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Algorithms and Complexity for Temporal and Spatial Formalisms

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Abstract

The problem of computing with temporal information was early recognised within the area of artificial intelligence, most notably the temporal *interval algebra* by Allen has become a widely used formalism for representing and computing with qualitative knowledge about relations between temporal intervals. However, the computational properties of the algebra and related formalisms are known to be bad: most problems (like *satisfiability*) are NP-hard. This thesis contributes to finding restrictions (as weak as possible) on Allen's algebra and related temporal formalisms (the *point-interval algebra* and extensions of Allen's algebra for metric time) for which the satisfiability problem can be computed in polynomial time.

Another research area utilising temporal information is that of *reasoning about action*, which treats the problem of drawing conclusions based on the knowledge about actions having been performed at certain time points (this amounts to solving the infamous *frame problem*). One paper of this thesis attacks the computational side of this problem; one that has not been treated in the literature (research in the area has focused on *modelling* only). A nontrivial class of problems for which satisfiability is a polynomial-time problem is isolated, being able to express phenomena such as concurrency, conditional actions and continuous time.

Similar to temporal reasoning is the field of *spatial reasoning*, where spatial instead of temporal objects are the field of study. In two papers, the formalism *RCC-5* for spatial reasoning, very similar to Allen's algebra, is analysed with respect to tractable subclasses, using techniques from temporal reasoning.

Finally, as a spin-off effect from the papers on spatial reasoning, a technique employed therein is used for finding a class of *intuitionistic logic* for which computing inference is tractable.

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List of Papers

The thesis includes the following eight papers.

I. Peter Jonsson, Thomas Drakengren and Christer Bäckström.
The Computational Complexity of Relating Points with Intervals on the Real Line.

The paper is a revised and extended version of another paper:
Peter Jonsson, Thomas Drakengren and Christer Bäckström.
A Complete Classification of Tractability in the Point-Interval Algebra. In J. Doyle and L. Aiello, editors, *Proceedings of the 5th International Conference on Principles of Knowledge Representation and Reasoning, KR '96*, pages 352–363, Cambridge, MA, USA October 1996. Morgan Kaufmann.

II. Thomas Drakengren and Peter Jonsson, 1997.
Twenty-one Large Tractable Subclasses of Allen's Algebra.
Artificial Intelligence, 93(1–2):297–319.

The paper is an extended version of another paper:
Thomas Drakengren and Peter Jonsson.
Maximal Tractable Subclasses of Allen's Interval Algebra:
Preliminary Report. In *Proceedings of the 13th (US) National Conference on Artificial Intelligence, AAAI '96*, Portland, OR, USA, August 1996.

III. Thomas Drakengren and Peter Jonsson, 1997.
Eight Maximal Tractable Subclasses of Allen's Algebra with Metric Time.
Journal of Artificial Intelligence Research, 7:25–45.

IV. Thomas Drakengren and Peter Jonsson, 1997.
Towards a Complete Classification of Tractability in Allen's Algebra.
In *Proceedings of the Fifteenth International Joint Conference on Artificial Intelligence, IJCAI '97*, Nagoya, Japan, August 1997.

V. Thomas Drakengren and Marcus Bjäreland.
Reasoning about Action in Polynomial Time.

The paper is a version of the following paper, but with full proofs:
Thomas Drakengren and Marcus Bjäreland, 1997.

Reasoning about Action in Polynomial Time.
In *Proceedings of the Fifteenth International Joint Conference on Artificial Intelligence, IJCAI '97*, Nagoya, Japan, August 1997.

VI. Peter Jonsson and Thomas Drakengren, 1997.
RCC-5 and its Tractable Subclasses.
Journal of Artificial Intelligence Research, 6:211–221.

VII. Peter Jonsson and Thomas Drakengren.
Qualitative Reasoning about Sets Applied to Spatial Reasoning.

VIII. Thomas Drakengren and Peter Jonsson.
Reasoning about Set Constraints Applied to Tractable Inference in Intuitionistic Logic.