ELECTRONIC NEWSLETTER ON
REASONING ABOUT ACTIONS AND CHANGE

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Today

Discussion about the ontology of time: Pat Hayes today has a detailed answer to the points that Jixin Ma made recently. He also objects to my headline summary of his previous statement, so I'd better not try to summarize this one...

Debates

Ontologies for time

From: Pat Hayes

Sorry, I wasnt sufficiently clear, and my carelessness in using intuitive phrasing led to misunderstanding.

First, in my view there is no single answer to many of the issues that Jixin raises. One can make various choices, each internally consistent but not consistent with the others. (That is why I called the cited paper a 'catalog' of time theories, rather than a single theory of time.) This freedom means that one must be clear which alternative one is using, as confusion follows when one tries to put together bits and pieces of incompatible views. (For example, the 1990 AIJ critique of Allen's account by Galton (wrongly) assumes that Allen's intervals are sets of points on the real line.) Having said this, however, there does seem to be a simple, basic, account which can be extended in various ways to produce all the other alternatives, and this core theory is the one I was referring to.

Second, I dont agree with Erik's introduction of my note as putting intervals before points. As Allen and I showed some time ago, the choice is arbitrary, since points can be transparently defined in an interval theory and vice versa, so the choice of either one as somehow more basic is, er, pointless; and one gets a more useful account simply by allowing them both as primitive. (Actually, if anything, the simple theory I outlined seems more to rely on points as basic, since an interval there is completely defined by its two endpoints and has no other structure, and all the temporal relations between intervals can be inferred from the total ordering of points.)

Jixin asks:

> *However, it is not clear what's the exact meaning of "taking points and intervals as basic". Are they both taken as primitive temporal objects, or, as Allen suggests, points are thought as places where intervals meet each other?*

Both. These arent incompatible alternatives. The basic idea in the 'simple' theory is essentially Allen's, that points are meeting-places. Still, there's no harm in being able to mention these meeting-places as real objects, and doing so makes it easier to say quite a lot of things, such as 'when' some change happens. Clock times seem to be associated more naturally with points than intervals, for example.

> *2. Pat argues that "the question is whether it makes sense to say that something is true at points". However, his argument is quite confused: in the first place, he claims "truths hold not at points but during intervals" (as for the case when one insists that interval endpoints are "in" the interval). Later, he states "Some truths may be instantaneous, ie true only* **at points***; others make sense only when asserted to hold during noninstantaneous intervals".*

(In the above I was careless at the place marked by boldface, sorry. I should have said 'pointlike interval'. It gets hard to speak about this stuff clearly in English, since I need to distinguish our intuitive notion of 'point' from the way that a particular theory encodes this intuition, and different theories do it differently. I will use scare-quotes to refer to the intuitive concept.)

> *So, what's the answer to the question "whether it makes sense to say that something is true at points"?*

There is no (single) answer: one can craft the theory to suit various different intuitions on matters like these. The way I prefer, myself, is to say that propositions hold only during intervals, so that it is simply ill-formed to assert a proposition of a single point; but to allow the possibility of pointlike intervals, of the form $\langle t,t \rangle$, to be the temporal durations of propositions which are (intuitively) thought of as happening at a single 'point'. Or, put another way, some intervals may consist of just a single point, and some points may completely fill an interval. These pointlike intervals are the way that (this version of) the theory encodes the times when instantaneous truths hold.

This doesnt require us to say that *every* point fills an interval, notice: since 'interval' is a basic predicate, it is perfectly consistent to say `(not (interval <t,t>))`; this would entail, for example, that nothing changed at that particular time. But it allows us to consider the proposition that a tossed ball's vertical velocity is zero, and assert that it is true at a single 'point', ie formally, that its interval of truth was pointlike. And since it is easy to characterise pointlike in the theory:

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((pointlike i) iff ((begin i) = (end i)))
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one can, for example, say something like

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((illuminated i) or (dark i)))  implies (not (pointlike i))
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so that the light is neither on nor off AT the switching point. In this theory, every proposition has a 'reference interval' during which it is true, and a proposition might not be true of subintervals of its reference interval. (Though some propositions might be. This kind of distinction has often been made in the linguistic literature. Note however that this intuition is basically incompatible with the idea that an interval is identical to the set of the points it contains.)

3. Pat's claims that one may identify interval $\langle t,t\rangle$ with point t, or distinguish $\langle t,t\rangle$ from t, or even forbid instantaneous intervals completely. However, what's the choice? Do we need points (instantaneous intervals) or not?

We certainly need something corresponding to 'points', I agree. I meant only that the formal theory can be crafted in the way Ive outlined above, or alternatively by identifying the pointlike intervals with their endpoints, and allowing a proposition to hold at a single point. This is in many ways more intuitively transparent but it is formally a bit more awkward, since pointlike isnt definable any more, and one has to put in special axioms forbidding points to meet each other. The 'reference interval' of a proposition could now be a single point in the theory. This is essentially the theory that Allen and I described in our 1985 IJCAI paper, though it takes a little work to see it.

Let's consider the case that we do (in fact, for general treatments, we **do** *need them). For this case, Pat's states that if $meets(\langle a,b\rangle,\langle b,b\rangle) \wedge meets(\langle b,b\rangle,\langle b,c\rangle)$ then $meets(\langle a,b\rangle,\langle b,c\rangle)$. (In fact, it seems in Pat's formulation, we always have $meets(\langle a,b\rangle,\langle b,c\rangle) \wedge meets(\langle a,b\rangle,\langle b,b\rangle)$ etc., since the intervals are uniquely defined by their endpoints).*

Yes, exactly. Interval relations are comletely determined by endpoint orderings,and Allen's huge transitivity table can be painstakingly derived from the assumption of total ordering. That's all it amounts to, in fact.

Below are some problems with this formulation:

I. As noted by Pat himself, "an instantaneous interval meets itself", though the "basic" points are totally ordered. How to characterise the relation between them? Pat's gives a suggestion: to characterise them without mentioning points explicitly. Then, what's the relationship between points and intervals?

The relations are quite simple and transparent: intervals lie between endpoints, and points have intervals extending between them. Self-meeting is the interval-interval relation corresponding to equality in the point ordering. Again, if one has an intuitive objection to self-meeting intervals, then one can take the second alternative mentioned earlier. (All these alternatives are got by extending the basic theory.)

II. How to define other relationships between intervals like those introduced by Allen? For instance, it is intuitive to say that $meets(\langle a,b\rangle,\langle b,c\rangle) \wedge meets(\langle b,c\rangle,\langle c,d\rangle) \rightarrow before(\langle a,b\rangle,\langle c,d\rangle)$. However, in this case, one would have both $meets(\langle a,b\rangle,\langle b,b\rangle)$ and $before(\langle a,b\rangle,\langle b,b\rangle)$, and hence "meets" and "before" would not be exclusive to each other.

True, and indeed the Allen relations only have their usual transitivity properties when applied to intervals which are nonpointlike and forward-oriented. Of course both these are properties expressible in the theory, so that the Allen transitivity relationships can be stated there, suitably qualified. (When the alternative extension axioms are added, the qualifications become tautologous.)

BTW, the claim that "meets" and "before" being exclusive is "intuitive" depends on how one's intuition is formed. Part of what I learned by having to construct alternative formalisations is that intuition is very malleable. Having gotten used to pointlike intervals, I dont find this exclusivity condition at all intuitive.

> *III. By saying meets*$(\langle a,b\rangle,\langle b,b\rangle)$, *meets*$(\langle b,b\rangle,\langle b,c\rangle)$, *and meets*$(\langle a,b\rangle,\langle b,c\rangle)$, *one can only express the first case, that is case a), but not the other two cases, that is case b) and case c), as we demonstrated in our former arguments.*

But these cases only make sense if one thinks of interval and points in the usual mathematical way, which is exactly what Im suggesting we don't need to do. We can get almost everything we need just from the ordering structure: we don't need to get all tied up in distinguishing cases which can only be formally stated by using all the machinery of real analysis.

> *4. Pat argues that "I'm largely in agreement with Jixin about points and intervals, although I dont think its got anything to do with knowledge". But it does. In fact, as pointed out by Pat himself, "if you want to know whether the light was on or not, you have to say which interval you are talking about". In other words, if the (additional) knowledge of "which interval you are talking about" is given (e.g., in terms of which interval is open/closed at the switching point, or in terms of the corresponding meets relations - "knowledge"?), we can say whether the light was on or not.*

Again I was careless in using the word "knowledge", sorry. I should have said: in order to answer the question whether the light is on or off, one has to specify the interval with respect to which this question is posed. On this view, the truth or otherwise of a proposition is only meaningful with respect to certain intervals. I dont mean that the facts are determined by knowing more about the details of the interval, but that the question is a different question when asked about one interval than when asked about another, and for some intervals in may be simply meaningless. Is the light on or off at (exactly) 3.00 pm? The only way to answer this is to find a suitable non-pointlike interval of light or darkness completely surrounding 3.00 pm, because 'being on' is the kind of proposition that requires a nonpointlike reference interval.

This has nothing to do with whether an interval is open or closed: in fact, there is no such distinction in this theory. It only arises in a much more complicated extension which includes set theory and an extensionality axiom for intervals.

> *5. Pat also argues that his formulation is simpler (and elegant). In what aspects, compared with which formulation?*

Perhaps I should have said, of all the various formalized temporal theories I have ever examined in detail, which amounts now to maybe 25 or so, this seems to distill out the essence. The others can all be described as extensions of this one (some a little artifically, but mos tof them quite naturally.) The conventional picture of intervals as sets of points carries with it a lot of excess conceptual baggage, and removing this gives a theory which is simple and intuitive (once you get used to it :-), and is a sound formal 'core' which can be extended to give many other theories.

> *It seems that it still needs a lot of axioms to characterise the formal structure, especially when issues such as density, linearity, boundness, etc, are to be addressed.*

Yes; any theory needs to be extended, of course, to deal with density, boundedness, etc., but again a merit of this very simple framework is that it can be transparently extended in these different ways more or less orthogonally to each other. One can establish unboundedness with one very obvious axiom (there's always a future and past to any timepoint) and density is also very easy. Lack of density, ie discrete time, is harder; in fact, theres a sense in which no first-order theory can describe this, since it assumes the integers. But again, this is a matter of adding one (rather complicated) induction axiom, in a way that is mathematically very ordinary. Or, alternatively, one can just assume that the integers are defined elsewhere, and declare that every point has an integer 'date', which gives the theory implicitly used by most temporal databases. It can even be extended into the standard real line, if you wish, by distinguishing 'open' and 'closed' intervals as triplets of the form $\langle point, interval, point \rangle$.

The theory is basically linear in its nature, since it assumes timepoints are totally ordered. One can easily weaken it to allow partial orders, but then the extensions involving density, etc.,, get rather tricker. I think the universe is deterministic in any case, so linearity doesnt bother me :-)

Pat Hayes

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