Take-Home Exercises 3(4)
Discrete Mathematics (HKGAB4) 1998:
Functions and Boolean Algebra

February 1997

1. Let $A = \{hej, hopp, hu, la, beebop\}$, $B = \{0, 1\}$ and $C = \{11, 22, 33, 44\}$.

   (a) Define a function that is onto, but not one-to-one, with sets A, B or C.

   (b) Define a function that is one-to-one, but not onto, with sets A, B and C.

   (c) Define a one-to-one correspondence.

   (d) Give an identity function.

   (e) Enumerate all functions from C to$B$.

2. Let $A = \{l, m, n, o, p\}$ and $F = \{\langle l, m\rangle, \langle m, n\rangle, \langle n, l\rangle, \langle o, p\rangle, \langle p, o\rangle\}$ and $G = \{\langle l, p\rangle, \langle m, o\rangle, \langle n, n\rangle, \langle o, l\rangle, \langle p, m\rangle\}$.

   (a) What is $F \circ G \circ F(m)$?

   (b) What is $(F \circ G \circ F)^{-1}(m)$?

   (c) What is $F^{-1} \circ G^{-1} \circ F^{-1}(m)$?

   (d) Give the function defined by $F \circ G^{-1}$.

   Remember that, if $y = f(x)$ then $x = f^{-1}(y)$.

3. Calculate the following boolean expressions.

   (a) $(1 \vee 0)^* \vee (1 \wedge 0)$.

   (b) $((1 \wedge 1) \vee (0 \wedge 0))^* \vee (1^* \vee 0)$.

4. Give a solution to the boolean equation

$$1 = (a \vee b^*) \wedge (b \vee c^*) \wedge (a^* \vee b \vee c).$$

5. Show that

$$(a \vee b^*) \wedge (b \vee c^*) = ((a \wedge b) \vee (a \wedge c^*)) \vee (b^* \wedge c^*).$$

using the laws of boolean algebra.

6. Simplify

$$(a \vee (a^* \wedge b))^* \wedge (b \vee (c^* \wedge a))$$

as much as possible.