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Protocol of the discussion during May, 1998 on

Ontologies of Time

Edited by: Erik Sandewall
Linköping University, Sweden

From: Pat Hayes on 3.5.1998

A comment on Sergio's reply to Jixin:

> *An interval from the real-line is an ordered set of real numbers limited by its end-points, which are not necessarily included in the set.*

It seems from this that the set of intervals is supposed to include open, half-open and closed intervals; is that right? (Or do you mean to say that there may be some doubt about whether a particular interval does or does not include its endpoints? If the latter, this is not the usual notion of 'interval' as used in real analysis, and you need to explain further.)

> *Suppose S consists of intervals from the real line. Assume $\langle s1, t1\rangle \in A$ and $\langle s2, t2\rangle \in B$, intervals in S. We say that $\langle s1, t1\rangle \leq \langle s2, t2\rangle$ iff $t1 \leq s2$. The strict order relation $<$ is an abbreviation for $\leq$* and *$\neq$.*

It follows then that for intervals, $\leq$ implies $<$ except for pointlike intervals (single-point closed intervals) since if $t1 \leq s2$, the intervals $\langle s1, t1\rangle$ and $\langle s2, t2\rangle$ cannot be equal unless $s1 = t1 = s2 = t2$.

> *Suppose now that $\langle s1, t1\rangle < \langle s2, t2\rangle$. The axiom of completeness states the existence of $\xi \in S$ such that $\langle s1, t1\rangle \leq \xi \leq \langle s2, t2\rangle$.*

Consider the closed intervals $[p, q]$ and $[q, r]$ with $p < q < r$. These satisfy $\leq$ and hence satisfy $<$, but there is no interval between them. Hence, your axiom is false for intervals on the real line.

Pat Hayes

From: Jixin Ma on 3.5.1998

To Sergio, who wrote:

> *The (temporal) domain S, as I meant, may consist either of time-points xor of time-intervals (exclusive "or").*
>
> *An interval from the real-line is an ordered set of real numbers limited by its end-points, which are not necessarily included in the set.*
>
> *Suppose S consists of intervals from the real line. Assume $\langle s1, t1\rangle \in A$ and $\langle s2, t2\rangle \in B$, intervals in S. We say that $\langle s1, t1\rangle \leq \langle s2, t2\rangle$ iff $t1 \leq s2$. The strict order relation $<$ is an abbreviation for $\leq$ logical-and $\neq$.*

> *Suppose now that $\langle s1, t1\rangle < \langle s2, t2\rangle$. The axiom of completeness states the existence of $\xi \in S$ such that $\langle s1, t1\rangle \leq \xi \leq \langle s2, t2\rangle$. I reply in advance to your next question: "Why did you write $\leq$ instead of $<$?". The reply is that $\leq$ means "less or equal", that is ξ may not be equal to $t1$ or $s2$, but it can do so. Note that since ξ belongs to S, then ξ is an interval. This is also meant as a reply to your question about the dividing instant problem.*

First of all, what you wrote in the above didn't solve the Dividing Instant Problem at all!

Anyway, you have claimed that "The (temporal) domain S may consist either of time-points or (**exclusive-or**) of time-intervals", and "an interval from the real-line is an ordered set of real numbers limited by its end-points, which are not necessarily included in the set." In this case, can your intervals be "pointlike"? That is, for an interval $\langle s, t\rangle$, is s allowed to be equal to t? In other words, can a set representing an interval be a singleton? As I suggested in my former response, the anwser has to be yes (see below). That is, if your domain S contains non-pointlike intervals, then, to satisfy the so-called completeness property, the domain S must contains singletons (or namely points!) **as well**. Therefore, all my former questions for you still apply.

I have shown in my former message that if your domain S contains intervals, it must contain points as well. However, you claimed that S does not consist both of time-points and intervals since you specially claimed that your "or" is **exclusive-or**). I would like to use your notation to show this again.

In fact, you define the (partial) relation "$\leq$" as follows: $\langle s1, t1\rangle \leq \langle s2, t2\rangle$ iff $t1 \leq s2$. Consider the case that interval $\langle s1, t1\rangle$ in A and interval $\langle s2, t2\rangle$ in B, satisfying $\langle s1, t1\rangle \leq \langle s2, t2\rangle$, **and** $t1 = s2$ (this is a valid case according to your definition). To fulfill the completeness property, there exists a ξ in domain S such that $\langle s1, t1\rangle \leq \xi \leq \langle s2, t2\rangle$. Let $\xi = \langle s, t\rangle$. Again, by the definition of "$\leq$" between intervals, we have $t1 \leq s$ and $t \leq s2$. However, remember $t1 = s2$, we infer that it is impossible for $s < t$. Therefore, we reach that $s = t$. That is ξ must be a point (pointlike)!

By the way, it seems that your description of the axiom of completeness is not a first-order one.

Jixin

From: Sergio Brandano on 3.5.1998

In ENRAC 24.4.1998 I made a typing mistake. I wrote: "ξ may not be equal to $t1$ or $s2$, but it can do so", while it should obviously be: "assuming $\xi = \langle \xi_1, \xi_2\rangle$, then ξ_1 may not be equal to $t1$ and ξ_2 may not be equal to $s2$, but they can do so".

In reply to Jixin Ma (ENRAC 23.4 and 24.4 1998) – completion:

> *So, you think intervals are* not needed*? Anyway, our arguments...*

I am actually skeptic about the need of a temporal domain which includes time-intervals. There are many convincing arguments that a temporal domain consisting of time-points is good enough in many different situations

(Newtonian mechanics and Thermodynamics, for instance, as well as Sandewall's underlying semantics for $\mathcal{K}$-**IA**), and I see no reason why I should pursue a different path.

> *... about the convenience of using intervals are based on the belief of the need of them.*

This is why I originally asked for some convincing argument(s) for the plausibility of this approach. According to the standard scientific methodology, in fact, we shall build on top of already existent solutions, and be consistent. Just to make an example, suppose one refuses a classical notion (continuity?), and encounters the problems that this notion was used to solve (the dividing instant?); it is surely not consistent to justify the need for a novel approach via the claim that the problems he encountered can not be solved by the notion he just refused. The notion of semi-continuity, for instance, has dignity, and its plausibility is far to be based on the belief that continuity is not needed... The case of time-intervals is clearly safer; one may simply give a preliminary example and show some objective advantages when using time-intervals instead of time-points.

> > Premise: It is evident that if you assume the axiom of completeness, the domain S can just be continuous, while if you do not assume the axiom of completeness then S is necessarily discrete.

> *Wrong! Even if you do not asssume the axiom of completeness, it is still not nessarily discrete.*

Yes, I agree. I realize I wrote that sentence having in mind the basic time structure on my paper. The question holds properly if you do not assume any axiom of density other than the one I stated.

Concerning the dividing instant problem, which seems to summarize what is left from your objections, please read below.

In reply to Pat Hayes (ENRAC 24.4.1998):

As posted in my original message, I have not yet seen any explanation why an alternative notion of continuous structure is needed at all?

Probably, in order to prevent any misunderstanding, I should have included an additional sentence like "... is needed at all, within the search of those non-monotonic logics whose purpose is to formalize common sense reasoning when reasoning about actions and change", but I thought it was evident, as the title of this Newsletter reminds.

In particular, in the same message, I asked to give at least one convincing argument on the need of a notion which is an alternative to the classical one, along the lines: "the problem P of temporal reasoning about actions and change can not be solved adopting the axiom of completeness", or "the axiom of completeness is too strong an assumption for our purposes; axiom A is better suited, because..." (*)

You and Jixin Ma proposed the "dividing instant problem", apropos of the problem of switching on the light, and argued the axiom of completeness inadequate for solving that problem. The formulation I gave in ENRAC 24.4.1998, with today's minor adjustment, gives the evidence on how the axiom of completeness is, instead, safe with respect to the dividing instant problem. You and Jixin based your argument on the fact that I do not allow the domain S to hold points "and" intervals, so that if S admits just intervals then the dividing point p can not exist. I refuted that argument by simply observing an interval from the real line may have equal end-points.

You also gave other examples, but you did not explain how they relate to the world of "Reasoning about Actions and Change". In particular, and I somehow repeat myself, it is not evident that one needs a temporal domain with non-homogeneous continuity (let me say it is even less evident the need of the imaginary number i in our temporal structure). Does there exist at least one representative problem of reasoning about actions and change that can not be solved adopting the axiom of completeness, so that to justify a temporal domain with non-homogeneous continuity? (and I repeated (*))

You also gave an informal argument on the plausibility of a temporal structure which formalizes the perceived smooth flux and perceived fast flux of time (ENRAC 21.4.1998). I refuted that plausibility with my contribution to ENRAC 23.4.1998.

(Is it really "free of context" to you ?)

Best Regards

Sergio

From: Sergio Brandano on 4.5.1998

In reply to Pat Hayes (ENRAC 3.5.1998)

> An interval from the real-line is an ordered set of real numbers limited by its end-points, which are not necessarily included in the set.

> *It seems from this that the set of intervals is supposed to include open, half-open and closed intervals; is that right? (Or do you mean to say that there may be some doubt about whether a particular interval does or does not include its endpoints? If the latter, this is not the usual notion of 'interval' as used in real analysis, and you need to explain further.)*

The former case is the one I meant.

You posed a good question, which may call into the present debate the possible relations between epistemological and ontological assumptions, at least within the "Features and Fluents" framework.

If we assume the epistemological assumption $\mathcal{K}$ (accurate and complete information about actions), then occurrences of actions are also supposed to give no doubtful information whether the scheduled time interval where they are supposed to be performed does or does not include its endpoints, so that the latter case from the quoted text must not hold. Probably the case may hold within "Mo".

> Suppose S consists of intervals from the real line. Assume $\langle s1, t1\rangle \in A$ and $\langle s2, t2\rangle \in B$, intervals in S. We say that $\langle s1, t1\rangle \leq \langle s2, t2\rangle$ iff $t1 \leq s2$. The strict order relation $<$ is an abbreviation for $\leq$ and $\neq$.

> *It follows then that for intervals, $\leq$ implies $<$ except for pointlike intervals (single-point closed intervals) since if $t1 \leq s2$, the intervals $\langle s1, t1\rangle$ and $\langle s2, t2\rangle$ cannot be equal unless $s1 = t1 = s2 = t2$.*

$\leq$ does not necessarily imply $<$, as in the case of $\langle 2,5\rangle \leq \langle 5,9\rangle$, which is a valid case with respect to $\leq$.

You are right concerning the case whether $\langle s1,t1\rangle$ may be equal to $\langle s2,t2\rangle$, but this does not really affects the axiom of completeness and, into the slightest question, it may be easily fixed.

> Suppose now that $\langle s1,t1\rangle < \langle s2,t2\rangle$. The axiom of completeness states the existence of $\xi \in S$ such that $\langle s1,t1\rangle \leq \xi \leq \langle s2,t2\rangle$.

> *Consider the closed intervals $[p,q]$ and $[q,r]$ with $p < q < r$. These satisfy $\leq$ and hence satisfy $<$, but there is no interval between them. Hence, your axiom is false for intervals on the real line.*

The closed intervals $[p,q]$ and $[q,r]$, with $p < q < r$, do not fulfill the relation $\langle p,q\rangle < \langle q,r\rangle$, hence they do not make a valid counterexample.

Best regards
Sergio

From: Jixin Ma on 7.5.1998

In ENRAC 3.5 (980521), Sergio wrote:

> *I am actually skeptic about the need of a temporal domain which includes time-intervals. There are many convincing arguments that a temporal domain consisting of time-points is good enough in many different situations (Newtonian mechanics and Thermodynamics, for instance, as well as Sandewall's underlying semantics for $\mathcal{K}$-**IA**), and I see no reason why I should pursue a different path.*

You said here "a temporal domain consisting of time-points is good enough in many different situations". Is this ("many") a convincing argument for general treatments? Anyway, the fact that "you see no reason why **you** should pursue a different path" does not mean others don't see/have the reason (see below).

> ... about the convenience of using intervals are based on the belief of the need of them.

> *... According to the standard scientific methodology, in fact, we shall build on top of already existent solutions, and be consistent. Just to make an example, suppose one refuses a classical notion (continuity?), and encounters the problems that this notion was used to solve (the dividing instant?); it is surely not consistent to justify the need for a novel approach via the claim that the problems he encountered can not be solved by the notion he just refused.*

Have you applied the above arguments to that one proposed by youself? Sorry, I am here again using your question to ask you.

Anyway, while I (and many others) have seen the convenience of using intervals, I can also see the need of them. In fact, there have been quite a lot of examples (**many**) in the literature that demonstrated the need of time-intervals (or time-periods). Haven't you ever encountered any one of them? Or you simply cannot see anyone of them is convincing?

All right, let's just have a look at the example of throwing a ball up into the air. As I showed in ENRAC 1.4 (98033) (one may disagree with this), the motion of the ball can be modelled by a quantity space of three elements: going-up, stationary, and going-down. Firstly, or at least, we can see here the convenience of using intervals. In fact, we can conveniently associate the property that "the ball changes its position" with some time-intervals. Secondly, let's see if we indeed need time-intervals. Without the notion of time-intervals (neither primitive nor derived from time-points), can you just associate such a property with time-points? Yes, we may associate it with a pair of points. However, this doesn't mean that the property holds at these points. What it really means is that the property holds for the time periods denoted by the pair of points. Aren't these time periods in fact time intervals?

It is important to note, up to now in the above, I just talked about the need of the notion of intervals. As for how to characterise intervals (e.g., are intervals taken as primitive or derived structures from time-points?) is another important issue, and this issue, again, has been addressed in the literature for a long time.

The Point Is: while we were/are discussing/arguing about some broader issues on temporal ontology, you just jumped in and asked "why an alternative notion of continuous structure is needed at all?" First of all, the "continuity" (or more truly, density) is not the main issue we are talking about. The fundamental question is if we need to address and how to address time intervals. Based on such a discussion, in the case that intervals are taken as temporal primitive, then, we are talking about how to characterise some corresponding issues including dense/discrete structures. But your questions and arguments/replies do not seem to follow this. As stated in the former replies from both Pat and myself, first of all, the dense structure does not have to be characterised in terms of the only form of the so-called "axiom of completeness". Also, in the case where time-intervals are involved (even they are still point-based, let alone in the case they are taken as primitive), such an axiom doesn't simply apply. In fact, I have shown this twice with different notations in this discussion. I will point out more problems in detail below in my response to your reply to Pat.

> *Concerning the dividing instant problem, which seems to summarize what is left from your objections, please read below.*

As I already stated, your approach does not solve the DIP at all. In fact, it seems that you don't realise the DIP in the way as we are talking about (see below).

> In reply to Pat Hayes (ENRAC 24.4.1998):
>
> *As posted in my original message, I have not yet seen any explanation why an alternative notion of continuous structure is needed at all?*

Still not yet?

> *You and Jixin Ma proposed the "dividing instant problem", apropos of the problem of switching on the light, and argued the axiom of completeness inadequate for solving that problem. The formulation I gave in ENRAC 24.4.1998, with today's minor adjustment, gives the evidence on how the axiom of completeness is, instead, safe with respect to the dividing instant problem. You and Jixin based your argument on the fact that I do not allow the domain S to hold points*

"and" intervals, so that if S admits just intervals then the dividing point p can not exist. I refuted that argument by simply observing an interval from the real line may have equal end-points.

You claimed already that your domain S contains points or (exclusive-or) intervals? To fulfill the axiom of completeness, you *must* allow your intervals to be possibly some singletons (i.e., a set of single *point*). In other words, if your S contains intervals, it should also contain singletons (points). The real problem is that even if you allow your intervals to be singletons, the Dividing Instant Problem is still there, and in fact more obviously. Do you agree with this?

The closed intervals $[p, q]$ and $[q, r]$, with $p < q < r$, do not fulfill the relation $\langle p, q\rangle < \langle q, r\rangle$, hence they do not make a valid counterexample.

Pat's example becomes invalid **only after** you made the "minor adjustment" that replaces the relation $\leq$ in your hypothesis $\langle s1, t1\rangle \leq \langle s2, t2\rangle$ by $<$, that is $\langle s1, t1\rangle < \langle s2, t2\rangle$. (Is this an alternative?)

It follows that you do need alternation, doesn't it? (Note that this is just for the case when you construct intervals out of points. In the case where intervals are taken as primitive, the need of such alternative is indeed more conceptually necessary). However, your adjustment is not enough, or you haven't reached the proper form for general treatments. In fact, you need to address the issue regarding different cases. To see this, you may just consider the difference between the case where at least one of $\langle s1, t1\rangle$ and $\langle s2, t2\rangle$ is "closed" at $t1$ $(= s2)$, and the case where both $\langle s1, t1\rangle$ and $\langle s2, t2\rangle$ are "open" at $t1$ $(= s2)$. In the former case, you need use $<$ in the hypothesis; otherwise, Pat's example will be a valid counterexample. In the latter case, you need use $\leq$ in the hypothesis; otherwise, your axiom cannot prevent a "gap" between $\langle s1, t1)$ and $(s2, t2\rangle$, that is, there is no guarantee that the singleton $[t1, t1]$ is contained in S (Do you think this is consistent with the "classical" concept of contiunity?).

Jixin

From: Sergio Brandano on 8.5.1998

In reply to Jixin Ma (ENRAC 7.5.1998)

Pat's example becomes invalid **only after** *you made the "minor adjustment" that replaces the relation $\leq$ in your hypothesis $\langle s1, t1\rangle \leq \langle s2, t2\rangle$ by $<$, that is $\langle s1, t1\rangle < \langle s2, t2\rangle$. (Is this an alternative?)*

Ex falso sequitur quodlibet!

The only one "minor adjustment" I made consists in the first four lines of my contribution to ENRAC 3.5.1998, where no inequality appears at all. Concerning the hypothesis, notice what I wrote in ENRAC 24.4.1998:

> Suppose now that $\langle s1, t1\rangle < \langle s2, t2\rangle$. The axiom of completeness states the existence of $\xi \in S$ such that $\langle s1, t1\rangle < \xi < \langle s2, t2\rangle$.

In fact, Pat quoted me correctly in ENRAC 3.5.1998.

So, you do need alternation, don't? (And this is just for the case ...

The axiom of completeness imposes $\leq$, so no "alternation" is needed. The reason why I wrote $<$ instead of $\leq$ is simply due to my need to stress the example, since the case $\langle s1, t1\rangle = \langle s2, t2\rangle$ is trivial.

> *...when you construct intervals out of points. In the case where intervals are taken as primitive, the need of such alternative is indeed more conceptually necessary). However, your adjustment is not enough, or you haven't reached the proper form for general treatments. In fact, you need address the issue regarding different cases. To see this, you may just consider the difference between the case where at least one of $\langle s1, t1\rangle$ and $\langle s2, t2\rangle$ is "closed" at $t1$ $(= s2)$, and the case where both $\langle s1, t1\rangle$ and $\langle s2, t2\rangle$ are "open" at $t1$ $(= s2)$. In the former case, you need use $<$ in the hypothesis; otherwise, Pat's example will be a valid counterexample. In the latter case, you need*

... at least one is closed. So we have, since $s2 = t1$:

1. $[s1, t1] \leq [t1, t2]$
2. $[s1, t1] \leq (t1, t2]$
3. $[s1, t1) \leq [t1, t2]$

where $\xi = [t1, t1] \in S$ in all cases.

Note I used $\leq$, as required by the axiom of completeness. If I use $<$, as you recommend, then all cases *trivially* fail.

Pat's example:

- trivially fails when using $<$,
- trivially succeeds ($\xi = [q, q]$) when using $\leq$.

> *use $\leq$ in the hypothesis; otherwise, your axiom cannot not prevent a "gap" between $\langle s1, t1)$ and $(s2, t2\rangle$, that is, there is no guarantee that the singleton $[t1, t1]$ is contained in S (Do you think this is consistent with the "classical" concept of contiunity?).*

... the latter case. So we have, since $s2 = t1$:

4. $\langle s1, t1) \leq (t1, t2\rangle$

where $\xi = [t1, t1] \in S$. I used $\leq$ here too.

So, the axiom of completeness has no problems with your examples.

Concerning the first part of your message, as you wrote in it, it was entirely based on the DIP problem and the above argument-examples.

> *while I (and many others) have seen the convenience of using intervals, I can also see the need of them. In fact, there have been quite a lot of examples (many) in the literature that demonstrated the need of time-intervals (or time-periods). Haven't you ever encountered any one of them? Or you simply cannot see anyone of them is convincing?*

The problem about intervals is whether one needs to introduce them into the temporal domain, and the few argument-examples I encountered are far from being convincing. Furthermore, in this debate, you and Hayes proposed the DIP, and I refuted it.

Does there exist at least one problem (within R.A.C) that needs to introduce intervals into the temporal domain?

The other problem is:

Does there exist at least one problem (within R.A.C.) that can not be solved with a continuous temporal domain, so as to justify a temporal domain with non-uniform continuity?

This debate aims at generality, and surely does not aim at completeness of case examples. If many examples do exist, then this is the proper debate where at least the most representative of them should appear "naked" under the spotlight, for general benefit. On the other hand, I note that more than two weeks have now passed since my criticism, and no such representative example appeared.

Sergio

From: Jixin Ma on 12.5.1998

Reply to Sergio Brandano (ENRAC 8.5.1998)

> *The only one "minor adjustment" I made consists in the first four lines of my contribution to ENRAC 3.5.1998, where no inequality appears at all. Concerning the hypothesis, I remind you what I wrote in ENRAC 24.4.1998:*

When you presented the (classical) axiom of completeness (ENRAC 23.4.1998), you used $\leq$ in the hypothesis (You are now still using it, see below). But for the case where elements in domain S are just intervals, you used $<$ instead (otherwise, Pat's example is valid, see below).

> *The axiom of completeness imposes $\leq$, so no "alternation" is needed at all. The reason why I wrote $<$ instead of $\leq$ is simply due to my need to stress the example, since the case $\langle s1, t1\rangle = \langle s2, t2\rangle$ is trivial. If you like to check, the reference is ENRAC 24.4.1998.*

Again, if you re-claim that the axiom of completeness imposes $\leq$, then Pat's example is valid (see below).

> ...when you construct intervals out of points. In the case where intervals are taken as primitive, the need of such alternative is indeed more conceptually necessary). However, your adjustment is not enough, or you haven't reached the proper form for general treatments. In fact, you need address the issue regarding different cases. To see this, you may just consider the difference between the case where at least one of $\langle s1, t1\rangle$ and $\langle s2, t2\rangle$ is "closed" at $t1$ ($= s2$), and the case where both $\langle s1, t1\rangle$ and $\langle s2, t2\rangle$ are "open" at $t1$ ($= s2$). In the former case, you need use $<$ in the hypothesis; otherwise, Pat's example will be a valid counterexample. In the latter case, you need

> *... at least one is closed. So we have, since $s2 = t1$:*
>
> 1. $[s1, t1] \leq [t1, t2]$
> 2. $[s1, t1] \leq (t1, t2]$
> 3. $[s1, t1) \leq [t1, t2]$
>
> *where $\xi = [t1, t1] \in S$ in all cases.*
>
> *Note I used $\leq$, as required by the axiom of completeness. If I use lt, as you recommend, then all cases* trivially *fail.*
>
> *Pat's example:*

- *trivially fails when using $<$,*
- *trivially succeeds ($\xi = [q,q]$) when using $\leq$.*

This is exactly what I wanted to show and have shown a few times now. That is, in the case where your domain S contains intervals, to fulfill the axiom of completeness, S has to contain singletons (single points) as well, not as you specially claimed that S contains points or (exclusive-or) intervals. My observation that Pat's example would be valid is under your assumption that the domain S refuses to take both intervals and singletons (points). I think when Pat gave the example, he also followed this assumption of yours. (Actually, Pat did specially claim that "unless you allow intervals consisting of a single point" when he gave the example in ENRAC 24.4.1998).

> use $\leq$ in the hypothesis; otherwise, your axiom cannot not prevent a "gap" between $\langle s1, t1)$ and $(s2, t2\rangle$, that is, there is no guarantee that the singleton $[t1, t1]$ is contained in S (Do you think this is consistent with the "classical" concept of contiunity?).

> *... the latter case. So we have, since $s2 = t1$:*
>
> *4.* $[s1, t1) \leq (t1, t2]$
>
> *where $\xi = [t1, t1] \in S$. I used $\leq$ here too.*

This is exactly what I have suggested in my message to you (see above).

> *The problem about intervals is whether one needs to introduce them into the temporal domain, and the few argument-examples I encountered are far from being convincing. Furthermore, in this debate, you and Hayes proposed the DIP, and I refuted it.*

The DIP was proposed much earlier in the literature. What you have done, is just a **re**-Writing of a model where the DIP arises. Could you please check carefully what exactly is the problem and if it can be solved by your formulation?

Jixin

From: Pat Hayes on 17.5.1998

[S.B.]

> *Suppose S consists of intervals from the real line. Assume $\langle s1, t1\rangle \in A$ and $\langle s2, t2\rangle \in B$, intervals in S. We say that $\langle s1, t1\rangle \leq \langle s2, t2\rangle$ iff $t1 \leq s2$.* The strict order relation $<$ is an abbreviation for $\leq \wedge \neq$.

[P.H.]

> It follows then that for intervals, $\leq$ implies $<$ except for pointlike intervals (single-point closed intervals) since if $t1 \leq s2$, the intervals $\langle s1, t1\rangle$ and $\langle s2, t2\rangle$ cannot be equal unless $s1 = t1 = s2 = t2$.

[S.B.]

> *$\leq$ does not necessarily imply $<$, as in the case of $\langle 2, 5\rangle \leq \langle 5, 9\rangle$, which is a valid case with respect to $\leq$.*

Clearly, $\langle 2,5\rangle$ is not equal to $\langle 5,9\rangle$, ie $\langle 2,5\rangle \neq \langle 5,9\rangle$. That is, both $\leq$ and $\neq$ hold between those intervals. According to Sergio's definition (in italics above) it follows that the relation $<$ must hold between them. In general, if p is not equal to q, then the intervals $\langle p,q\rangle$ and $\langle q,r\rangle$ cannot be equal, so must be $\neq$, but are also $\leq$, and therefore must be $<$. The rest of his message in ENRAC 4.5 (98042) makes the same error, and the subsequent confusion has been noted by Jixin in the later discussion.

Sergio's point seems to be that one can describe the line in terms of conventional open and closed intervals in such a way that no 'gaps' appear, so that the 'interval' between the open intervals (a, b) and (b, c) is the closed interval $[b]$ containing a single point. Yes, of course: that is not at issue. We are not claiming to have found some basic flaw in conventional real analysis. The question is whether this standard mathematical view of the line is the most suitable for capturing linguistic intuition or for action reasoning. For example, we want to be able to assert that a light is off before time t and on after time t without having to commit ourselves to its being either on or off **at** that time, but also without sacrificing the assumption that it is always either on or off. Of course we could just decide that periods of off-ness, say, shall be left-open-right-closed, or some other convention: but this is arbitrary, ad-hoc and theoretically unsatisfactory, since the intuition we would like to capture is that the question (of the light being on or off at b) simply doesnt arise: it just *goes* on then, that's all. That is the intuition which Allens interval calculus, where intervals can simply *meet*, is intended to capture. The fact that this calculus violates Sergios pet axiom doesnt seem to be a very important matter for further discussion.

In any case, many users of temporal ontologies do not wish to assume continuity or even density, for reasons of their own, and so a general-purpose temporal ontology should therefore not make such unnecessarily strong assumptions as Sergio's 'completeness' axiom. Temporal database technology usually assumes times are discrete, for example.

Pat Hayes

From: Sergio Brandano on 24.5.1998

In reply to Pat Hayes [ENRAC 17.5]

> *Sergio's point seems to be that one can describe the line in terms of conventional open and closed intervals in such a way that no 'gaps' appear, so that the 'interval' between the open intervals (a, b) and (b, c) is the closed interval $[b]$ containing a single point.*

In my example, the temporal domain S consists *exclusively* of intervals from the real-line and no total order relation was imposed, so that the interpretation for which one can describe the line with that domain is surely not correct. If one restricts S to the case of intervals from the positive real-line, for example, then S is a tree with a continuum of branches.

> *The question is whether this standard mathematical view of the line is the most suitable for capturing linguistic intuition or for action reasoning. For example, we want to be able to assert that a light is off before time t and on after time t*

In your approach the switching action takes place at time t, while the process of turning on the light does not necessarily require a single time-point.

Anyway, according to the classical approach (linear point-based temporal-domain) one may easily capture the proper intuition with a simple function of time:

```
 \[ to\_switch(s,\tau,t) =
\cases{ {\rm off}               & if $\tau = s$       \cr
                {\rm on} \vee {\rm off} & if $\tau \in (s,t)$ \cr
                {\rm on}                & if $\tau = t$}
 \]
```

where the interval $[s, t]$ is the switching interval, which may also be point-like, of course. The switching interval is properly that time-interval where the action of switching on the light is performed. Please note that s and t may also be variables.

Now, assume the light is initially "off". Then it will remain "off" until s, because of the assumption of inertia. From s to t the value will change according to the definition of the switching action, while at t the light will be surely "on". The light will be "on" at t and at every timepoint after t until another action will occur to change state. As simple as this...

> *In any case, many users of temporal ontologies do not wish to assume continuity or even density, for reasons of their own, and so a general-purpose temporal ontology should therefore not make such unnecessarily strong assumptions as Sergio's 'completeness' axiom. Temporal database technology usually assumes times are discrete, for example.*

(a) When implementing whatever formalism on a resource-bounded machine, compromises are never enough, otherwise we shall bound ourselves when designing them. On the other hand, I like to observe that humans formulated the notion of continuity in their mind, although humans too are resource-bounded. It is also true that not every human knows about continuity, but some do. In any case the notion of continuity does exist, at least on paper.

(b) The problem that comes when aiming at generality (as in this debate?), is one shall include all possible cases, instead of excluding those which are of no interest for someone. Personally, I like to consider problems within Newtonian physics as being an important part of Reasoning about Actions and Change, where continuity plays an fundamental role when idealizing the physical world. Furthermore a continuous temporal domain subsumes the case of integer time, as well as the case of rational time, so that it is general enough to embrace all possible cases.

Therefore, in my opinion, the axiom of completeness is a good assumption, at least within the classical point-based approach. The purpose of my example with S as an ad-hoc example to show the axiom of completeness adequate as well for an interval-based temporal domain. As an immediate consequence of that result, the dividing instant problem is then solved for those interval-based formalisms where this axiom will be properly adopted. Still remains the question whether at least one problem exists that needs to introduce intervals into the temporal domain. Personally, I think this problem does not really exist.

Finally, I would like to resume Jixin's original statements in ENRAC 13.3:

(1) For general treatment, both intervals and points are needed.

(2) To overcome the so-called Dividing Instant Problem, that is the problem in specifying whether intervals are "open" or "closed" at their ending-points, both intervals and points should be treated as primitive on the same footing.

My reply to (1) was: [ENRAC 8.5]

> Does there exist at least one problem (within R.A.C) that needs to introduce intervals into the temporal domain?

My reply to (2) was the example with S, where just intervals are needed.
Best Regards Sergio

From: Erik Sandewall on 24.5.1998

Pat,
You wrote (ENRAC 17.5]:

> *The question is whether this standard mathematical view of the line is the most suitable for capturing linguistic intuition or for action reasoning. For example, we want to be able to assert that a light is off before time t and on after time t without having to commit ourselves to its being either on or off* **at** *that time, but also without sacrificing the assumption that it is always either on or off.*

The most natural way of dealing with such a situation, it seems to me, is to admit that one's axioms allow two kinds of models: those where the light is on at time t, and those where it is off. The assumption that light is either on or off holds in each of the models, but the axioms don't imply one or the other.

The alternative that has occurred in the present discussion, and proposed as a way of dealing with the "dividing instant problem" is the use of a *punctuated timeline* where the domain of timepoints is chosen e.g. as Re - B where Re is the real numbers and B is a finite set of "breakpoints", typically chosen as the times where actions start or end. If we ignore questions of how this domain is specified in terms of axioms or how intervals are formally defined, this seems to be the essence of how one would deal with the DIP without going to *very* unconventional concepts of time.

Given that the standard view already exists, it seems worthwhile to understand what the concrete reasons would be for replacing it with the punctuated timeline ontology. Apart from the purely subjective reasons (that is, some people *prefer* to do it that way) I wonder what *results* have been obtained using punctuated or other nonstandard ontologies, and which are not trivially translatable back to the standard view. The following would seem to be interesting *results* for this purpose:

- Expressiveness in a concrete scenario. This ought to be a scenario that can't be properly expressed in the standard ontology, but which can be expressed in the nonstandard one.
- Algorithms or other implementation techniques. Is there some algorithm that works with a punctuated timeline and that outperforms those that don't?

- Validation and range of applicability results. These are results stating that or when a certain entailment method is correct, that is, it obtains exactly the intended models viz conclusions. Have some such results been obtained using nonstandard ontologies?

Please feel free to add more categories to the list. (In the methodology discussion at the NRAC workshop at IJCAI last year we got to more than ten such categories).

I realize of course the purely philosophical and/or logical interest in analyzing different possible concepts of time, but from an AI point of view there is no point in pursuing an approach if it doesn't deliver any results. I haven't been able to see any indications of such concrete results from the present discussion or from the articles that have been referenced in it. (That most work in temporal databases uses discrete time doesn't say anything about the choice between a standard or a punctuated real timeline, does it?)

Although I'd prefer to see algorithmic results or range of applicability results, a few words about expressiveness since after all that's also a relevant type of achievement. One nice scenario for hybrid change (that is, continuous and discrete) is the *impact problem* that Persson and Staflin used in their ECAI 1990 paper [c-ecai-90-497]. It goes like this: two solid spheres B and C are lying side by side on a horizontal surface, B to the left of C. A third sphere, A, comes rolling from the left and hits B. As we know from physics, the result of the impact is that A stops, B stays, and C starts moving towards the right. The representation of this particular exercise has been solved in 1990, but is there now some harder one that makes essential use of punctuated time ontology?

Erik

References:

[c-ecai-90-497] Tommy Persson and Lennart Staflin. *A Causation Theory for a Logic of Continuous Change.* Proc. European Conference on Artificial Intelligence, 1990, pp. 497-502.