definitions using inductive definitions.

\mathcal{AL} (Semantics)

$C, D \rightarrow A$ | (atomic concept)

\top | (universal concept) $\top^{\mathcal{J}} = \Delta^{\mathcal{J}}$

\perp | (bottom concept) $\perp^{\mathcal{J}} = \emptyset$

$\neg A$ | (atomic negation) $(\neg A)^{\mathcal{J}} = \Delta^{\mathcal{J}} \setminus A^{\mathcal{J}}$

$C \cap D$ | (conjunction) $(C \cap D)^{\mathcal{J}} = C^{\mathcal{J}} \cap D^{\mathcal{J}}$

$\forall R.C$ | (value restriction) $(\forall R.C)^{\mathcal{J}} =$

$\{a \in \Delta^{\mathcal{J}} \mid \forall b.(a,b) \in R^{\mathcal{J}} \rightarrow b \in C^{\mathcal{J}}\}$

$\exists R.T$ | (limited existential
quantification) $(\exists R.T)^{\mathcal{J}} = \{a \in \Delta^{\mathcal{J}} \mid \exists b.(a,b) \in R^{\mathcal{J}}\}$

\mathcal{ALC} (Semantics)

$$(\neg C)^{\mathcal{J}} = \Delta^{\mathcal{J}} \setminus C^{\mathcal{J}}$$

$$(C \cup D)^{\mathcal{J}} = C^{\mathcal{J}} \cup D^{\mathcal{J}}$$

$$(\geq n R)^{\mathcal{J}} = \{a \in \Delta^{\mathcal{J}} \mid \# \{b \in \Delta^{\mathcal{J}} \mid (a,b) \in R^{\mathcal{J}}\} \geq n\}$$

$$(\leq n R)^{\mathcal{J}} = \{a \in \Delta^{\mathcal{J}} \mid \# \{b \in \Delta^{\mathcal{J}} \mid (a,b) \in R^{\mathcal{J}}\} \leq n\}$$

$$(\exists R.C)^{\mathcal{J}} = \{a \in \Delta^{\mathcal{J}} \mid \exists b \in \Delta^{\mathcal{J}} : (a,b) \in R^{\mathcal{J}} \wedge b \in C^{\mathcal{J}}\}$$

Semantics

Individual i

$$i^{\mathcal{J}} \in \Delta^{\mathcal{J}}$$

Unique Name Assumption:

$$\text{if } i_1 \neq i_2 \text{ then } i_1^{\mathcal{J}} \neq i_2^{\mathcal{J}}$$

Semantics

An interpretation \mathcal{J} is a model for a terminology T iff

$$C^{\mathcal{J}} = D^{\mathcal{J}} \text{ for all } C = D \text{ in } T$$

$$C^{\mathcal{J}} \subseteq D^{\mathcal{J}} \text{ for all } C \subseteq D \text{ in } T$$

$$C^{\mathcal{J}} \cap D^{\mathcal{J}} = \emptyset \text{ for all (disjoint } C \ D) \text{ in } T$$

Semantics

An interpretation \mathcal{I} is a model for a knowledge base $\langle T, A \rangle$ iff

\mathcal{I} is a model for T

$a^{\mathcal{I}} \in C^{\mathcal{I}}$ for all $C(a)$ in A

$\langle a^{\mathcal{I}}, b^{\mathcal{I}} \rangle \in R^{\mathcal{I}}$ for all $R(a,b)$ in A

Semantics - acyclic Tbox

$\text{Bird} = \text{Animal} \cap \forall \text{Skin.Feather}$

$\Delta^{\mathcal{J}} = \{\text{tweety}, \text{goofy}, \text{fea1}, \text{fur1}\}$

$\text{Animal}^{\mathcal{J}} = \{\text{tweety}, \text{goofy}\}$

$\text{Feather}^{\mathcal{J}} = \{\text{fea1}\}$

$\text{Skin}^{\mathcal{J}} = \{\langle \text{tweety}, \text{fea1} \rangle, \langle \text{goofy}, \text{fur1} \rangle\}$

$\rightarrow \text{Bird}^{\mathcal{J}} = \{\text{tweety}\}$

Exercise - Homework for credits

Create an interpretation for:

Team $\subseteq \geq 2$ hasMember

Large-Team = Team $\cap \geq 10$ hasMember

Sports-team = Team $\cap \forall$ hasMember.Athlete

Football-Team = Team $\cap \geq 11$ hasMember

$\cap \forall$ hasMember.Football-player

Football-player \subseteq Athlete

Football-Team(Real_Madrid)

hasMember(Real_Madrid, Eden_Hazard)

Semantics - cyclic Tbox

$\text{QuietPerson} = \text{Person} \cap \forall \text{Friend}.\text{QuietPerson}$
($A = F(A)$)

$\Delta^{\mathcal{J}} = \{\text{john}, \text{sue}, \text{andrea}, \text{bill}\}$

$\text{Person}^{\mathcal{J}} = \{\text{john}, \text{sue}, \text{andrea}, \text{bill}\}$

$\text{Friend}^{\mathcal{J}} = \{\langle \text{john}, \text{sue} \rangle, \langle \text{andrea}, \text{bill} \rangle, \langle \text{bill}, \text{bill} \rangle\}$

→ $\text{QuietPerson}^{\mathcal{J}} = \{\text{john}, \text{sue}\}$

→ $\text{QuietPerson}^{\mathcal{J}} = \{\text{john}, \text{sue}, \text{andrea}, \text{bill}\}$

Semantics - cyclic Tbox

Descriptive semantics: $A = F(A)$ is a constraint stating that A has to be some solution for the equation.

- Not appropriate for defining concepts
- Necessary and sufficient conditions for concepts

Human = Mammal \cap \exists Parent

$\cap \forall$ Parent.Human

Semantics - cyclic Tbox

Least fixpoint semantics: $A = F(A)$ specifies that A is to be interpreted as the smallest solution (if it exists) for the equation.

- Appropriate for inductively defining concepts

$DG = \text{EmptyDG} \cup \text{Non-Empty-DG}$

$\text{Non-Empty-DG} = \text{Node} \cap \forall \text{Arc. Non-Empty-DG}$

$\text{Human} = \text{Mammal} \cap \exists \text{Parent} \cap \forall \text{Parent. Human} \rightarrow$
 $\text{Human} = \perp$

Semantics - cyclic Tbox

Greatest fixpoint semantics: $A = F(A)$ specifies that A is to be interpreted as the greatest solution (if it exists) for the equation.

- Appropriate for defining concepts whose individuals have circularly repeating structure

$FoB = Blond \cap \exists Child.FoB$

$Human = Mammal \cap \exists Parent \cap \forall Parent.Human$

$Horse = Mammal \cap \exists Parent \cap \forall Parent.Horse$

$\rightarrow Human = Horse$

Open world vs closed world semantics

Databases: closed world reasoning

database instance represents one interpretation

→ absence of information interpreted as negative information

“complete information”

query evaluation is finite model checking

DL: open world reasoning

Abox represents many interpretations (its models)

→ absence of information is lack of information

“incomplete information”

query evaluation is logical reasoning

Open world vs closed world semantics

hasChild(Jocasta, Oedipus)

hasChild(Jocasta, Polyneikes)

hasChild(Oedipus, Polyneikes)

hasChild(Polyneikes, Thersandros)

patricide(Oedipus)

¬ patricide(Thersandros) *(not represented in DB)*

Does it follow from the Abox that

$\exists \text{hasChild.}(\text{patricide} \sqcap \exists \text{hasChild.} \neg \text{patricide})(\text{Jocasta})$?



DL REASONING



Example

Teams have at least two members, while large teams have at least 10 members. Sports teams are teams which have only athletes as members. A football team is a team which has at least 11 members and all the members are football players. Football players are athletes. Real Madrid is a football team that has Eden Hazard as a member.

Example

Team $\subseteq \geq 2$ hasMember

Large-Team = Team $\cap \geq 10$ hasMember

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Football-player \subseteq Athlete

Football-Team(Real_Madrid)

hasMember(Real_Madrid, Eden_Hazard)



Example

Every team has at least 2 members

Every large team is a team and has at least 10 members

Every sports team is a team and has only athletes as members

Every football team is a team and has at least 11 members
and has only football players as members



Example

Every team has at least 2 members

Every large team is a team and has at least 10 members

Every sports team is a team and has only athletes as members

Every football team is a team and has at least 11 members
and has only football players as members

Reasoning:

Every football team is a large team

Every football team is a sports team



Example

Real Madrid is an instance of football team

Real Madrid has member Eden Hazard

Example

Reasoning:

Real Madrid is an instance of football team

Real Madrid is an instance of large team

Real Madrid is an instance of team

Real Madrid is an instance of sports team

Real Madrid has at least 11 members

All members in Real Madrid are football players

All members in Real Madrid are athletes

Real Madrid has member Eden Hazard

Eden Hazard is an instance of football player

Eden Hazard is an instance of athlete



Reasoning services

- Satisfiability of concept
- Subsumption between concepts
- Equivalence between concepts
- Disjointness of concepts

- Classification

- Instance checking
- Realization
- Retrieval
- Knowledge base consistency

Reasoning services

- Reduction to subsumption
 - C is unsatisfiable iff C is subsumed by \perp
 - C and D are equivalent iff C is subsumed by D and D is subsumed by C
 - C and D are disjoint iff $C \cap D$ is subsumed by \perp
- The statements also hold w.r.t. a Tbox.

Reasoning services

- Reduction to unsatisfiability
 - C is subsumed by D iff $C \cap \neg D$ is unsatisfiable
 - C and D are equivalent iff
 - both $(C \cap \neg D)$ and $(D \cap \neg C)$ are unsatisfiable
 - C and D are disjoint iff $C \cap D$ is unsatisfiable
- The statements also hold w.r.t. a Tbox.

Tableau algorithms

- To prove that C subsumes D:
 - If C subsumes D, then it is impossible for an individual to belong to D but not to C.
 - Idea: Create an individual that belongs to D and not to C and see if it causes a contradiction.
 - If **always** a contradiction (clash) then subsumption is proven. Otherwise, we have found a model that contradicts the subsumption.

Tableau algorithms

- Based on constraint systems.
 - $S = \{ x: \neg C \cap D \}$
 - Add constraints according to a set of propagation rules
 - Until clash or no constraint is applicable

Tableau algorithms – de Morgan rules

$$\neg \neg C \rightarrow C$$

$$\neg (A \cap B) \rightarrow \neg A \cup \neg B$$

$$\neg (A \cup B) \rightarrow \neg A \cap \neg B$$

$$\neg (\forall R.C) \rightarrow \exists R.(\neg C)$$

$$\neg (\exists R.C) \rightarrow \forall R.(\neg C)$$

Tableau algorithms – constraint propagation rules

- $S \rightarrow \cap \{x:C_1, x:C_2\} \cup S$

if $x: C_1 \cap C_2$ in S

and either $x:C_1$ or $x:C_2$ is not in S

- $S \rightarrow \cup \{x:D\} \cup S$

if $x: C_1 \cup C_2$ in S and neither $x:C_1$ or $x:C_2$ is in S , and $D = C_1$ or $D = C_2$

Tableau algorithms – constraint propagation rules

- $S \rightarrow_{\forall} \{y:C\} \cup S$

if $x: \forall R.C$ in S and xRy in S and $y:C$ is not in S

- $S \rightarrow_{\exists} \{xRy, y:C\} \cup S$

if $x: \exists R.C$ in S and y is a new variable and there is no z such that both xRz and $z:C$ are in S

Example

- ST: Tournament

$\cap \exists \text{ hasParticipant.Swedish}$

- SBT: Tournament

$\cap \exists \text{ hasParticipant.}(\text{Swedish} \cap \text{Belgian})$

Example 1

- SBT \Rightarrow ST?
- $S = \{ x:$
 - $\neg(\text{Tournament} \cap \exists \text{ hasParticipant.Swedish})$
 - $\cap (\text{Tournament}$
 - $\cap \exists \text{ hasParticipant.}(\text{Swedish} \cap \text{Belgian}))$
 - $\}$

Example 1

- $S = \{ x:$
 $(\neg \text{Tournament}$
 $\cup \forall \text{ hasParticipant.} \neg \text{Swedish})$
 $\cap (\text{Tournament}$
 $\cap \exists \text{ hasParticipant.} (\text{Swedish} \cap \text{Belgian}))$
 $\}$

Example 1

\cap -rule:

- $S = \{$
 - $x: (\neg \text{Tournament}$
 - $\cup \forall \text{hasParticipant}.\neg \text{Swedish})$
 - $\cap (\text{Tournament}$
 - $\cap \exists \text{hasParticipant}.\text{Swedish} \cap \text{Belgian}),$
 - $x: \neg \text{Tournament}$**
 - $\cup \forall \text{hasParticipant}.\neg \text{Swedish},$**
 - $x: \text{Tournament},$**
 - $x: \exists \text{hasParticipant}.\text{Swedish} \cap \text{Belgian}$**

Example 1

\exists -rule:

- $S = \{$
x: (\neg Tournament $\cup \forall$ hasParticipant. \neg Swedish)
 \cap (Tournament
 $\cap \exists$ hasParticipant.(Swedish \cap Belgian)),
x: \neg Tournament
 $\cup \forall$ hasParticipant. \neg Swedish,
x: Tournament,
x: \exists hasParticipant.(Swedish \cap Belgian),
x hasParticipant y, y: (Swedish \cap Belgian)
}

Example 1

\cap -rule:

- $S = \{x: (\neg \text{Tournament} \cup \forall \text{ hasParticipant.} \neg \text{Swedish})$
 $\cap (\text{Tournament}$
 $\cap \exists \text{ hasParticipant.}(\text{Swedish} \cap \text{Belgian})),$
 $x: \neg \text{Tournament} \cup \forall \text{ hasParticipant.} \neg \text{Swedish},$
 $x: \text{Tournament},$
 $x: \exists \text{ hasParticipant.}(\text{Swedish} \cap \text{Belgian}),$
 $x \text{ hasParticipant } y, y: (\text{Swedish} \cap \text{Belgian}),$
 $y: \text{Swedish}, y: \text{Belgian} \quad \}$

Example 1

U-rule, choice 1

- $S = \{ x: (\neg\text{Tournament} \cup \forall \text{hasParticipant}.\neg \text{Swedish})$
 $\cap (\text{Tournament}$
 $\cap \exists \text{hasParticipant}.\text{Swedish} \cap \text{Belgian}),$
 $x: \neg\text{Tournament} \cup \forall \text{hasParticipant}.\neg \text{Swedish},$
 $x: \text{Tournament},$
 $x: \exists \text{hasParticipant}.\text{Swedish} \cap \text{Belgian},$
 $x \text{ hasParticipant } y, y: (\text{Swedish} \cap \text{Belgian}),$
 $y: \text{Swedish}, y: \text{Belgian},$
 $x: \neg\text{Tournament}$
 $\}$

→ clash

Example 1

U-rule, choice 2

- $S = \{x: (\neg \text{Tournament} \cup \forall \text{ hasParticipant}.\neg \text{Swedish})$
 $\cap (\text{Tournament}$
 $\cap \exists \text{ hasParticipant}.\text{Swedish} \cap \text{Belgian}),$
 $x: \neg \text{Tournament} \cup \forall \text{ hasParticipant}.\neg \text{Swedish},$
 $x: \text{Tournament},$
 $x: \exists \text{ hasParticipant}.\text{Swedish} \cap \text{Belgian},$
 $x \text{ hasParticipant } y, y: (\text{Swedish} \cap \text{Belgian}),$
 $y: \text{Swedish}, y: \text{Belgian},$
 $x: \forall \text{ hasParticipant}.\neg \text{Swedish}$
}

Example 1

choice 2 – continued

\forall -rule

- $S = \{$
- x: (\neg Tournament $\cup \forall$ hasParticipant. \neg Swedish)
- \cap (Tournament $\cap \exists$ hasParticipant.(Swedish \cap Belgian)),
- x: \neg Tournament $\cup \forall$ hasParticipant. \neg Swedish,
- x: Tournament,
- x: \exists hasParticipant.(Swedish \cap Belgian),
- x hasParticipant y, y: (Swedish \cap Belgian),
- y: Swedish, y: Belgian,
- x: \forall hasParticipant. \neg Swedish,
- y: \neg Swedish**
- $\}$

→ clash

Example 2

- $ST \Rightarrow SBT?$
- $S = \{ x:$
 - $\neg (\text{Tournament}$
 - $\cap \exists \text{hasParticipant.}(\text{Swedish} \cap \text{Belgian}))$
 - $\cap (\text{Tournament} \cap \exists \text{hasParticipant.Swedish})$
 - $\}$

Example 2

- $S = \{ x:$
 $(\neg \text{Tournament}$
 $\cup \forall \text{hasParticipant.}(\neg \text{Swedish} \cup \neg \text{Belgian}))$
 $\cap (\text{Tournament} \cap \exists \text{hasParticipant.Swedish})$
 $\}$

Example 2

\cap -rule

- $S = \{$
 $x: (\neg \text{Tournament}$
 $\cup \forall \text{hasParticipant} . (\neg \text{Swedish} \cup \neg \text{Belgian}))$
 $\cap (\text{Tournament} \cap \exists \text{hasParticipant} . \text{Swedish}),$
 $x: (\neg \text{Tournament}$
 $\cup \forall \text{hasParticipant} . (\neg \text{Swedish} \cup \neg \text{Belgian}))$,
 $x: \text{Tournament},$
 $x: \exists \text{hasParticipant} . \text{Swedish}$
 $\}$

Example 2

\exists -rule

- $S = \{$
 - $x: (\neg \text{Tournament}$
 - $\cup \forall \text{ hasParticipant.}(\neg \text{Swedish} \cup \neg \text{Belgian}))$
 - $\cap (\text{Tournament} \cap \exists \text{ hasParticipant.Swedish}),$
 - $x: (\neg \text{Tournament}$
 - $\cup \forall \text{ hasParticipant.}(\neg \text{Swedish} \cup \neg \text{Belgian})),$
 - $x: \text{Tournament},$
 - $x: \exists \text{ hasParticipant.Swedish},$
 - $x \text{ hasParticipant } y, y: \text{Swedish}$**
 - $\}$

Example 2

U –rule, choice 1

- S = {
 - x: (\neg Tournament
 - U \forall hasParticipant.(\neg Swedish U \neg Belgian))
 - \cap (Tournament \cap \exists hasParticipant.Swedish),
 - x: (\neg Tournament
 - U \forall hasParticipant.(\neg Swedish U \neg Belgian)),
 - x: Tournament,
 - x: \exists hasParticipant.Swedish,
 - x hasParticipant y, y: Swedish,
 - x: \neg Tournament**}

→ clash

Example 2

U \neg -rule, choice 2

- $S = \{$
 - $x: (\neg \text{Tournament}$
 - $\cup \forall \text{ hasParticipant.}(\neg \text{Swedish} \cup \neg \text{Belgian}))$
 - $\cap (\text{Tournament} \cap \exists \text{ hasParticipant.Swedish}),$
 - $x: (\neg \text{Tournament}$
 - $\cup \forall \text{ hasParticipant.}(\neg \text{Swedish} \cup \neg \text{Belgian})),$
 - $x: \text{Tournament},$
 - $x: \exists \text{ hasParticipant.Swedish},$
 - $x \text{ hasParticipant } y, y: \text{Swedish},$
 - $x: \forall \text{ hasParticipant.}(\neg \text{Swedish} \cup \neg \text{Belgian})$**
- $\}$

Example 2

choice 2 continued

\forall -rule

- $S = \{$
 $x: (\neg \text{Tournament}$
 $\cup \forall \text{ hasParticipant.}(\neg \text{Swedish} \cup \neg \text{Belgian}))$
 $\cap (\text{Tournament} \cap \exists \text{ hasParticipant.Swedish}),$
 $x: (\neg \text{Tournament}$
 $\cup \forall \text{ hasParticipant.}(\neg \text{Swedish} \cup \neg \text{Belgian})),$
 $x: \text{Tournament},$
 $x: \exists \text{ hasParticipant.Swedish},$
 $x \text{ hasParticipant } y, y: \text{Swedish},$
 $x: \forall \text{ hasParticipant.}(\neg \text{Swedish} \cup \neg \text{Belgian}),$
 $\mathbf{y: (\neg \text{Swedish} \cup \neg \text{Belgian})}$
 $\}$

Example 2

choice 2 continued

U-rule, choice 2.1

- $S = \{$
 - $x: (\neg \text{Tournament}$
 - $U \forall \text{ hasParticipant.}(\neg \text{Swedish } U \neg \text{Belgian}))$
 - $\cap (\text{Tournament} \cap \exists \text{ hasParticipant.Swedish}),$
 - $x: (\neg \text{Tournament}$
 - $U \forall \text{ hasParticipant.}(\neg \text{Swedish } U \neg \text{Belgian})),$
 - $x: \text{Tournament},$
 - $x: \exists \text{ hasParticipant.Swedish},$
 - $x \text{ hasParticipant } y, y: \text{Swedish},$
 - $x: \forall \text{ hasParticipant.}(\neg \text{Swedish } U \neg \text{Belgian}),$
 - $y: (\neg \text{Swedish } U \neg \text{Belgian}),$
 - $y: \neg \text{Swedish}$**
- $\} \rightarrow \text{clash}$

Example 2

choice 2 continued

U-rule, choice 2.2

- $S = \{$
 - $x: (\neg \text{Tournament}$
 - $U \forall \text{ hasParticipant.}(\neg \text{Swedish } U \neg \text{Belgian}))$
 - $\cap (\text{Tournament} \cap \exists \text{ hasParticipant.Swedish}),$
 - $x: (\neg \text{Tournament}$
 - $U \forall \text{ hasParticipant.}(\neg \text{Swedish } U \neg \text{Belgian})),$
 - $x: \text{Tournament},$
 - $x: \exists \text{ hasParticipant.Swedish},$
 - $x \text{ hasParticipant } y, y: \text{Swedish},$
 - $x: \forall \text{ hasParticipant.}(\neg \text{Swedish } U \neg \text{Belgian}),$
 - $y: (\neg \text{Swedish } U \neg \text{Belgian}),$
 - $y: \neg \text{Belgian}$**
 - $\} \rightarrow \text{ok, model}$

Complexity - languages

- Overview available via the DL home page at <http://dl.kr.org>

Example tractable language:

$A, T, \perp, \neg A, C \cap D, \forall R.C, \geq n R, \leq n R$

Reasons for intractability:

choices, e.g. $C \cup D$

exponential size models,

e.g. interplay universal and existential quantification

Reasons for undecidability:

e.g. role-value maps $R=S$

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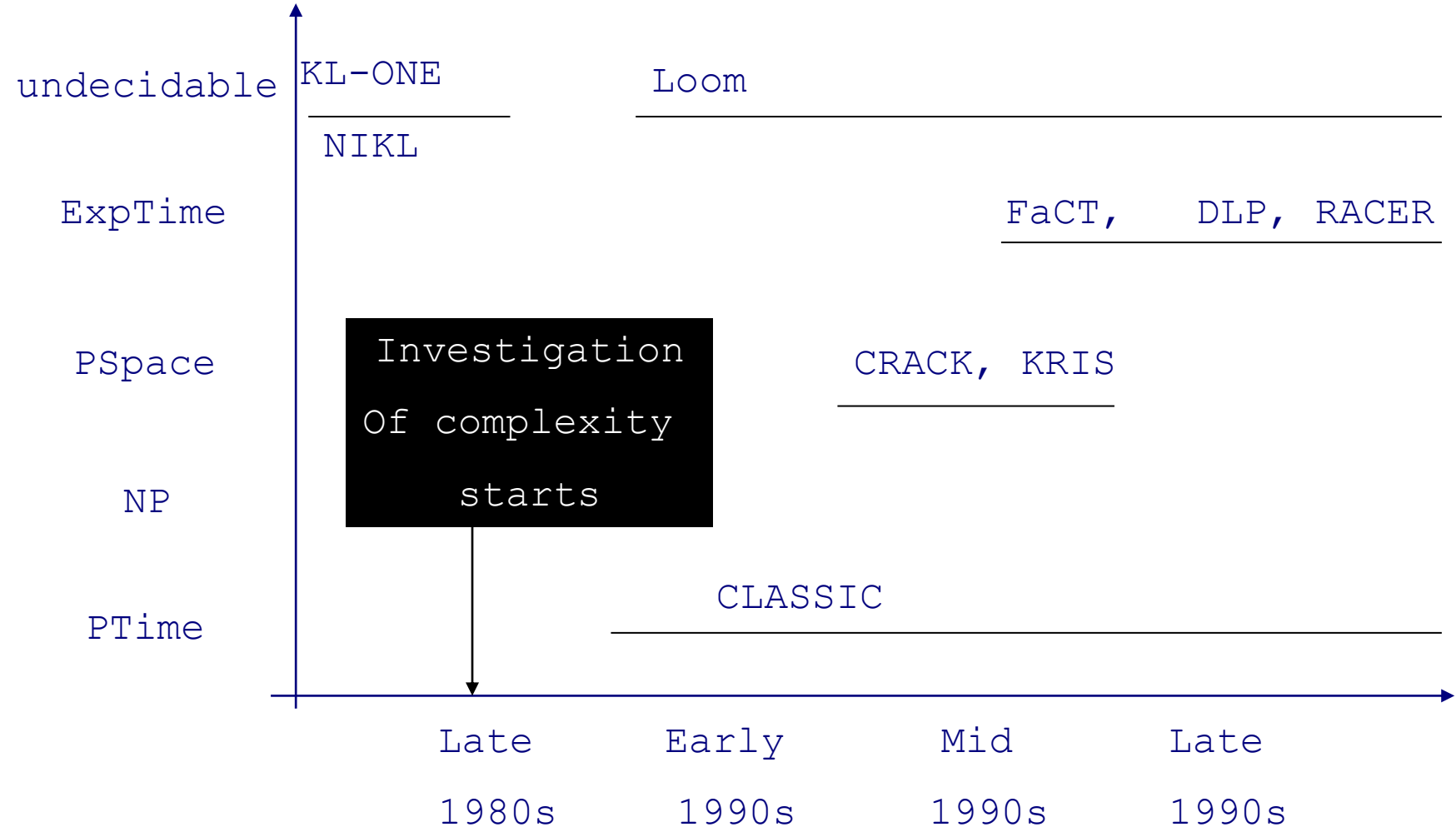
Reasons for undecidability:

e.g. role-value maps $R=S$



DL SYSTEMS

Systems



Systems

- Overview available via the DL home page at <http://dl.kr.org>
- Current systems include: CEL, Cerebra Enginer, FaCT++, fuzzyDL, HermiT, KAON2, MSPASS, Pellet, QuOnto, RacerPro, SHER



Extensions

- Time
- Defaults
- Part-of
- Knowledge and belief
- Uncertainty (fuzzy, probabilistic)



DL AND THE WEB

OWL

- OWL-Lite, OWL-DL, OWL-Full: increasing expressivity
- A legal OWL-Lite ontology is a legal OWL-DL ontology is a legal OWL-Full ontology
- OWL-DL: expressive description logic, decidable
- XML-based
- RDF-based (OWL-Full is extension of RDF, OWL-Lite and OWL-DL are extensions of a restriction of RDF)

OWL-Lite

- **Class**, subClassOf, equivalentClass
- intersectionOf (only named classes and restrictions)
- **Property**, subPropertyOf, equivalentProperty
- domain, range (global restrictions)
- inverseOf, TransitiveProperty (*), SymmetricProperty, FunctionalProperty, InverseFunctionalProperty
- allValuesFrom, someValuesFrom (local restrictions)
- minCardinality, maxCardinality (only 0/1)
- **Individual**, sameAs, differentFrom, AllDifferent

(*) restricted

OWL-DL

- **Type separation** (class cannot also be individual or property, property cannot be also class or individual), Separation between DatatypeProperties and ObjectProperties
- **Class –complex classes**, subClassOf, equivalentClass, *disjointWith*
- intersectionOf, *unionOf*, *complementOf*
- **Property**, subPropertyOf, equivalentProperty
- domain, range (global restrictions)
- inverseOf, TransitiveProperty (*), SymmetricProperty, FunctionalProperty, InverseFunctionalProperty
- allValuesFrom, someValuesFrom (local restrictions), *oneOf*, *hasValue*
- minCardinality, maxCardinality
- **Individual**, sameAs, differentFrom, AllDifferent

(*) restricted

OWL2

- OWL2 Full and OWL2 DL
- OWL2 DL compatible with SROIQ
- Punning
 - IRI may denote both class and individual
 - For reasoning they are considered separate entities

OWL2 profiles

- OWL2 EL (based on EL++)
 - Essentially intersection and existential quantification
 - SNOMED CT, NCI Thesaurus
- OWL2 QL (“query language”)
 - Can be realized using relational database technology
 - RDFS + small extensions
- OWL2 RL (“rule language”)

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