Tuples (sequences)

It is difficult to express *order* with sets,
since e.g. $\{1,2\} = \{2,1\}$.
New Concept: *tuple* (sequence),
can have both order and duplicates. We write:
$\langle x_1, x_2, x_3, \ldots, x_n \rangle$,
which is a tuple of lenght n (n-tuple).
If $n = 2$ the tuple is sometimes called a *pair*.

Equivalence between Tuples

When are tuples equivalent?
$\langle x_1, x_2, \ldots, x_m \rangle = \langle y_1, y_2, \ldots, y_n \rangle$ iff
$m = n$, $x_1 = y_1$, $x_2 = y_2$, $\ldots$, $x_m = y_n$.
Thus, we can see that $\langle 1,2 \rangle \neq \langle 2,1 \rangle$, just as we wanted!

Tuples as (Simple) Propositions

We can, e.g., represent $35 < 1729$ with
$\langle 35, 1729 \rangle$, since the order between the numbers matter.
Moreover: Pelle is Stina's father with
$\langle$Pelle, Stina$\rangle$,
or that book b_1 is between books b_2 and b_3 with
$\langle b_1, b_2, b_3 \rangle$.

Relations: Tuples in Sets

"Is the Father of" is a relation between two people.
We can represent relations as sets of n-tupler.
E.g. $Father = \{\langle Pelle, Stina \rangle, \langle Adam, Kain \rangle, \ldots\}$.
"Pelle is the father of Stina, Adam is the father of Kain,...".
We can write $\langle Pelle, Stina \rangle \in Far$,
or $Far(Pelle, Stina)$.

Relations: Example

$<$ ("Less than") is a relation.
$<= \{\langle 0,1 \rangle, \langle 0,2 \rangle, \langle 1,2 \rangle, \ldots\}$. We write $< (0,1)$,
or $0 < 1$.
Take a book shelf, and let
Between $= \{\langle b_1, b_2, b_3 \rangle | b_1$ is between b_2 och $b_3\}$.
We have that *Mellan*(x,y,z) is true iff x is between the books y and z.

Relations: Different Ways of Writing

- $\langle x_1, x_2, \ldots, x_n \rangle \in R$, e.g. $\langle Pelle, Stina \rangle \in Father$.
- $R(x_1, x_2, \ldots, x_n)$, e.g. $Far(Pelle, Stina)$ (predicate notation).
- xRy, e.g. $0 < 1$.

Cartesian Product (cross product)

Let A and B be sets.
The *Cartesian product* of A and B
is define as
$A \times B = \{\langle a,b \rangle | a \in A$ och $b \in B\}$,
i.e. the set of tuples with one element from each set.
This can be generalized to n sets,
for n-tuples.

Cartesian Product: Example

Let $A = \{1,2,3\}$ and $B = \{a,b\}$. We get
$A \times B = \{\langle 1,a \rangle, \langle 1,b \rangle, \langle 2,a \rangle, \langle 2,b \rangle, \langle 3,a \rangle, \langle 3,b \rangle\}$
and
$B \times B = \{\langle a,a \rangle, \langle a,b \rangle, \langle b,a \rangle, \langle b,b \rangle\}$.
Moreover: $A \times \emptyset = \emptyset \times A = \emptyset$.
Q: How many elements does $A \times B$ have?