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Today

The discussion between Pat Hayes and Jixin Ma on the ontology of time continues:

Debates

Ontologies for time

From: Jixin Ma

Pat wrote:

> *Jixin's own intuition, for example, seems to agree with McCarthy's that time is continuous, and yet also finds the idea of contiguous atomic 'moments' (intervals with no interior points) quite acceptable. But you can't have it both ways: if moments can meet each other, then there might not be a point where the speed is exactly 60mph, or the ball is exactly at the top of the trajectory. If time itself is discrete, then the idea of continuous change is meaningless. Appealing to a kind of raw intuition to decide what axioms 'feel' right lands one in contradictions. (That was one motivation for the axiom in Allen's and my theory, which Jixin found "awkward", that moments could not meet. The other was wanting to be able to treat moments as being pointlike. That was a mistake, I'll happily concede.)*

First of all, as Pat noted (see below), our theory is similar (in fact, it is specially stated to be an extension/revision) to that of Allen and Hayes ([c-ijcai-85-528], [j-ci-5-225]). As an extension, it allows not only intervals and moments, but also points (however, it dose not necessarily have to include points or moments). As a revision, Allen and Hayes' constraint that moments could not meet each other is replaced by "a point cannot meet another point", that is there must be an interval (or a moment) standing between any two points (if these two point are explicitly expressed). However, basically, the theory doesn't commit the time structure as (left-, right-, left-and-right) linear, (left-, right-, left-and-right) branching, continuous or discrete, etc., although all these can be characterised by means of some corresponding extra axioms.

Yes, in this time theory, atomic moments (AND points) are acceptable. However, they are just ACCEPTABLE, but not necessarily to be everywhere over the time. The theory only claims that a time element is either an interval (or specially, a moment) or a point. If one insists on using moments/points somewhere over the time, they can be explicitly expressed there. For somewhere else over the time, it may be the case that each time element is a decomposable interval. That is, there may be no moments/points at all. It is also consistent to have a time structure where each time element is either an decomposable interval or a point, or even a time structure where each time element is a decomposable interval. In fact, generally speaking, the basic time structure may be neither dense nor discrete anywhere, or may be continuous over some parts and discrete over other parts. This depends on what you want to express and what extra axiom you would impose.

Pat wrote

> *Galton's intuitions are clearly based on thinking of intervals as sets of points. He takes it as simply obvious, for example, that there is a distinction between open and closed intervals.*

I still cannot find where Galton claimed this. Actually, before I answered Pat's last contribution (Newsletter ENRAC 27.3 (98030), I had re-read Galton's paper and phoned Galton to confirm about this. Yes, a distinction between open and closed intervals did appear in Galton's paper, but I found it was only used to demonstrate the corresponding problem in thinking of intervals as sets of points, and hence as the reason for him (and for James and Pat?) to take intervals as primitive. Anyway, this issue doesn't really affect this discussion.

Pat wrote

> > Some advantages of this time theory are: (1) It retains Allen's appealing characteristics of treating intervals as primitive which overcomes the Dividing Instant Problem.
>
> *See above. But in any case this doesn't overcome the problem. Allen's treatment allows lights to just come on, but it doesnt provide anywhere for the ball to be motionless.*

So, that is why we need to extend Allen and Hayes' treatment by allowing time points as primitive as well. On the one hand, treating time intervals as primitive avoids the question of whether intervals are open or closed; on the other hand, allowing time points (as primitive) provides means for expressing instantaneous phenomena such as "the ball is motionless at a point".

Pat wrote

> (2) It includes time points into the temporal ontology and therefore makes it possible to express some instantaneous phenomenon, and adequate and convenient for reasoning correctly about continuous change. (3) It is so basic that it can be specified in various ways to subsume others. For instance, one may simply take the set of points as empty to get Allen's interval time theory, or specify each interval, say T, as $\langle$ *T-left*, *T-right*$\rangle$ where *T-left* $\leq$ *T-right*, to get that one Pat prefers.

> *Not quite right. In my simple theory, T-left isnt before T-right, it equals it.*

But, the relation *T-left* $\leq$ *T-right* does already include *T-left* $=$ *T-right*, that is *T-left* equals *T-right*. In the case *T-left* $<$ *T-right*, T is an interval; in the case *T-left* $=$ *T-right*, T is a pointlike interval.

Pat wrote

> *Yes, exactly, although there is no need to use this formal strategy, as I explain in the time catalog section 1. Briefly,* ⌜Holds-In P i⌝ *is true just when i is a subinterval of a reference interval j where* ⌜Holds-On P j⌝.
>
> *Again, it is largely an aesthetic judgement, but I find Galton's hold-on vs. hold-in distinction awkward and unintuitive. (It suggests that there are two different 'ways to be true'.)*

Yes. And in Ma et al's paper [j-cj-37-114], some examples are given to show the problem with Galton's distinction between *hold-on* and *hold-in*. Also, it is claimed that the fundamental reason for the problem is that Galton wanted to characterise the fact that a proposition holds for an interval in terms of that the proposition holds for every point **within** the interval.

Pat wrote

> *Well, it depends on what axioms one assumes! Perhaps I have been speaking too carelessly about the 'usual mathematical way'. Heres my intuition: the standard account of the continuum seems* forced *to resolve the dividing point problem by deciding which interval contains the point, distinguishing open from closed intervals, because it* identifies *an interval with a set of points. (So if both intervals 'contain' the point, the intervals must intersect.) One can take points as basic or intervals as basic or both as primitive; that's irrelevant, but the crucial step is that (set-of-points = interval) identification. Thats exactly what I want*

to avoid. My point is only that if we abandon that idea (which is only needed for the formal development of analysis within set theory, a rather arcane matter for us), then there is a way to formalise time (using both intervals and points as primitive, if you like) which neatly avoids the problem.

It is **also** exactly what the approach of treating both intervals and points as primitive wants to avoid - it avoids to "identify" an interval with a set of points. **In addition**, such an approach allows expressions of all the three cases shown in my former discussion (Newsletter ENRAC 13.3 (98027)), without thinking of intervals and points in the usual mathematical way, or "identify an interval with a set of points"

Pat wrote

> In the time theory where both intervals and points are taken as primitive, we can (if we like) talk about the open and closed nature of intervals with some **knowledge** being available. This kind knowledge can be given in terms of the Meets relation, rather than some "much more complicated extension which includes set theory and an extensionality axiom for intervals". In fact, we can define that:
>
> - interval I is left-open at point P iff $Meets(P, I)$
> - interval I is right-open at point P iff $Meets(I, P)$
> - interval I is left-closed at point P iff there is an interval I' such that $Meets(I', I) \wedge Meets(I', P)$
> - interval I is right-closed at point P iff there is an interval I' such that $Meets(I, I') \wedge Meets(P, I')$...

That certainly seems to be an elegant device. (Though the definitions have nothing to do with knowledge; all Jixin is saying is that the definitions of open and closed can be given in terms of MEETS. As Allen and I showed in our old paper, the entire theory can be reduced to MEETS.) However, in order to be nontrivial, it must be that points 'separate' meetings, ie if $meets(I, P)$ *and* $meets(P, J)$ *then* $\neg meets(I, J)$*, right? For if not, all left-open intervals are also left-closed, etc. This seems to make 'points' similar to our old 'moments': in fact, if Jixin's theory predicts* $meets(P, Q) \rightarrow P = Q$ *for points* P *and* Q*, then I'll lay odds it is isomorphic to our moments theory. One of the main observations in our paper was that with the no-meets axiom, one can map moments to points with no change to the theorems provable.*

On the other hand, if the theory allows distinct points to MEET, I'd be interested to know how it is able to map smoothly to a con-

ventional account of the continuum, since that is provably impossible. One-point closed intervals exist everywhere on the real line, but no two of them are adjacent. Atomic adjacent times (whatever we call them) are pretty much a definition of discrete time models, and are incompatible with density, let alone continuity.

Yes. If $Meets(I, P) \wedge Meets(P, J)$ then $\neg Meets(I, J)$. Actually, in our theory, If $Meets(I, P) \wedge Meets(P, J)$ then $Before(I, J)$, where $Before(I, J) \rightarrow \neg Meets(I, J)$, since all the 13 relations are exclusive to each other. Therefore, it is impossible for an interval to be both left-open and left-closed.

Also, in our theory, $Meets(P, Q)$ implies that at least one of P and Q is an interval (or a moment) - they cannot be both points.

In addition, it is important to note that the constraint that a point cannot meet another point makes it is possible to establish a consistency checker for temporal database systems (see Knight and Ma's 1992 paper [j-aicom-5-75]).

Yes. As claimed earlier in this dicussion and actually pointed out in our published paper, our theory is in fact an extension to that of Allen and Hayes. Our points are quite like Allen and Hayes' moments - they cannot meet each other. However, on the one hand, points are fundamentally different from moments - points have no duration while moments do have, no matter how small they are. Therefore, it is more convenient to use points than moments in modelling some instantaneous phenomenon, especially in the case where duration reasoning is involved. On the other hand, if moments are simply mapped to points, how to express the **real** moments, i.e., non-decomposable intervals with positives duration (like the "seconds" example given in my last discussion)?

Jixin

References:

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