1 Thesis Overview

1.1 Topic

The topic of this thesis is the study of computational complexity of temporal and spatial reasoning[1]. Much of the work within the area of Artificial Intelligence (and elsewhere) has been recognised as including the notions of time and space, naturally, since these are fundamental aspects of the real world. It is clear that capabilities of modelling and computing with facts about time and space will be needed for any reasonably flexible system interacting with its environment. The areas of temporal and spatial reasoning in AI have developed from this recognition.

In this thesis, the focus is on *temporal reasoning*, and the results on spatial reasoning can largely be considered as by-products of the research on temporal reasoning. Vila [1994] has written an overview of the general uses of temporal reasoning in AI.

The thesis contributes to the problem of *computing* with temporal and spatial information, as opposed to the task of *modelling* temporal and spatial phenomena. The modelling problem consists of deciding what are the ideal primitive concepts and what is the appropriate formalism to use for a given application. Once it is decided how to model a class of phenomena, methods can be devised for computing facts given information expressed in the chosen formalism. Attacking the problem of computation certainly requires a choice of modelling formalism, and in order for the solutions to the computational problems to be *relevant*, here formalisms that are well established and uncontroversial have been used, at least for the temporal case, where the basis is Allen's interval algebra [Allen, 1983], both in its original form and extended with constructs for expressing metric temporal information. For the spatial case, however, the modelling language used is the spatial algebra, RCC-5 [Bennett, 1994], taken from the RCC approach to spatial reasoning [Randell and Cohn, 1989; Randell *et al.*, 1992b], about which there is still some controversy whether or when it correctly models spatial phenomena [Lemon, 1996].

[1] Instead of *temporal reasoning*, sometimes the expression *temporal constraint reasoning* is used to distinguish the field from that of *reasoning about action*, which deals with the problem of inferring what propositions hold at what time points when it is known that certain actions have been performed (see e.g. [Sandewall, 1994] and paper V of this thesis). I think that this use is unfortunate, since in line with this, any reasoning task having a temporal component could be called *temporal reasoning*.

There are fundamental computational problems associated with temporal and spatial formalisms, including the following:

- Given some information, checking whether some temporal or spatial relation is *entailed* from it, that is, being a logical consequence thereof.

- Checking whether some information is *satisfiable*, that is, checking whether there exists a model that satisfies the information.

Cleary, there is a connection between the two problems, and indeed, they are polynomially equivalent[2] for Allen's algebra [Golumbic and Shamir, 1993], and an analogous proof would easily establish the same result for the RCC-5 algebra. Another reasoning problem that is not as "logical" as the above problems is that of finding a polynomially-sized representation from which models can be extracted in polynomial time [Golumbic and Shamir, 1993].

This thesis is mainly concerned with the *satisfiability* problem. However, it is important to note that although the above two problems are equivalent for the full algebras, this does not necessarily hold for restricted subsets of the algebras (see e.g. the end of the discussion section in paper II).

It was early recognised that computing even with moderately expressive temporal formalisms such as Allen's interval algebra is difficult (the same holds for spatial formalisms); in particular, the polynomial-time algorithm originally proposed by Allen [1983] was designed to be incomplete in order to make the computation reasonably fast. However, this is a very dangerous way of attacking computational obstacles, unless the algorithm is accompanied with some kind of guarantee, like a class for which it is correct. This was not the case for Allen's algorithm.

The computational problems inspired several researchers to apply methods from the field of *complexity theory* to temporal reasoning (and recently, also to spatial reasoning), since the field addresses precisely the correctness of problems related to how difficult they are to compute. Results obtained from this approach include the following:

- It is a polynomial-time problem to check satisfiability (and entailment) for the *point algebra*, i.e. where point variables are related by the relations $<$, $>$, $\leq$, $\geq$, $=$ and $\neq$ [Ladkin and Maddux, 1994].

[2] That is, one problem can be solved in polynomial time using at most a polynomial number of calls to the other, and vice versa.

- Satisfiability for Allen's algebra is NP-complete [Vilain and Kautz, 1986], i.e. it is very unlikely that there exists a polynomial algorithm for the problem.

- Satisfiability for the *point-interval algebra* defined by Vilain [1982], where the only relations are those relating a point with an interval, is NP-complete [Meiri, 1991].

- There exists a *maximal tractable*[3] *subclass* of Allen's algebra, the *ORD-Horn subclass* [Nebel and Bürckert, 1995], i.e. satisfiability is tractable for that subclass, and is tractable for no proper superclass of it. Furthermore, it is the *unique* maximal subclass containing all *basic relations.*

- There exists one maximal tractable subclass in the spatial algebra RCC-5 [Renz, 1996] and one in the spatial algebra RCC-8 [Renz, 1996; Renz and Nebel, 1997].

An important assumption that underlies most of the above results is that variables (taking for example values of temporal intervals or spatial regions) are allowed to be *unrelated*, and this also holds for the results of this thesis. Relaxing that assumption yields more tractable classes [Golumbic and Shamir, 1993].

An area that is related to temporal reasoning is the area of *reasoning about action* (see e.g. [Sandewall, 1994] for an introduction), which is the topic of paper V. In addition to treating relations between temporal entities, *propositions* being true at time points and *actions* which have been executed are also modelled in this field, giving rise to the infamous *frame problem* (the problem of concisely expressing what does *not* change when it is known what *does* change) [McCarthy and Hayes, 1969]. However, emphasis of the research in this area has almost exclusively been on *modelling*, in contrast to the temporal reasoning field. One attempt to address the computational issues of the area is that of Schwalb *et al.* [1994], but it cannot be considered satisfactory, since their "solution" is to sacrifice the completeness requirement of making inferences, but providing no other criterion for correctness[4]. Furthermore, they do not correctly deal with the frame problem.

[3]The expression "tractable problem" is often used in place of "polynomial-time problem".

[4]Note that any algorithm for entailment that yields no conclusions at all is sound.

1.2 Contributions

This thesis contributes in line with the above results, first by finding new maximal tractable subclasses for temporal and spatial reasoning, thus providing knowledge about the borderline between tractable and intractable reasoning, and second by exhibiting the first tractable class for reasoning about action known in the literature (to my knowledge), and also providing some intractability results in this area. In addition, a new tractable class of *intutionistic logic* is presented; this result was obtained as a by-product of the results on spatial reasoning.

1.3 About the Thesis

The structure of the thesis is simple: this introductory chapter describes the context of and summarises the eight papers, which in turn constitute the rest of the thesis. The papers can be read in any order, but I recommend that papers II and III be read before paper IV, and paper VII before paper VIII.

All of the papers are intended to be reasonably self-contained, presupposing knowledge about discrete mathematics and some theory of computational complexity. (An good introduction to complexity theory is that of Papadimitriou [1994].) Due to this fact, some papers overlap slightly, but this should hopefully not be too disturbing.

Concerning the amount of work put into the papers by the author, it should be noted that the order of the authors reflects this (if indeed it can be quantified), in that the first author of a paper has done most of the research reported therein, and so on for the remaining list of authors. Of course, I am responsible for any errors in the papers.

1.4 Brief Summary of the Papers

Paper I

It is known in the literature that testing satisfiability of a set of point variables over real numbers related with the relations $<$, $>$, $\leq$, $\geq$, $=$ and $\neq$, that is, using the *point algebra* [Vilain, 1982], can be done in polynomial time [Ladkin and Maddux, 1994; van Beek and Cohen, 1990; Gerevini *et al.*, 1993], and that there are several tractable special cases of checking satisfiability using Allen's interval algebra [van Beek, 1989; van Beek, 1990; Golumbic and Shamir, 1993; Nebel and Bürckert, 1995] (the general case is NP-complete [Vilain and Kautz, 1986]). However, not much is known about the computational properties of the *point-interval algebra* [Meiri, 1991], where the only relations allowed are those between a *point* and an *interval*, except for the general satisfiability problem being NP-complete [Meiri, 1991].

Paper I classifies every possible subset of the point-interval algebra into having an NP-complete or polynomial satisfiability problem, respectively. It is proved that there are exactly five maximal tractable subclasses. This represents a step towards obtaining tractable subclasses of Meiri's *qualitative algebra*, which includes the expressive power of the point algebra, the point-interval algebra and the Allen interval algebra in one formalism. An interesting feature of the method of proof employed, is that it requires computer support for testing approximately $2.44 \cdot 10^5$ cases.

Paper II

In Paper I, we could directly attack the problem of finding the complete set of tractable subalgebras of the point-interval algebra. However, this is possible only due to size of the algebra: it contains only 32 relations, making the total number of potential subclasses $2^{32} \approx 4.3 \times 10^9$, which would be theoretically possible to enumerate on a computer. Allen's algebra, on the other hand, contains 8192 relations, yielding $2^{8192} \approx 10^{2466}$ subclasses, clearly being impossible to enumerate on today's computers. Therefore, a less systematic approach to finding tractable subclasses could be excused.

For the Allen algebra, only one maximal tractable class was previously known, the *ORD-Horn* subclass by Nebel and Bürckert [1995], containing 868 relations. This algebra extends all known tractable classes, except for one four-element set found by Golumbic and Shamir [1993]. Paper II expands our knowledge of tractable and maximal tractable subclasses, by defining twenty-one new tractable subclasses whose satisfiability problems are solvable in linear time, all of which are considerably larger than the ORD-Horn class, nine of them being maximal tractable, and the remaining ones nonmaximal (these are extended in paper III, but to the price of using an $O(n^2)$ time algorithm). Each algebra contains exactly three basic relations, and four of them contain the relation $(\prec\ \succ)$, "before or after", which is needed for expressing the important notion of *sequentiality*. Also the tractable class by Golumbic and Shamir is subsumed by these algebras.

Paper III

This paper combines two directions in temporal reasoning: the search for tractable subalgebras of Allen's algebra, and providing temporal formalisms with the possibility of expressing not only *qualitative* temporal information, but also *metric* information (such as statements like $\frac{3}{4}t - 0.6s \geq 3 \vee x \neq 17$). Eight new subclasses of Allen's algebra are defined, and they are all proved to be maximal tractable. It is proved that all nonmaximal algebras of paper II are included in these algebras. Six of the algebras contain exactly five basic relations each, and two of them contain three basic relations each. Furthermore, we use the new linear-programming approach to temporal reasoning by Jonsson and Bäckström [1996] in order to provide these algebras with metric temporal information: four of the algebras can have metric information on *interval starting points* and the remaining ones on *interval ending points*. It is also proved that we cannot have both, without losing tractability, except for the already known case of *Horn DLRs* [Jonsson and Bäckström, 1996], which provides the ORD-Horn algebra with metric temporal information.

Paper IV

This paper sets papers II and III in perspective, by proving a *partial* classification of tractability in Allen's algebra, in contrast to the *complete* classification of paper I. Given the results of papers II and III, an important question is whether the algebras found are just a few out of a multitude of algebras of comparable expressivity. Using intensive computer support (equivalent to 40 CPU weeks on a Sun SPARC 10 station), we show that the answer to this question has to be answered negatively, in the following precise way: any tractable subclass of Allen's algebra that is not a subset of either the ORD-Horn class or one of the algebras presented in papers II and III, cannot contain more than three basic relations, which have to be $\equiv$, b and the converse of b, where b is one of d, o, s or f. Furthermore, there is no remaining tractable subclass containing the relation $(\prec\ \succ)$, required for expression the property of *sequentiality*. Thus any remaining tractable subclass is necessarily of weak expressivity concerning basic relations and sequentiality. Note that this result also entails a complete classification of tractable algebras containing the basic relation $\prec$.

Paper V

As mentioned in the introduction, the motivation for investigating the problem of reasoning about time in separation has been that the concept of time is used in a large number of artificial intelligence applications. Paper V is the only temporal reasoning paper in this thesis which does not consist *solely* of a temporal component: here we can compute with *propositions* which are true at time points. The paper achieves two goals in this: first, a propositional temporal logic is developed, and second, this logic is extended to model the so-called *common-sense law of inertia*. Logics of this kind are mostly referred to as "logics of action of change", and in modelling power this logic has to be compared to such logics. However, the emphasis of this paper is *not* the traditional one in this field, that of modelling, but that of analysing computational complexity: no analysis of that kind has yet been done in the literature for such logics. It is proved that both in the basic temporal logic and in the extension for reasoning about action, the satisfiability problems are NP-complete, but perhaps more important, nontrivial tractable classes are found in both, capable of modelling phenomena like concurrent actions, continuous time and conditional results of actions. The expressivity of the temporal component is equal to that of Horn DLRs [Jonsson and Bäckström, 1996], which means that qualitative as well as metric time can be used.

One interesting (although not surprising) conclusion to draw from this paper is that sometimes a separation of temporal and non-temporal components is not the best way to attack the computational problems involving time: we obtain the tractable cases by *coding* the logics into Horn DLRs, and not by treating truth-value propositions and temporal propositions separately. It is not at all clear that a separated approach could handle this as efficiently.

Paper VI

In this paper, we apply methods from results in temporal reasoning on a related field, that of *spatial* reasoning. Also in this field, formalisms like Allen's algebra, the point-interval algebra and the point algebra have been invented to model qualitative spatial phenomena. In this paper, we take an existing such formalism from the RCC approach to spatial reasoning [Randell and Cohn, 1989; Randell *et al.*, 1992b], in particular the RCC-5 algebra [Bennett, 1994], and classify every subset of it (there are 32 relations in it, so the effort is comparable to that of paper I) into having a tractable or NP-complete satisfiability problem. Only one maximal tractable subclass of this algebra was known previously [Renz, 1996], which in turn built on earlier results by Nebel [1995]. In the paper we find the remaining three maximal tractable classes.

Paper VII

This paper relates the temporal and spatial reasoning areas: generalising the method of proof employed by Jonsson and Bäckström [1996] for proving that satisfiability for Horn-DLRs is tractable, we prove that for a corresponding Horn class of *reasoning about nonempty sets*, satisfiability checking can be done in polynomial time. Then we apply this result to finding a simpler algorithm for the previously known maximal tractable class of the RCC-5 algebra for spatial reasoning [Renz, 1996]. The existing polynomial algorithm involves complicated transformations using modal logic, whereas our algorithm is based on simple set-theoretical manipulations. In addition, by approaching the problem from the more abstract set-theoretical level, it is possible to express statements about spatial relationships that are not expressible in the RCC-5 algebra alone. Furthermore, some topological properties of a tractable subclass of the RCC-5 algebra are investigated, showing that satisfiability of that subclass is equivalent to satisfiability in three-dimensional space. This complements the results by Grigni *et al.* [1995], which show that satisfiability in two-dimensional space for a restricted version of the same subclass is NP-complete.

Paper VIII

This paper is somewhat more loosely related to the central topic of the thesis, that of reasoning about temporal and spatial relations, than the rest of the papers. It connects mainly to paper VII, by providing a new tractable class of tractable reasoning about sets. The difference from paper VII is that we investigate the case when empty sets are also allowed as set variables, and it turns out that the Horn class found in paper VII has an NP-complete satisfiability problem under this assumption. A new Horn-like class (being a subset of the class of paper VII) is further defined, which is proved to have a tractable satisfiability problem also when empty sets are allowed. The technique for proving tractability is very similar to that of paper VII, and was originally employed by Jonsson and Bäckström [1996].

Using a connection between topological spaces and *intuitionistic logic* established by Tarski [1956], we use the tractable class to obtain a subclass of intuitionistic logic for which inference is tractable. This result is somewhat related to spatial reasoning: in a more expressive approach to spatial reasoning than the RCC-5 algebra, the RCC-8 algebra [Bennett, 1994], a different tractable class of intuitionistic logic was used in order to find a tractable class [Nebel, 1995] (extended by Renz and Nebel [1997]), and it is possible that this class can be used in a similar way, to find new tractable subclasses of the RCC-8 algebra.

2 The Papers in Context

This section describes a broader perspective of the papers than the one given in Section 1.4.

2.1 Papers I–IV: Temporal Reasoning

Outside the area of Artificial intelligence, much work has been done concerning the *modelling* of temporal phenomena using various kinds of temporal logics (see e.g. Gabbay *et al.* [1994] for an overview). However, the *computational* issues dealt with are usually on the level of establishing *decidability* or *axiomatisability*, due to the expressiveness of the logics involved. Therefore, since the emphasis of this thesis is on finding polynomial-time algorithms for reasoning about time, these logics merit less attention here.

Within the area of Artificial Intelligence, early attempts at computing with temporal information are those of Bruce [1972] and Kahn and Gorry [1977]. The former seems to be the first one defining what constitute the basic relations of Allen's algebra. Both of these present an implemented system for reasoning about temporal information, but there is no evidence of the existence of correctness proofs for their algorithms (which are not even reported), and at least for the latter, the intended semantics for the formalism employed seems to be unclear.

Allen [1983] seems to be one of the first to take formalisation, computation and correctness seriously, by giving a precise definition of his algebra of interval relations, providing an algorithm for computing the closure of an interval network (that is, finding the strongest possible implied relation between two interval variables), and showing *incompleteness* of the proposed algorithm. In Allen's paper, the reason for emphasising the *algebraic operations* (making it an algebra) on interval relations (*composition*, *intersection* and *converse* of relations) was primarily for the operation of his particular closure algorithm. Interestingly, the property of being an algebra was later exploited for a completely different purpose, pioneered by Nebel and Bürckert [1995] and frequently used in papers I, II, III, IV and VI of this thesis: it turns out that the satisfiability problem for interval networks using only a restricted set S of interval relations is NP-complete if and only if the satisfiability problem using the set S' of interval relations is, where S' is the least set of relations containing S and being closed under the algebraic operations (this is a different kind of *closure*).

Soon, the computational properties of Allen's algebra were investigated by Vilain and Kautz [1986]: the satisfiability problem is NP-complete, and so is the problem of computing the closure of an interval network. This means that it is very unlikely that there exist polynomial-time (let alone efficient) algorithms for these problems. Essentially three lines of attack have been pursued in response to this fact:

- Not solving the intended problem, by designing the algorithms as being incomplete and hoping that they will work for most cases anyway. Allen's original algorithm [Allen, 1983] is an example of this approach.

- Trying to design fast algorithms for the full algebra, concentrating on the performance on certain randomly-generated instances and on benchmark problems which are naturally occurring in, for instance, molecular biology [Valdés-Pérez, 1987; Ladkin and Reinefeld, 1992; van Beek and Manchak, 1996; Mitra and Loganantharaj, 1996; Nebel, 1996]. Of course, these algorithms cannot solve the full problem efficiently, but it appears that improvement is made towards solving instances which are of somewhat practical size. However, the difficulty of empirically analysing algorithms (see e.g. [Johnson, 1996]) will probably require more investigation.

- Restricting the set of relations, proving that the problem for the restricted set of relations is a polynomial-time one. This is a more theoretical approach than the former, since no experimental data is required for establishing results. It can also be argued that it might be appropriate to investigate what efficient algorithms exist before a general-purpose superpolynomial algorithm is employed.

The papers of this thesis follow the third approach, which was indeed the first one to be pursued: Vilain and Kautz [1986] found that when the *point* *algebra* [Vilain, 1982] is used (that is, relating time point variables with the relations $<$, $\leq$, $=$, $\geq$, $>$ and $\neq$), satisfiability can be solved in polynomial time (the algorithm was improved by Gerevini *et al.* [1993] to a linear-time one), and expressing interval relations in terms of these yields a fragment of Allen's algebra for which computing satisfiability is polynomial; the *pointisable subalgebra*. However, it was not clear whether this subalgebra could be extended further, until Nebel and Bürckert [1995] defined a superset thereof, the *ORD-Horn* algebra, which furthermore was shown to be maximal tractable, that is, impossible to extend without losing tractability, and also that no other subalgebras than those contained in the ORD-Horn algebra can both be tractable and contain all the basic relations. Also Golumbic and Shamir [1993] defined several tractable subclasses of Allen's algebra (and provided NP-completeness results) using graph-theoretic methods. One of their tractable classes is not included in the ORD-Horn class (there are also other tractable classes, but these cannot express the relation "unrelated", as discussed above).

When time intervals or points are related using Allen's algebra or the point-algebra, only *qualitative* information can be expressed, that is, we cannot express, for instance, that a time point comes five seconds after some other time point. In order to remedy this, several different formalisms allowing such expressiveness have been developed and analysed [Dechter *et al.*, 1991; Meiri, 1991; Kautz and Ladkin, 1991; Koubarakis, 1992; Lassez and McAloon, 1992]. By the main result of Jonsson and Bäckström [1996], all tractable classes defined in these papers (except for two by Meiri, involving nonstandard entities such as discrete domains and multi-intervals) are unified into a single tractable formalism based on linear programming, that of *Horn-DLRs*, which also includes the ORD-Horn algebra (this result was independently found by Koubarakis [1996], but using a less transparent algorithm). The contributions of papers I–IV fall mainly within this category of results. Related to papers I and IV is also the complete classification by Pe'er and Shamir [1997] of tractable subclasses of a *subset* of Allen's algebra (using the same kind of *macro relations* as Golumbic and Shamir [1993]) with a nonstandard semantics where all intervals are required to have the same length.

We should also mention that much research is being done on finding special-purpose data structures for making reasoning practically efficient even though a polynomial-time algorithm is already known (note that the requirement of polynomial-time algorithms is not a sufficient condition for practical usefulness), mostly for the point-algebra [Ghallab and Alaoui, 1989;

Dean and McDermott, 1987; Dorn, 1992; Schrag *et al.*, 1992; Delgrande and Gupta, 1996].

2.2 Paper V: Reasoning about Action

Early in the history of Artificial Intelligence, McCarthy [1958] introduced logic as a means for representing knowledge. Soon, several problems were discovered with this approach; except for the inefficiencies involved with running a theorem prover for computing consequences of facts [Green, 1969], the representation issues themselves provided some problems, notably the infamous *frame problem*, identified by McCarthy and Hayes [1969]. The frame problem is how to represent what does *not* change in the world, when it is known that certain actions affecting the world have been performed, and maybe some additional observations are to be taken into account. Since the emphasis of this thesis is not on modelling but on computing, it serves no purpose to describe the development of these modelling formalisms, but it is interesting to note that after several papers proposing solutions to the frame problem, Hanks and McDermott [1986] pointed out that none of these was correct, using the *Yale shooting problem* (thereafter a prototypical example for reasoning about action) as a diagnostic example. As a reaction to this, some researchers fortunately started to be more systematic [Sandewall, 1994; Gelfond and Lifschitz, 1993; Kartha and Lifschitz, 1994; Lin and Shoham, 1991], by starting to define clearly what problem was to be solved, and under which assumptions the approach would hold as a correct solution to the frame problem. This so-called *systematic approach* (as opposed to the previously used *example-based approach*) is followed in paper V, of course, by stating mathematically what problem is to be solved and proving that this is indeed done.

In this area, the focus of interest has generally been that of *modelling* rather than computation, probably by the impact of the complications surrounding the frame problem, and by the fact that formalisms employed are almost always at least as expressive as first order logic, where computing entailment is known to be undecidable [Boolos and Jeffrey, 1989]. To our knowledge, no complexity analyses of such formalisms have yet appeared in the literature[5].

[5] Liberatore [1997] has performed a complexity analysis of the language $\mathcal{A}$ (which is less expressive than that used in the paper) for reasoning about action [Gelfond and Lifschitz, 1993]; the publication is however unrefereed.

2.3 Papers VI-VII: Spatial Reasoning

The problem of spatial reasoning is slightly more involved than that of temporal reasoning: whereas the choice of ontological entities in temporal reasoning is relatively obvious (points or intervals; sometimes perhaps multi-intervals or sets of time points [Meiri, 1991; Ladkin, 1986a; Ladkin, 1986b]), this is not the case for spatial reasoning. For instance, we have the following aspects to decide on: How many *dimensions* (two or three[6]) are to be used? Is the *form* or *orientation* of objects to be represented? How can (qualitative) spatial relations be naturally chosen? For a discussion of such issues, see Freksa and Röhrig [1993].

In this thesis we shall only consider spatial reasoning in its form closest related to temporal reasoning formalisms like Allen's algebra, and not the more general (and more imprecise) kind of reasoning as, for instance, *naïve physics* [Hayes, 1979; Hayes, 1985], which concerns representing knowledge about physical processes without using standard physics.

The so-called RCC approach by Randell, Cohn and Cui [Randell and Cohn, 1989; Randell *et al.*, 1992b; Bennett, 1994] is a first-order formalism for expressing knowledge about spatial relationships, with two limited sub-formalisms, named RCC-5 and RCC-8. These formalisms are defined as algebras like the Allen interval algebra, containing five and eight basic relations, respectively, but of different expressive power. Essentially, RCC-5 expresses relations between nonempty sets, whereas RCC-8 instead has sets in a topological space as its ontological primitive, thus distinguishing the border of a region from the rest of it. Until recently, the computational properties of this approach were largely unknown; existing methods were either not known to be complete [Egenhofer, 1991; Hernàndez, 1994] or very inefficient [Randell *et al.*, 1992b; Bennett, 1994]. Even computing a transitivity table posed a computational problem [Randell *et al.*, 1992a]! Nebel [1995] presented the first results on the computational complexity of these formalisms, showing that consistency-checking of the basic relations of RCC-8 can be done in polynomial time, and Grigni *et al.* [1995] classified the complexity of several cases when models are required to be two-dimensional. Renz [1996] extended Nebel's result, providing one maximal tractable subclass of RCC-8 (see also Renz and Nebel [1997]) and also one of RCC-5. Paper VI identifies the remaining tractable subclasses of RCC-5, and paper VII presents an algorithm for reasoning about arbitrary nonempty sets, yielding as a side-effect a new, simpler algorithm for the maximal tractable RCC-5 subclass of Renz [1996].

[6] One-dimensional spatial reasoning seems to collapse to temporal reasoning.

Furthermore, we cast some light on the semantical foundations of RCC-5 by showing that for a particular subclass, satisfiability is equivalent to satisfiability in $\mathbb{R}^3$ (see Lemon [1996] for a criticism of the semantics of various spatial logics).

3 Paper VIII: Intuitionistic Logic

Intuitionistic logic represents an attempt to formalise the ideas arising from Brouwer's criticism of the foundations of mathematics [Heyting, 1971; Dragalin, 1988]. Not much is known about the computational properties thereof, except for inference being PSPACE-complete in the propositional case [Statman, 1979] (there also exists an $O(nlogn)$-space procedure [Hudelmaier, 1993]), and the tractable class used by Nebel [1995] to find a tractable class of the RCC-8 algebra for spatial reasoning. In paper VIII, we improve on this by finding a new tractable subclass of intuitionistic logic, using a technique for reasoning about sets similar to that of paper VII. Furthermore, the results on reasoning about sets are interesting on their own, as are the corresponding results of paper VII.

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