Orderings (Partial Orders)

Weak order: transitive, reflexive and antisymmetric
Strict order: transitive, irreflexive and asymmetric

Weak Order: Examples

- $\leq$ on $\mathbb{N}$
- $\subseteq$ on sets

Strict Order: Examples

- $<$ on $\mathbb{N}$
- $\subset$ on sets
- *Descendant*(x, y) on human beings
- *LowerRank*(x, y) on soldiers

How Are the Types of Orders Connected?

Let R be a strict order on a set A.
Then $R \cup id_A$ is a weak order.
Let R be a weak order on a set A.
Then $R - id_A$ is a strict order.

Orderings: Concepts (1)

Let R be an order.
If $R(x, y)$, we say that x *precedes* y, and that y *succeeds* x.
If $R(x, y)$, with $x \neq y$, and there is no z such that $z \neq x$, $z \neq y$, $R(x, z)$ and $R(z, y)$, we say that x *immediately precedes* y.

Orderings: Concepts (2)

An element $x \in A$ is *minimal*, iff no *other* element in A precedes x (according to R).
An element $x \in A$ is *maximal*, iff no *other* element in A succeeds x (according to R).

Examples

- 0 is minimal according to $<$ on $\mathbb{N}$, but there is no maximal element.
- All childless people are minimal according to *Descendant* on M and Adam and Eve are maximal (well, ...).
- All Privates are minimal accroding to *LoerRank* and ÖB (english?) is maximal.

Orderings: Concepts (3)

An element is *least* in A iff it *precedes* every *other* element in A.
An element is *greatest* in A iff it *succeeds* every *other* element in A.

Examples

- 0 is least according to $<$ on N there exists no greatest element.
- $\emptyset$ is least according to $\subseteq$ on $\mathcal{P}(A)$ and A is greatest.
- There exists no least or greatest element according to *Descendant*.
- There is no least element according to *LowerRank*, but ÖB is greatest.

Total Order (Linear Order)

A *connected* order is called a total order (or linear order).
"Connected" means that for every pair of elements x and y, either x precedes y or y precedes x.

Examples

- $<$ on N is a total order.
- $\subseteq$ on $\mathcal{P}(A)$ is not a total order.
- *StandsInFrontOf*(x, y) r is total order on people in a queue.
- Neither *Descendant*, nor *LowerRank* are total orders.

What Have We Done So Far? (Week 1)

- Sets with operations
- Relations: Sets of tuples, with operations and properties
- Special kinds of relations: Equivalence relations and orderings

What is left? (Week 2)

- Functions: what are they good for, properties, and so on
- Boolean algebra
- Infinite sets and ways of defining them: Recursive definitions
- Proofs by induction and recursive functions, connections to recursive definitions