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Protocol of the discussion during July, 1998 on

Ontologies of Time

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From: Pat Hayes on 7.7.1998

Erik wants to know why we should bother to consider punctuated times. Well, I guess my first answer would be: ask the people who use the Allen theory. I think it has something to do with natural language understanding. But more concretely, I disagree with Erik's way of thinking. I start with axioms and ask what models they have. Erik starts with models (the real line, for example) and assumes that we somehow have access to the 'standard' ones. But of course we don't: there is no complete computationally enumerable theory of the standard real line. All we have are axiomatic theories.

(In any case, why is the real line 'standard' for time? If anything, quantum physics suggests that time may be more like a kind of everywhere-punctutuated continuum (imagine two copies $R1$, $R2$ of R and order them by: x before y iff $x = y \wedge x \in R1 \wedge y \in R2 \vee x < y$. I find that once one starts playing around with 'nonstandard' models of axioms, the idea that R is 'standard' seems more and more implausible. There are a *lot* of different ways that time could be.)

> The question is whether this standard mathematical view of the line is the most suitable for capturing linguistic intuition or for action reasoning. For example, we want to be able to assert that a light is off before time t and on after time t without having to commit ourselves to its being either on or off **at** that time, but also without sacrificing the assumption that it is always either on or off.

> *The most natural way of dealing with such a situation, it seems to me, is to admit that one's axioms allow two kinds of models: those where the light is on at time t, and those where it is off. The assumption that light is either on or off holds in each of the models, but the axioms don't imply one or the other.*

That is one way to think about it. In fact, given any 'punctuated' model, one can always fill it out by putting points at the ends of the intervals and choosing arbitrarily whether or not the light is on (or whatever) at those points. So there are models on the reals even of a theory with punctuated models. But now I return to my earlier point. If we start with the theory and ask, what are its models?, we do get these models on the reals, but we also get rather simpler models which fit the axioms more naturally (and can even be considered to be minimal in a kind of complexity ordering which minimises the size of the domain by cutting out 'surplus' points.) So if you are using this theory and feel uncomfortable with 'nonstandard' models, you can, if you wish, pretend that you are always talking about the real line, though sometimes vaguely (for example, you might not know if an interval is half-open or truly open.) But adopting that stance doesn't magically make the 'nonstandard' models vanish: on the contrary, it is a *theorem* that one can never make the nonstandard models go away. Even set theory has nonstandard models. For myself, I like to know what the nonstandard models are: I find it gives me greater insight into what my axioms really say, as opposed to what I wish they would say.

Erik asks for examples of what can be done with punctuated-time models. I don't know what he means. It is theories that do things, not their models.

Pat Hayes

[7.7.1998] **PS.** Here are the axioms of my proposal for a core theory. (The theory can be reformulated without explicitly mentioning intervals, but more awkwardly; and also without explicitly mentioning points, although even more awkwardly and using a little elementary set theory. Its much easier to allow both: but notice that an interval is defined completely by its endpoints.)

1. Points are totally ordered by Before.
2. Every proposition is true over a set of reference intervals.
3. There is a function from pairs of points p, q to the interval $[p, q]$.
4. Intervals Meet only if they share an endpoint, and this point is the unique point When they meet.
5. If p is Before q is Before r, then q is In $[p, r]$
6. $[p, q]$ is a Subinterval of $[r, s]$ if
$(p = r \wedge q \ \mathsf{In}\ [r, s])\vee$
$(p\ \mathsf{In}\ [r, s] \wedge q = s)\vee$
$(p\ \mathsf{In}\ [r, s] \wedge q\ \mathsf{In}\ [r, s])$
or

6a p $\mathsf{BeforeEq}$ q iff (p Before $q \vee p = q$)

6b $[p, q]$ is a Subinterval of $[r, s]$ if (r Before $p \wedge q$ $\mathsf{BeforeEq}$ $s)\vee(r$ $\mathsf{BeforEq}$ $p \wedge$ q Before s)

7. Points have unique Clocktimes.
8. The Clocktime at point q is the Clocktime at p plus the Duration of the interval $[p, q]$.

This theory allows pointlike intervals $[p, p]$ of zero duration, but it doesnt insist that anything be true at such an interval. It also allows 'backward' intervals with negative duration. It allows time to consist of atomic intervals with no subintervals, and it also allows time to be continuous, or any mixture thereof. It allows instantaneous truths and it also allows light-on/off cases with meeting reference intervals. It allows a proposition to be true of an interval but not of its subintervals. All of these variations can be locked in or out by asserting extra conditions using the vocabulary already given.

Now, this theory has models on the reals, naturally. It also has models on the rationals, the integers, and many other more exotic structures, including various 'punctuated' models. (Same theory, different models.) If you relax the total ordering to a partial one it even has models consistent with special relativity. It can be extended to Allenian thinking (by changing

4 and 6 to iff), or it can be extended to Sergioian continuity (the intervals are closed: open intervals are the sets of points In an interval), or to analyse temporal databases (every point has a unique 'next' point with no points between.) Take your pick; but I think you will always find a least this minimal amount of structure in any theory. (Suggested counterexamples welcome!)

From: Jixin Ma on 7.7.1998

To Erik,

In [ENRAC 98047, 24.5.1998], you put the following questions:

> *What are examples of concrete problems that cannot (conveniently) be represented using a "standard" view of the time axis as e.g. timepoints = the real numbers; time-intervals = sets of points each defined by a lower bound a and an upper bound b.*
>
> *Given that both timepoints and time-intervals are needed to have sufficient expressivity, what is the best way of defining those concepts and their relation?*

The Dividing Instant Problem is a typical problem with the approach of simply constructing time intervals from points (such as reals, rationals or integers), e.g., by means of defining an intervals as a set of points (usually written as a pair of points, denoting the lower bound and the upper bound, i.e., the ending-points) respectively. The DIP is in fact equivalent to the question of whether time intervals include their ending-points or not. If all intervals include their ending-points, then adjacent intervals would have ending-points in common. Hence, if two adjacent intervals correspond to states of truth and falsity of a given fluent, there will be a point at which the fluent is both true and false. Similarly, if all intervals exclude their ending-points, there will be points at which the truth or falsity of some fluents is undefined. Another alternative is to take point-based intervals as semi-open (e.g., all intervals include their left ending-points, and exclude their right ones) so that they may sit conveniently next to one another. However, on the one hand, since this approach insists that every interval contains only a single ending-point, the choice of which ending-point of intervals should be included/excluded seems arbitrary, and hence unjustifiable and artificial. On the other hand, although the approach may offer a solution to some practical questions, there are some other critical questions which remain problematical (examples can be found in AI(42), or Computer Journal 37(2 & 10)). The fundamental reason is that in a system where time intervals are all taken as semi-open, it will be difficult to represent time points in an appropriate structure so that they can stand between intervals conveniently.

In your message to Pat appeared in the same issue, you wrote:

> The question is whether this standard mathematical view of the line is the most suitable for capturing linguistic intuition or for action reasoning. For example, we want to be able to assert that a light is off before time t and on after time t without having to commit ourselves to its being either on or off at that time, but also without sacrificing the assumption that it is always either on or off.

> *The most natural way of dealing with such a situation, it seems to me, is to admit that one's axioms allow two kinds of models: those where the light is on at time t, and those where it is off. The assumption that light is either on or off holds in each of the models, but the axioms don't imply one or the other.*

Exactly! In fact, as shown in my message (ENRAC 13.3.1998), the approach of treating both intervals and points as primitive on the same footing allows modelling the following *three* cases:

1. the light changes from state "Off" to state "On", and the light is "On" at the switching point t;
2. the light changes from state "Off" to state "On", and the light is "Off" at the switching point t;
3. the light changes from state "Off" to state "On" (but whithout the modelling of the state of the light at the switching point t).

It is important to note that, the treatment here for case 3) is different from the approach which characterises intervals as sets of points. Here, it is not forced to say that there is a point between the Light-Off interval and Light-On interval.

However, it seems that by some careful and proper treatments, we may also reach the same results by defining the concepts of intervals based on points. The key point here is, in addition to the concept of lower and upper bounds for point-based intervals, the concept of left type and right type for intervals needs to be addressed as well. What follows is the skeleton of the structure:

1. P is a partially-ordered set of points;
2. *Type* is a two-member set $\{open, closed\}$;
3. An interval i is defined as a quaternion $\langle p1, p2, l, r\rangle$ such that:
   - $p1 \leq p2$
   - l and r belong to *Type*
   - if $p1 = p2$ then $l = r = closed$

Therefore, if we take point, p, as identical to closed point-like interval (p,p,closed,closed), the set of point can be taken as a subset of the set of intervals.

The relation between intervals, *Meets*, is defined as:
$Meets(\langle p1', p2', l', r'\rangle, \langle p1'', p2'', l'', r''\rangle)$ iff $p2' = p1'' \wedge \neg(r' = l'')$

It is easy to see that such a Meets relation defined here represents the relationship that an interval is immediately before another interval (no matter wether the set of points is further characterised as dense or discrete), without any other intervals standing between them. All other temporal relations, such as *Before*, *Starts*, *During*, etc., can be derively defined in terms of the *Meets* relation.

Additional issues such as dense/discrete, linear/nonlinear structure, and so on, may be characterised by means of extra axioms.

Also, some extra constraints about the internal structure of intervals may be imposed.

It has been checked that such a point-based time model has the expressive power of that based on both intervals and points as primitive. For instance, it can also express all the above three cases successfully. In fact, it seems that the two are equivelent!

Jixin

From: Erik Sandewall on 11.7.1998

Pat Hayes wrote:

> *Erik wants to know why we should bother to consider punctuated times... But more concretely, I disagree with Erik's way of thinking. I start with axioms and ask what models they have. Erik starts with models (the real line, for example) and assumes that we somehow have access to the 'standard' ones. But of course we don't: there is no complete computationally enumerable theory of the standard real line. All we have are axiomatic theories.*
>
> *...*
>
> *Erik asks for examples of what can be done with punctuated-time models. I don't know what he means. It is theories that do things, not their models.*

I have no problems with theories that allow both punctuated and non-punctuated real time models. However, since punctuated time is proposed as a solution to the "Dividing Instant Problem", and since accordingly some or all non-punctuated models suffer from such a DIP, I'd like to understand what concrete difficulties are obtained in a computational system that admits non-punctuated time, and even relies on it.

This question of mine arises in a cognitive robotics perspective, which may in fact be distinct from the perspectives of commonsense reasoning or of natural language understanding. In cognitive robotics, it's of paramount importance to be able to deal with hybrid scenario descriptions, combining qualitative and quantitative estimates of duration, distances, consumption of resources such as fuel and battery power, and so on.

Consequently, we are led to use state-of-the-art algorithms for dealing with these kinds of constraints, including, in particular, temporal constraint solving in the tradition of Dechter-Meiri-Pearl and the more recent developments that are used on linear programming. All of these methods make use of standard arithmetic operations such as addition and multiplication, and don't make any exception for them not being everywhere defined. Therefore, it would seem adventurous to use them in a context where the time line is assumed to be punctuated, so that for some numbers in the (e.g. real) domain there does not exist any corresponding point in time.

Now, back to Pat's comments.

> *Erik asks for examples of what can be done with punctuated-time models. I don't know what he means. It is theories that do things, not their models.*

isn't it more exact to say that software does things? From the point of view of the algorithms mentioned above, it just doesn't matter whether and how the (e.g. real) numbers are axiomatized. However, it does matter that addition is always defined, which is why it is convenient to assume the use of "standard" models of time.

If software is identified only with theorem provers, then of course theories are of primary interest. But would you seriously propose using a theorem prover plus an axiomatization of the time domain as a computational instrument?

Some axioms and some deduction is needed, for sure, but suppose I develop and use a hybrid system that combines algorithms with Pat's core theory. Is there something that can then go wrong? Can I obtain some unwarranted conclusions? Will I miss some warranted conclusions? Or, will I get into inordinate difficulties when writing my remaining scenario description axioms? (At least *one concrete example* of such problems would be useful).

On the other hand, if no such problems have been reported and none can be found, they why is all this fuss about the so called DIP? Why can't we just take Pat's core theory, observe that it allows both the integers and the reals as timepoint domains, and that in addition it allows a whole lot of other domains that noone needs to care about from a practical point of view?

Or, to reverse the question - *concretely why* do people care about those other models? On this topic, Pat has referred to others:

> *Erik wants to know why we should bother to consider punctuated times. Well, I guess my first answer would be: ask the people who use the Allen theory. I think it has something to do with natural language understanding.*

and in another context to the temporal database community. Fine, but what is it that they can do using non-"standard" time that can't be done using "standard" time?

Jixin's answer to that question was:

> *The Dividing Instant Problem is a typical problem with the approach of simply constructing time intervals from points (such as reals, rationals or integers), e.g., by means of defining an intervals as a set of points...*
>
> *The fundamental reason is that in a system where time intervals are all taken as semi-open, it will be difficult to represent time points in an appropriate structure so that they can stand between intervals conveniently.*

I had actually asked for something more concrete: a scenario that can't be expressed, or a scenario query that requires inordinately long completion time, for example. Is it possible to be more specific about in what sense it "will be more difficult" to represent time points that stand between intervals?

As an aside: Jixin also wrote:

> *However, it seems that by some careful and proper treatments, we may also reach the same results by defining the concepts of intervals based on points. The key point here is, in addition to the concept of lower and upper bounds for point-based intervals, the concept of left type and right type for intervals needs to be addressed as well. What follows is the skeleton of the structure:*

1. *P is a partially-ordered set of points;*
2. *Type is a two-member set {open, closed};*
3. *An interval i is defined as a quaternion seq(p1, p2, l, r) such that:*
   - $p1 \leq p2$
   - *l and r belong to Type*
   - *if $p1 = p2$ then $l = r = closed$*

Yes, but how does this differ from the standard notions of open and closed intervals? It would appear that

```
(a,b,open,open) = (a,b)
(a,b,open,close) = (a,b]
```

and so on. Are there some models where your fourtuple intervals can not be reduced to standard definitions, such as

```
(a,b) = {x | a < x < b}
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and so on? Do those "nonstandard" models have some interesting properties?

Returning to the use of algorithms that are incompatible with punctuated time lines: I realize, of course, that computation on quantified estimates of durations and resources is rarely used in commonsense reasoning, at least as studied in A.I. Therefore, researchers in CSR may not find the algorithms mentioned above very useful. However, it would certainly be an advantage, from a general scientific point of view, if a common framework could be found for reasoning about actions in both CSR, natural language understanding, and cognitive robotics. Doing this would also facilitate the design of combined systems covering all of those aspects. However, one constraint in such a search for a common ground is that cognitive robotics needs the full set of reals (or some other dense domain) for the time axis. My question is therefore: **is there something in those other areas that *can not* be rendered using full real time?** In other words, is it the case that these different areas require intrinsically different and mutually inconsistent extensions of a core theory such as the one proposed by Pat in the previous Newsletter?

From: Jixin Ma on 14.7.1998

In ENRAC 11.7 (98054), Erik wrote:

> *I had actually asked for something more concrete: a scenario that can't be expressed, or a scenario query that requires inordinately long completion time, for example. Is it possible to be more specific about in what sense it "will be more difficult" to represent time points that stand between intervals?*

As I put in my former arguments, for many practical cases (e.g., in cognitive robotics perspectives), the Dividing Instant Problem may be just simply dealt with by means of taking point-based intervals as semi-open. However, from the perspectives of philosophy, or commonsense reasoning, this approach seems arbitrary in deciding which end the intervals should include/exclude. The choice has to be artificial and hence unjustified. For example, there is nothing to justify the reason why the light MUST BE off (or on) at the switching point, although, practically, one may just artificially take a choice which will make the robot work well in most cases (But who can prove, even practically, there is no problem for ALL cases?).

As for the sense of "it will be difficult to represent time points in an appropriate structure so that they can stand between intervals conveniently", what I mean here is, if all time intervals are taken as semi-open and therefore they can sit next to one another, it would br difficult to allow time point to stand between intervals, since it is not appropriate to represent time points with the semi-open structure.

> However, it seems that by some careful and proper treatments, we may also reach the same results by defining the concepts of intervals based on points. The key point here is, in addition to the concept of lower and upper bounds for point-based intervals, the concept of left type and right type for intervals needs to be addressed as well. What follows is the skeleton of the structure:
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> 1. P is a partially-ordered set of points;
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> 3. An interval i is defined as a quaternion $seq(p1, p2, l, r)$ such that:
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>   - l and r belong to *Type*
>   - if $p1 = p2$ then $l = r = closed$

Yes, but how does this differ from the standard notions of open and closed intervals? It would appear that

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(a,b,open,open) = (a,b)
(a,b,open,close) = (a,b]
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and so on. Are there some models where your fourtuple intervals can not be reduced to standard definitions, such as

```
(a,b) = {x | a < x < b}
```

and so on? Do those "nonstandard" models have some interesting properties?

First of all, in the quaternion form $\langle p1, p2, l, r\rangle$, the left-type, l, and right-type, r, are allowed to be unspecified. Therefore, without knowing what is the exactly value of r' and l'', we can still have

$Meets(\langle p1', p2', l', r'\rangle, \langle p1'', p2'', l'', r''\rangle)$

if we just have $r' \neq l''$. This allows one to express the case that the light changes from state "Off" to state "On" (but without modelling the state of the light at the switching point), that is the case 3 in the classfication given in my former contributions to the ETAI. However, if we insist on that the light is "On" at the switching point p2, then we have $r' = open$ and $l'' = closed$. This is the case 1. Similarly, we can express case 2, that is the light is "Off" at the switching point.

Secondly, for general treatments, intervals don't necessarily have to be sets of points, though they can be specified as such a model, e.g., as Erik suggested, $\langle a, b\rangle = \{x \mid a < x < b\}$. In fact, we can have various other models. For instance, we may specify that all the bounding points of time intervals are integers, while the internal points are all the reals bounded by the bounding points. In fact, we can have models which can be discrete, dense, or neither discrete nor dense. Also, we may have models which can be linear (one or both directions), parallel, circular, bounded (one or both sides). And so on.

In addition, both the point-based time theory using the quaternion structure, and the time theory that takes both intervals and points as primitive allow the expression of the case (case 4?) where a left-open interval Meets a point which, in turn, *Meets* a right-open interval (This open/closed nature would be expressed by the *Meets* relation when both intervals and points are taken as primitive).

What follows is an example of using the time model in modelling some critical problems:

Gelfond et al noted some of the limitations of the Situation calculus, and specially proposed an approach to represent actions with delayed effects. They use an action called *Wait* to deal with the delay between action A and its effect. That is: $Result(S0, A + Wait)$, where the duration of *Wait* equals the time delay. However, if we consider some critical cases, such as that one that throwing a ball into the air: While the ball is going up (say for just 8 seconds), the velocity is not zero (and again, not zero when the ball is going down). Only at the apex (the stationary point) where the ball is neither going up nor going down, the velocity becomes zero. Now, how to express the (delayed) effects of the action of throwing the ball? Specially, using the approach of Gelfond et al, one may use $Result(S0, Throw + Wait)$ to represent the situation 8 seconds after throwing the ball (here, we assume the duration of Wait is 8 seconds). However, in this result situation, is the velocity zero or not? The answer is not unique.

In fact, there are two situations, one is the situation where the ball is at the stationary point, another is the situation immediately after the stationary point. Both of them satisfy that the *Wait* action lasts for 8 seconds. Gelfond's approach seems unable to distinquish these two delayed effects. I have put this question to Pat in my former discussions with him, and he suspected (and I have the same feeling) that the real problem with the idea of Gelfond et al, like many applications of sitcalc, is that it doesn't give one a good way to distinguish 'instantaneous' situations from 'stable' ones (where eg the block is on the table).

However, if we associate actions with "typelised" point-based intervals, we may sucessfully express them. For the former, the associated interval's right type is "open", i.e., excluding the stationary point; while for the latter, the associated interval's right type has to be "closed", i.e., including the stationary point. For both cases, the left type may be unknown.

Jixin

From: Erik Sandewall on 27.7.1998

Pat and Jixin,

Here are some additional thoughts on ontologies for time.

1. For use within *cognitive robotics*, I take it from Jixin's latest message (ENRAC 14.7) that we can now safely ignore the dividing instant problem and use dense time (for example, the real numbers) as the time axis. The

only requirement is that we are careful about how the values of fluents are defined around their discontinuities (for continuous-valued fluents) or points of change (for discrete-valued fluents). In particular, no article in this category should need to be downgraded by a referee on the grounds that "the author has not considered the dividing instant problem".

2. For use within *commonsense reasoning*, I understand that both of you consider the DIP to still be an issue. I will use the term *dividing instant situation* when, for certain combinations of fluent and clocktime, one wishes to refrain from assigning a value. For example, one may choose to consider that the state of a switch be neither "on" nor "off" at the moment when it is thrown. According to Jixin's latest message,

> ... *However, from the perspectives of philosophy, or commonsense reasoning, this approach seems arbitrary in deciding which end the intervals should include/exclude. The choice has to be artificial and hence unjustified. For example, there is nothing to justify the reason why the light MUST BE off (or on) at the switching point, although, practically, one may just artificially take a choice which will make the robot work well in most cases...*

For Pat, if I understand him right, this is one of the reasons why he tries to find a technical approach that satsifies those who want to use a nonstandard concept of time (nonstandard relative to conventional engineering mathematics).

In order to make sense, Jixin's objection must refer not only to what conclusions are obtained, but also to the structure of the interpretations, since otherwise there ought to be no problem with the *multiple models* approach that I outlined in an earlier message (ENRAC 24.5): allow some models where the switch is "on" at the moment of change and some other models where the switch is "off" at that time, thereby avoiding getting a conclusion either way. Thus, the representational *dividing instant problem*, if I understand you right, is the problem of representing dividing instant situations in a formal system that is entirely similar to commonsense thinking.

But is it reasonable to impose this restriction on the formal semantics? From a classical AI point of view, along the lines of the Turing test, it ought to be sufficient that the intended commonsense *conclusions* are obtained from the formal system in the case of dividing instant situations. From an SB (simulation of behavior) point of view, on the other hand, one must ask whether it is appropriate to use logic at all for characterizing commonsense reasoning. Isn't people's actual reasoning *very* different from what we can render in logics of commonsense?

3. What is even more problematic, however, is that the punctuated time approach (the one concrete proposal for dealing with the DIP) actually runs into technical difficulties. Pat's proposed core theory (ENRAC 7.7) is defined with a distinction between "points" and "clocktimes"; each point has a corresponding clocktime, but it is not required for each clocktime to have a corresponding point. This makes it possible to assume normal arithmetic properties for clocktimes without imposing the same structure on points.

Jixin has observed that sometimes it is necessary to assume an intervening point between two intervals, for example, for being able to say that a ball that has been thrown vertically up into the air will be motionless (zero velocity) at a certain point in time. For some other actions or events it is desired not to have an intervening (time-)point, as is the case for two successive intervals, one where a switch was "on" and the next where the switch was "off". Sergio Brandano has already observed (ENRAC 23.4) that the timepoint domain will then be agent-specific. It will also be local to each scenario, so that if additional actions are added to the scenario description, one must change the timepoint domain accordingly. In fact, it even becomes necessary to revise the timepoint domain each time a *query* is asked for a scenario, which seems a bit odd.

I suppose one could get used to this mode of thinking, and to accept that time is in the mind of the beholder. However, here are two examples where the punctuated time approach leads to absurd conclusions.

A. Jim fires a model rocket and observes its flight. At the moment when it reaches the top of its trajectory, he turns a switch. If the flight of the rocket is modelled like Jixin proposes to model the throwing of a ball, then it requires that there exist a point for the clocktime when it is at its apex, but at the same time the fact that Jim turned the switch implies that there must not be any such point. Therefore, if the scenario description includes the statement that Jim turned the switch at the same (clock-)time as the rocket reached the apex, then it is semantically inconsistent.

B. Tom and Bob compete for eating icecream cones. They start with four cones each, and have to eat them as fast as possible, starting at the same time. The referee rings a bell when one of them has finished eating all his cones. If one of the contestants tries to cheat by dropping icecream on the ground, he also rings the bell in order to call off the contest. The bell sounds at time t. What conclusions can be drawn from this?

Consider the following ontology for this domain which is plausible if one uses punctuated time in order to deal with the DIP: 1. The ringing of the bell is momentary, that is, it occurs at a particular point in time. 2. The cone is supposed to still exist while it is being eaten, it does not exist after it has been consumed, and the clocktime at the end of the eating period does not have a corresponding point. This convention is made in order to avoid the dividing instant problem with respect to the existence of the icecream cone that has just been eaten. With these assumptions, it follows that one of the contestants cheated at time t, because in the case of an honest game, a point both exists and does not exist at the clocktime when the winner finishes his last cone and the bells rings.

It is easy to generate additional examples of the same kind. I don't see how one can get around them as long as punctuated time is allowed. The only reasonable approaches seem to be either (a) the *transitory value approach*, where one uses an additional fluent value for "undefined" or "transitory" for e.g. the state of a switch in the moment it is being thrown, or (b) the *multiple models approach* that I mentioned above. Both of these approaches are sufficiently expressive for dividing instant situations even when complete (unpunctuated) time is used. How else would you deal with these examples?

Erik

From: Jixin Ma on 31.7.1998

In Newsletter ENRAC 27.7 (98059), Erik wrote:

Jixin has observed that sometimes it is necessary to assume an intervening point between two intervals, for example, for being able to say that a ball that has been thrown vertically up into the air will be motionless (zero velocity) at a certain point in time. For some other actions or events it is desired not to have an intervening (time-)point, as is the case for two successive intervals, one where a switch was "on" and the next where the switch was "off". Sergio Brandano has already observed (ENRAC 23.4) that the timepoint domain will then be agent-specific. It will also be local to each scenario, so that if additional actions are added to the scenario description, one must change the timepoint domain accordingly. In fact, it even becomes necessary to revise the timepoint domain each time a query *is asked for a scenario, which seems a bit odd.*

First of all, it is important to note that, by taking both intervals and points as primitive, the time domain is general enough for various scenarios, and hence does not need to be revised when a "query" is asked for a special scenario. In fact, although the time line consists of both intervals and points, it is not required that between any two intervals there must be a point standing there. Therefore, the time domain itself allows the case of two successive intervals over some parts of the time line, that is an interval meets another interval without any point standing between them (this is the case 1 as I classified in ENRAC 13.3). But this does not necessarily exclude the case where a point "Finishes" the first interval or "Starts" the second (Please bear in mind that an interval "Meets" a point is treated as different with the case the interval is "Finished-by" the point; similarly, a point "Meets" an interval is different with the case that the point "Starts" the interval. See below). Therefore, if needed, it can also cope the other two cases, i.e., case 2 and case 3, by imposing futher information that a point "Finishes" the first interval, or "Starts" the second, respectively). In addition, it supports the case that an interval meets a point which in turn meets another interval (this is the case 4 which is needed in modelling the motion of the a ball being thrown vertically up into the air thrown). All these four cases are allowed by the same time theory, without the need to revise the time domain at all. In fact, one can have some of these scenarios appear somewhere over the time line, meanwhile with other scenarios appear over somewhere else. Specially, as Pat pointed out, one may even have the case where over somewhere actions or events are modelled as discrete, while over somewhere else (of the same time line) actions or events are modelled as continuous.

I suppose one could get used to this mode of thinking, and to accept that time is in the mind of the beholder. However, here are two examples where the punctuated time approach leads to absurd conclusions.

A. Jim fires a model rocket and observes its flight. At the moment when it reaches the top of its trajectory, he turns a switch. If the flight of the rocket is modelled like Jixin proposes to model the throwing of a ball, then it requires that there exist a point for the clocktime when it is at its apex, but at the same time the fact that Jim turned the switch implies that there must not be any such point. Therefore, if the scenario description includes the statement that Jim turned the switch at the same (clock-)time as the rocket reached the apex, then it is semantically inconsistent.

I think this example is similar to the following one as raised by Galton:

Suppose two lights, Green light and Red light, are both switched at the same time point P. By commonsense, we have that

(C) “GreenOff Meets GreenOn” and “RedOff Meets RedOn”.

In addition, assume that by some reason we impose that the Green light is, for instance, On, at the switching point P, how are we going to express the situation for the Red light? In other words, is the Red light “Off” or “On” at the switching point P? Galton was afraid that this would lead to the Deviding Instant Problem again.

Let me sort out this problem first and then show that Erik's example can be dealt with similarly.

Since we have no information about the the state of the Red light at the switch point P, we may just use two successive intervals, I1 and J1, to express the scenario as:

$$\begin{aligned} &Holds(RedOff, I1), \\ &Holds(RedOn, J1), \\ &Meets(I1, J1) \end{aligned} \qquad \text{(R)}$$

However, we do have the addition information that the Green light is on the switching point P, therefore, the expression for the situation of Green light could be:

$$\begin{aligned} &Holds(GreenOff, I2), \\ &Holds(GreenOn, P), \\ &Holds(GreenOn, J2), \\ &Meets(I2, P), \\ &Meets(P, J2) \end{aligned} \qquad \text{(G)}$$

And hence, we can express the assumed whole scenario as:

$$\begin{aligned} &Holds(GreenOff, I2), \\ &Holds(GreenOn, P), \\ &Holds(GreenOn, J2), \\ &Holds(RedOff, I1), \\ &Holds(RedOn, J1), \\ &Meets(I2, P), \\ &Meets(P, J2), \\ &Meets(I1, J1). \end{aligned} \qquad \text{(GR)}$$

where $I1 + J1 = I2 + P + J2$

Since (GR) allows us to express $P + J2$, i.e, the ordered union of P and $J2$, as a single interval $J2'$, such that $Holds(GreenOn, J2')$, it therefore provides a satisfactory expression consistent with (C).

However, if we in addition specially impose that, for instance, the Red light is Off at the swiching point P, we can still express the whole scenario as:

$$\begin{aligned} &Holds(GreenOff, I2), \\ &Holds(GreenOn, P), \\ &Holds(GreenOn, J2), \\ &Holds(RedOff, I2), \\ &Holds(RedOff, P), \\ &Holds(GreenOn, J2), \\ &Meets(I2, P), \\ &Meets(P, J2). \end{aligned} \qquad \text{(GR*)}$$

Again, since (GR*) allows us to express $P + J2$ as interval $J2'$, such that $Holds(GreenOn, J2')$, and also to express $I2 + P$ as interval $I2'$, such that *Holds*(*RedOff*, $I2'$), again, we reach a satisfactory expression consistent with (C) for the specified scenario.

Now, let's come back to Erik's example A:

First of all, I don't think the time theory criticized is semantically inconsistent. Yeah, for the modelling of the throwing of a ball, it requires that there exists a point referring to the apex. However, the fact that Jim turned the switch does not necessary imply that there must not be any such point, especially if one insists that "at the moment (point?) when it (the ball) reaches the top of its trajectory, he (Jim) turns a switch". I guess what Erik actually means here is that we don't have any information about the state of the object (a light?) being switched at the switching point. What we do know is just that the "On" state is immediately after the "Off" state. Therefore, similar to the treatment to the above "Two Lights Problem", we can express Erik's scenario as:

$Holds(BallGoingUp, I1)$,
$Holds(BallStationary, P)$,
$Holds(BallGoingDown, J1)$,
$Holds(LightOff, I2)$,
$Holds(LightOn, J2)$,
$Meets(I1, P)$,
$Meets(P, J1)$,
$Meets(I2, J2)$.

possibly with the constraint: $I2 + J2 = I1 + P + J1$

B. Tom and Bob compete for eating icecream cones. They start with four cones each, and have to eat them as fast as possible, starting at the same time. The referee rings a bell when one of them has finished eating all his cones. If one of the contestants tries to cheat by dropping icecream on the ground, he also rings the bell in order to call off the contest. The bell sounds at time t. What conclusions can be drawn from this?

Consider the following ontology for this domain which is plausible if one uses punctuated time in order to deal with the DIP: 1. The ringing of the bell is momentary, that is, it occurs at a particular point in time. 2. The cone is supposed to still exist while it is being eaten, it does not exist after it has been consumed, and the clocktime at the end of the eating period does not have a corresponding point. This convention is made in order to avoid the dividing instant problem with respect to the existence of the icecream cone that has just been eaten. With these assumptions, it follows that one of the contestants cheated at time t, because in the case of an honest game, a point both exists and does not exist at the clocktime when the winner finishes his last cone and the bells rings.

It is easy to generate additional examples of the same kind. I don't see how one can get around them as long as punctuated time is allowed. The only reasonable approaches seem to be either (a) the transitory value approach, *where one uses an additional fluent value for "undefined or "transitory" for e.g. the state of a switch in the moment it is being thrown, or (b) the* multiple models approach *that I mentioned above. Both of these approaches are sufficiently expressive for dividing instant situations even when complete (unpunctuated) time is used. How else would you deal with these examples?*

First of all, I don't agree with the claim that "a point both exists and does not exist at the clocktime when the winner finishes his last cone and the bells rings". Again, I think this claim was reached by means of confusing the two cases, that is, the case that an interval "Meets" a point, and the case that an interval was "Finished-by" a point. This is in fact the approach taken by many researchers such as Vilain (1982) and Parthasarathy (1990, 1994), and which actually leads to the problem. However, as I shown in ENRAC 1.4, these two cases should be taken as different since they play the critical roles in characterising the closed and open nature of intervals, and treating the corresponding problems.

Now, let me try to model this interesting example:

For Tom, we have:

$Holds(f1, I1)$,
$Holds(f2, I2)$,
$Holds(f3, I3)$,
$Holds(f4, I4)$,
$Meets(I1, I2)$,
$Meets(I2, I3)$,
$Meets(I3, I4)$

where $f1$ denotes that all the four cones have not been finished by Tom, and $f2$ denotes that just one cone has been finished by Tom, and so on.

Similarly, for Bob, we have:

$Holds(g1, j1)$,
$Holds(g2, j2)$,
$Holds(g3, J3)$,
$Holds(g4, J4)$,
$Meets(J1, J2)$,
$Meets(J2, J3)$,
$Meets(J3, J4)$

To express that Tom and Bob start at the same time, we have:

$Meets(I, I1)$,
$Meets(I, J1)$

Also, we have:

$Holds(RingingBell, t)$

If Tom finishes his last cone before Bob, that is:

$$Duration(I1 + I2 + I3 + I4) < Duration(J1 + J2 + J3 + J4)$$

then we have: $Meets(I4, t)$

Similarly, if Bob finishes his last cone before Tom, i.e.,

$$Duration(J1 + J2 + J3 + J4) < Duration(I1 + I2 + I3 + I4)$$

then $Meets(J4, t)$

As for the cheating: If

$$Holds(TomDroppingIcecream, K) \wedge During(K, I1 + I2 + I3 + I4)$$

then $Meets(K, t)$

Similarly, if

$$Holds(BobDroppingIcecream, L) \wedge During(L, J1 + J2 + J3 + J4)$$

then $Meets(L, t)$

I feel that's it, though I wonder if I missed some of the specifications of the example.

Jixin