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Today

Today: interventions by John McCarthy and Erik Sandewall concerning the ontology of time, and an answer by Sergio Brandano to Jixin Ma and Pat Hayes in the same debate.

Debates

Ontologies for time

From: John McCarthy

Pat Hayes wrote

> *In my view, axioms are tools which we can manipulate at will; they are not set in stone or somehow inevitable. Different formal accounts of time might be appropriate for different purposes or to capture different intuitions. (I agree with Jixin that it is useful to seek a common 'core' theory which can be extended in various ways to describe various possible more complex temporal structures; and that this theory will have to be rather weak.)*

and

> *A more mundane example is given by temporal databases, which usually assume in their basic ontology that time is discrete: for example, they routinely describe times as integers representing the number of milliseconds since the birth of Christ. (Of course, one can always insist that these are to be understood as being embedded in a continuum, but then what use is an axiom whose sole purpose is to insist that times exist which have no name and about which nothing can be asserted, other than that they exist?)*

If axioms are guaranteed to be used only in a particular program or set of programs, they need be no stronger than necessary.

As to the rhetorical "what use", suppose the theory is to tolerate the elaboration that two successive events, shooting Pat and his falling to the ground, occurred between successive ticks of the clock. If you guarantee that no such elaborations will be required or that you are willing to do major surgery on your theory should elaboration be required, then you are ok with a weak theory even if it is unextendable.

From: Sergio Brandano

In reply to Pat and Jixin.

I apologize for the length of this message, although it mainly consists of quoted text. As "skin perception", it seems to me my critique hits the target. The arguments of reply I received, in fact, are not as convincing as they were supposed to be. The details follow.

To Jixin

> *First of all, what do you mean "the classical one"? (the classical continuous time structure)? Does it refer to the classical physical model of time, where the structure is a set of points which is isomorphic to the real line?*

I can just quote myself ...

> Here in Pisa, we write "continuity" and we read "axiom of completeness", which is what everyone commonly means when speaking about (the founding notion of) continuity.
>
> At the ontological level, the notion of continuous time vs. discrete time is closely related to questions "Is the set of time elements dense or not?", and " Are there really time atoms?".

The word "continuity", even at the ontological level, can not be read as "continuous with some exception".

> *For a point-based model, the continuity is usually characterized as "Between any two points, there is a third"; while for an interval-based model (like that of Allen), it is characterized as "Every interval can be decomposed into two adjacent subintervals".*

The axiom of completeness states:

Let be A and B non empty subsets of S such that $a \leq b$ for all $a \in A$ and $b \in B$. Then there exists $\xi \in S$ such that $a \leq \xi \leq b$ for all $a \in A$ and $b \in B$.

Now, the set S, that is your domain, may consist as well either of time-points or time-intervals; S holds real numbers on the former case, intervals from the real line on the latter case.

> *As for general treatments, the Basic Time Structure* **does not** *have to impose the axiom of density or discreteness (Similar arquements apply to issues such as linear/non-linear, bounded/unbounded). Therefore, the time structure as a whole may be continuous or discrete, or neither continuous nor discrete.*

I agree with your premise: the Basic Time Structure does not have to impose the choice, in fact it leaves you free in that sense. As soon as you make the choice, then you obtain either a continuous structure or a discrete structure, just depending on this choice. I do not agree, instead, with your conclusion. If I leave you the freedom to choose, it does not mean the Structure is neither continuous nor discrete; it simply means you still have to make the choice.

> *Now, "why an alternative notion of continuous structure is needed at all"? It has been noted that temporal knowledge in the domain of aritifical intelligence, including "temporal reasoning about actions and change", is usually imcomplete, and using time intervals in many cases is more* `convenient` *and more in-keeping with common sense of temporal concepts than to use the classical abstraction of points. In fact, the notion of time intervals (or periods) has been introduced for a long time in the literature. In addition, in order to overcome/bypass the annoying question of if intervals are open or closed, various approaches have been proposed. An example is Allen's interval-based time theory. As for these time theories, the old (classical?) notion of continuity no longer simply applies.*

My question referred to what is *needed* rather than *convenient*. I understand it may be convenient, in some cases, to use intervals, but this is not pertinent with my criticism, which still holds.

Let me ask you a more stringent question.

Premise: It is evident that if you assume the axiom of completeness, the domain S can just be continuous, while if you do not assume the axiom of completeness then S is necessarily discrete.

Question: Suppose that you define your neither continuous nor discrete Temporal Structure. What is your domain S? What is your replacement for the axiom of completeness? Does this structure (provably) solve for at least one problem what can not be (provably) solved via the axiom of completeness? Can you give an example?

To Pat

> *Why cannot time be continuous in some places but discontinuous at others?*

Places? If we shall understand time like physicians understands the space, then things become considerably simpler: time can just be discrete, since space itself is fully discrete (the observable one). The use of real lines, in physics, is just a theoretical convenience. If by "place" we shall mean, instead, some point in a lattice (and we shall provide a convenient reference system for the branching-time case), then the case may hold. In part my question was, in fact, to insert in the actual discussion explicit and convincing arguments about the case (examples, counterexamples and axioms, are welcome).

> *There is no mathematical objection to such a structure, and it has been*

If a Temporal Structure exists in this sense, may I have a look at its domain (that is at the S domain, as stated above)? Can you post the reference together with the explicit case? But the primary question still remains whether is it needed at all.

> *You talk about a 'founding notion' of continuity as being that captured by the axiom of completeness. Here, in my view, you commit a philosophical error (especially in Pisa!) There are* intuitions *about continuity which one can try to capture in various formal ways, but there is no 'founding notion' of continuity other than those intuitions. ...*

What properly formalizes the notion of continuity is the axiom of completeness. Alternative notions are equivalent, until we speak about continuous domains. The point was whether one can have a continuous domain (that is the S I stated above) ... with exceptions.

Concerning the intuition, let me remind that the student who discovered the square root of 2 was killed (down the cliff), and no one was allowed to speak about ... "the fault of the god" for long time. Humans' common sense, to me, is something we shall not call too much.

> *century, famous mathematicians objected strongly to the view of the continuum as consisting of a set of points, for example. This modern perspective, now taught in high schools, is a modern invention, not a 'founding' notion. It is more recent than the gasoline engine, yet people have had intuitions about smoothness, instantaneity and continuity for eons. (Whether or not one agrees with me on this admittedly controversial point, it seems unwise to* identify *a mathematical property such as continuity with any kind of axiom until one has verified that no other axiom will do as well; and as I am sure Sergio knows, there are many alternative ways to axiomatize continuity.)*

... unwise ?

If another axiom exists, which *does as well*, then it is surely equivalent to the axiom of completeness, just because *it does as well.* Alternative notions are clearly equivalent, until we speak about continuous domains. The point here, instead, was whether one can have a continuous domain with exceptions, that is the claim I originally criticized.

In my view, axioms are tools which we can manipulate at will; they are not set in stone or somehow inevitable. Different formal accounts of time might

The point is what would you like to make out of it. If you do something I can do with a simpler approach and without arousing criticism, then my approach will have much more impact than yours, I think you may agree at least on this point.

be appropriate for different purposes or to capture different intuitions. (I agree with Jixin that it is useful to seek a common 'core' theory which can be extended in various ways to describe various possible more complex temporal structures; and that this theory will have to be rather weak.)

Concerning the core theory that you and Jixin are willing to obtain, I already developed a Basic Time Structure which may be of interest. It is as simple as I managed to design it, without un-useful complications. The structure works well in my case. you are welcome to read and comment my contribution, which may be found in my ETAI's reference.

Finally, concerning your examples:

The 'dividing point' problem which gave rise to this discussion would do. According to the modern account of the continuum, this point must exist, and since all intervals consist of points, the light is therefore either on or off at it. But it seems more natural, as well as formally simpler, to just say that the question is meaningless; perhaps (though this is no longer my own preference) because that point doesn't exist.

Why is it a convincing argument?

A more mundane example is given by temporal databases, which usually assume in their basic ontology that time is discrete: for example, they routinely describe times as integers representing the number of milliseconds since the birth of Christ. (Of course, one can always insist that these are to be understood as being embedded in a continuum, but then what use is an axiom whose sole purpose is to insist that times exist which have no name and about which nothing can be asserted, other than that they exist?)

Why is it a convincing argument ?

Best Regards Sergio

From: Erik Sandewall

Pat,

In answer to Sergio, you wrote

> *Why cannot time be continuous in some places but discontinuous in others?*

(Jixin answered along similar lines). I have no problems accepting that a *function of* time may be piecewise continuous, or that it may be undefined for some points along the time axis. However, it seems to me that there are several problems with saying that *time itself* is piecewise continuous (btw - do you mean piecewise dense?).

The first problem is with respect to motivation. For what reasons would Time suddenly skip over potential timepoints? If the reason is, as you wrote, that

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then exactly what events in the world would be allowed to contribute to the continuity faults? Does the next time I hit a key on my keyboard qualify? And what about the midpoint halfway between two continuity faults, is it also a continuity fault, recursively?

The other problem is with respect to the axiomatizations. Since your article "A catalog of temporal theories" characterizes the various theories through axiomatizations, I thought I'd go back to that article and check how you had done this formally. However I was not able to find it; the closest I got was the denseness axiom on page 15. If the intuitive notion is that time itself is continuous in some places but not in others, wouldn't it be natural to start with an axiomatization of continuous time (such as the real numbers) and then to proceed from there? For example, a domain of piecewise continuous time could be represented as a twotuple $\langle R, D\rangle$ where R is the real numbers and D is a "small" subset of it; the intention being that $R - D$ is the modified time domain in question. The notions of non-standard intervals could then be constructed as the natural next step.

Maybe I'm missing something - are constructs of this kind subsumed by the axioms in your report, or can they be inferred as theorems? Or why is this not the natural way of doing things?

Erik