Electronic News Journal on Reasoning about Actions and Change 1
Vol. 4: *00-00*, 30.6.2000
http://www.etaij.org/rac/notes/2000/01/

On the Adequacy of Modal Logic, II: A Response to McCarthy

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Abstract: *I respond to McCarthy's response to my comments on his article. In particular, I show how some of his challenges can be met.*

McCarthy [**?**] has argued that modal logic is inadequate for many purposes. I wrote a commentary on his paper [**?**)], to which McCarthy responded **?**). This is a response to his response. While I have tried to make it somewhat self-contained, it is probably best read after reading the original sequence of three (fortunately, short) articles.

1. McCarthy is certainly right when he observes (point 4 in **?**)) that it is not necessary to require that common knowledge be the transitive closure "everyone knows" in certain applications. It may suffice to take common knowledge to have certain properties and to include the transitive closure. If this is all that is required, then common knowledge can be captured in first-order logic. However, there are many applications (see, for example, **?**)) where we definitely want common knowledge to be the transitive closure.
2. Of course, common knowledge is expressible in ZFC (and much more besides). However, this comes at a cost, since ZFC is highly undecidable. This point brings out a much deeper philosophical difference between McCarthy and me. McCarthy wants robots to reason with a formal logic. I'm not sure that's either necessary or desirable. Humans certainly don't do much reasoning in a formal logic to help them in deciding how to act; it's not clear to me that robots need to either. Robots should use whatever formalisms help them make decisions. I do think logic is a useful tool for clarifying subtleties and for systems designers to reason about systems (e.g., for robot designers to reason about robots), but it's not clear to me that it's as useful for the robots themselves.
3. In his point 6, McCarthy **?** also argues that, when translating "Pat knows Mike's telephone number", the formula $knows(Pat, Telephone(Mike))$ corresponds more closely to natural language than does my $\exists x K_{Pat}(telephone\text{-}\#(Mike) = x$. It's hard to disagree with that. On the other hand, we are quite used to programming languages adding syntactic sugar to make it easy to express succinctly notions that look more complicated when expressed in unsugared syntax. I think this is an instance of the

same phenomenon. The apparent complexity of modal logic is due to the fact that it can express clearly subtle distinctions that are often lost in natural language—for example, the difference between Pat knowing that Mike has a telephone number and Pat knowing what that telephone number is. On a related matter, McCarthy translated my sentence into English as "There is a number concept x such that Pat knows that Mike's telephone number is x." I'm not sure why McCarthy talks about "number concept" rather than just "number". Concepts also play a significant role in **?**), but it's not clear to me why they are necesary. Certainly in my translations I do not distinguish between numbers and number concepts.

Finally, just a minor comment regarding non-rigid designators. McCarthy claims that to enusre that my sentence is not a tautology, I need *non-rigid designators*, that is, constants that take on different values at different possible worlds. Although I have no problem with non-rigid constants, in fact I don't need them in this case. All that I need is that the function *telephone-#* be non-rigid. It's fine if Pat and Mike are rigid. Non-rigid functions and predicates are quite standard in modal logic.

4. Later in point 6 McCarthy considers sentences such as "Nobody knows the trouble I've seen", and observes that this suggests that quantifying over knowers may need to be added to modal logic. This, in fact, has been done (see, for example, **?**)) Although I have not checked details, it does not seem to add any complexity to first-order modal logic without quantification over knowers.

5. Still later in point 6 McCarthy gives two sentences that he suggests will be hard to express using first-order modal logic. In fact, neither one is hard to express. The first is "Kepler knew that the number of planets was composite" **?**). Ignoring the past tense for a moment, this is simply

$$\exists x (K_{Kepler}(\mathit{Number\text{-}Planets} = x) \land \mathit{Composite}(x)). \tag{1}$$

Here I am assuming that K is interpreted as belief, where it is possible the belief something that happens to be false. There are standard modal logics (e.g., KD45 **?**)) for belief that have this property. The past tense also presents no difficulties. There are standard temporal modal logics with past-time operators **?**) that can express "at some point in the past".

Formula (1) implies that Kepler actually knew what the number of planets was. This was suggested by McCarthy's statement that "Kepler presumably thought the number of planets was 7" **?**). However, McCarthy has since indicated [private communication, 2000] that he did not want to presume that Kepler knew the number of planets, only that he knew that it was composite. In this case, the appropriate translation becomes

$$K_{Kepler} \exists x (\mathit{Number\text{-}Planets} = x) \land \mathit{Composite}(x)).$$

Moving the $\exists x$ inside the scope of the knowledge operator signals that Kepler did not know what the number of planets, but did know that, whatever it was, it was composite.

The second example is "I thought that your yacht was longer than it is". Again, ignoring the tense and treating K as belief, this becomes

$$\exists y \exists x((\mathit{Length\text{-}yacht} = y) \wedge (K_I(\mathit{Length\text{-}yacht} = x)) \wedge x > y),$$

if there is a specific length that the "I" in question actually believed to be the length of the yacht. (Note that there are no difficulties in having indexical names such as I as subscripts; this issue is dealt with in **??**).) On the other hand, if "I" didn't have a specific length that he believed to be the actual length of the yacht, a more appropriate translation would be

$$\exists y((\mathit{Length\text{-}yacht} = y) \wedge (K_I \forall x((\mathit{Length\text{-}yacht} = x)) \rightarrow x > y)).$$

The latter formula says that, whatever I believe the length of the yacht to be, it is greater than its actual length. Part of the power of modal logic comes from its ability to express such subtle distinctions.