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Autonomous Helicopter Control Using Linguistic and Model-Based Fuzzy Control

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Abstract

The work reported in this paper is aimed at designing a horizontal velocity controller for the unmanned helicopter APID MK-III by Scandicraft AB in Sweden. The controller is able of regulating high horizontal velocities via stabilization of the attitude angles within much larger ranges than currently available. We use a novel approach to the design consisting of two steps: first, a Mamdani-type of a fuzzy rules are used to compute for each desired horizontal velocity the corresponding desired values for the attitude angles and the main rotor collective pitch; second, using a nonlinear model of the altitude and attitude dynamics, a Takagi-Sugeno controller is used to regulate the attitude angles so that the helicopter achieves its desired horizontal velocities at a desired altitude. According to our knowledge this is the first time when a combination of linguistic and model-based fuzzy control is used for the control of a complicated plant such as an autonomous helicopter. The performance of the combined linguistic/model-based controllers is evaluated in simulation and shows that the proposed design method achieves its intended purpose.

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I. Introduction

The Wallenberg Laboratory for Information Technology and Autonomous Systems (WITAS) at Linköping University is involved in the development of a command and control system, supporting the operation of an autonomous helicopter. One platform of choice is the APID MK-III unmanned helicopter, by Scandicraft Systems AB (www.scandicraft.se). The WITAS operational environment is over widely varying geographical terrain with traffic networks and vehicle interaction of variable complexity, speed and density. APID MK-III is capable of autonomous take-off, landing, and hovering as well as of autonomously executing pre-defined, point-to-point flight executed at low-speed. The latter is insufficient since for the above operational environment much higher speed is desired. Thus, our goal is to achieve high-speed motion through stable "aggressive" manoeuvrability at the level of attitude control (pitch, roll, and yaw) and test it on the APID MK-III simulation environment.

Here we present a novel design method for horizontal velocity control based on the integration of a linguistic, Mamdani-type of fuzzy controller, and a model-based Takagi-Sugeno (TS) controller. The integrated controller achieves stabilization within much larger attitude angles and horizontal velocities than the ones used in APID MK-III. The approach, as shown in simulation, enables high-speed horizontal motion (in the range of $[-36, 100]$ km/h for backward/forward motion and $[-36, 36]$ km/h for sideward motion) and altitude stabilization. The ranges for the attitude angles that allow us to achieve this are within the intervals $-\pi/4 \leq \phi, \theta \leq \pi/4, -\pi \leq \psi \leq \pi$. The design of the integrated controller proceeds as follows:

1. given desired horizontal velocity at certain altitude, a set of Mamdani-type of linguistic rules computes desired attitude angles that help achieve this desired velocity at the given altitude. The rules are heuristic in nature and reflect the experience of a human "pilot" who is an expert in remotely controling the vehicle;
2. on the basis of Takagi-Sugeno (TS) model for the dynamics of both vertical motion and attitude angles, a TS control laws that achieve the desired attitude angles at the given altitude are designed.

In Sect. 2 we introduce the model of APID MK-III used for attitude/altitude control and the basic underlying assumptions used in its derivation. In Sect. 3 we present the synthesis approach to the design and analysis of the attitude/altitude TS controller. In Sect. 4 we describe the linguistic Mamdani-type of rules used in the derivation of desired set-points for the attitude angles and discuss the intuitions behind them. In Sect. 5 we provide results from simulation that illustrate the performance of the combined Mamdani and TS controllers. Section 6 presents conclusions and directions for future work.

II. The APID MK-III model

The mathematical model used for attitude and altitude control of APID MK-III is of the form (for details see [2]:

$$
\begin{aligned}
\ddot{z} &= \frac{1}{m}(F_D + F_g - k\omega^2\theta_0 c\phi c\theta), \\
\ddot{\phi} &= -a\dot{\phi} + dk\omega^2\phi_c\theta_0, \\
\ddot{\theta} &= -b\dot{\theta} - ek\omega^2\theta_c\theta_0, \\
\ddot{\psi} &= -c\dot{\psi} + f(\alpha_{tr} - \psi_0),
\end{aligned} \tag{1}
$$

where $c\phi = cos\phi$ and $s\phi = sin\phi$. The state vector is $(x, y, z, \phi, \theta, \psi, \dot{x}, \dot{y}, \dot{z}, \dot{\phi}, \dot{\theta}, \dot{\psi})$, i.e., positions, attitude angles and their respective velocities. The control inputs, are $(\phi_c, \theta_c, \theta_0, \alpha_{tr})$, i.e., these are the usual controls in terms of lateral and longitudinal cyclic pitches, collective pitches for the main and tail rotors. The first three equations describe the dynamics of translational motion in the inertial frame where $F_\circ$ are wind forces in north, east, and vertical directions; m is the helicopter body mass, F_g is the gravity force acting on the cabin, and ω is the main rotor RPM . The last three equations describe the dynamics of the rotational motion in the body frame. The coeficients $a = 38.7072$, $b = 10.1815$, $c = 0.434$ are derived from the expression of moment equations leading to the attitude angles' equations ($\psi_0 = 0.09$ is an offset term).

The assumptions underlying the above model are: (i) the variation of the rotor speed ω is constant as a consequence of maintaining constant throttle control at the nominal part of the power curve – the constant value of ω is implicit in the gain $k\omega^2 = 1703.46$, and (ii) the variations of the main rotor angles are small enough so that the magnitude of the main rotor force can be considered equal to the thrust force.

The uncertainty or unmodeled dynamics of the above model can be categorized as follows: (i) unmodeled aerodynamics – only the wind action, e.g., F_N, F_E, F_D on the body is considered, and the action of the tail rotor force on the angular accelerations is neglected; (ii) higher order dynamics such as rotor flapping dynamics is not considered at all, while the usually highly nonlinear link between the control inputs and servos of the main and tail rotors and governing equations are linearized and are implicit in the constant gains $k\omega^2 = 1703.4$, $dk\omega^2 = 223.5824$, $ek\omega^2 = 58.3258$, and $f = 31.9065$; and (iii) servo actuators are linked to the control inputs and are modeled by first-order transfer functions of the form $\dot{\delta} = -300\delta + 300u$ where u is any one of the control inputs and δ is a pseudo state variable.

The current control system for APID MK-III does not utilize large ranges of the rotor attitude angles. As a consequence this produces lower rate-of-change of the attitude angles ϕ, θ and ψ, and consequently the control is done on rather small ranges for these – all this reduces manoeuvrability w.r.t. these angles and consequently the speed of motion. In this context, the objective of our study is to design a horizontal velocity controller which acts on much larger ranges of the attitude angles, i.e., $-\pi/4 \leq \phi \leq +\pi/4, -\pi/4 \leq \theta \leq +\pi/4, -\pi \leq \psi \leq +\pi$, by utilizing the full range of the rotor attitude angles. The latter, for the purpose of this study, are in the interval $[-0.7, +0.7]$ rad.

III. Takagi-Sugeno controller

First, the nonlinearities in the control inputs of the nonlinear model from Sect. 2 are decoupled by adding first-order actuator transfer functions – as a result, these nonlinearities are moved into the state. The transformed model is then given as:

$$
\begin{aligned}
\dot{x}_1 &= x_5, \\
\dot{x}_2 &= x_6, \\
\dot{x}_3 &= x_7, \\
\dot{x}_4 &= x_8, \\
\dot{x}_5 &= \frac{1}{m}(F_D + F_g - 1703.4\cos(x_6)\cos(x_7)x_{11}), \\
\dot{x}_6 &= -38.7072\, x_6 + 223.5824\, x_9 x_{11}, \\
\dot{x}_7 &= -10.1815\, x_7 - 58.3258\, x_{10} x_{11}, \\
\dot{x}_8 &= -0.434\, x_8 + 31.9065\, x_{12} + 0.09, \\
\dot{x}_9 &= -300\, x_9 + 300\, u_1, \\
\dot{x}_{10} &= -300\, x_{10} + 300\, u_2, \\
\dot{x}_{11} &= -300\, x_{11} + 300\, u_3, \\
\dot{x}_{12} &= -300\, x_{12} + 300\, u_4,
\end{aligned} \tag{2}
$$

where x_1 and x_5 are the altitude and its velocity; $x_2, \ldots, x_4$ are the attitude angles, and $x_6, \ldots, x_8$ are their angular rates; $x_9, \ldots, x_{12}$ are the servo states. Furthermore, u_1, u_2, u_3, and u_4 are the commanded cyclic pitch and roll together with the main and tail rotor collective pitches. Note that ϕ_c, θ_c, θ_0 and α_{tr} now are state variables.

The above model is transformed into a TS model by a novel technique called *linear bounding* which not only approximates this nonlinear model exactly, but also drastically reduces the number of linear sub-models that constitute the TS model. In what follows we describe briefly this step of the design.

Consider again the model described in (2). The nonlinear terms to be linearized, so that the fuzzy system represents exactly the nonlinear system (2), are $x_9 x_{11}$ and $x_{10} x_{11}$ to be the terms to linearize in the attitude equations ($\dot{x}_6$ and $\dot{x}_7$), and $\cos(x_6)\cos(x_7)x_{11}$ to be linearized in the $\dot{x}_5$ equation. The state variables involved in these terms satisfy

$$
\begin{aligned}
x_6, x_7 &\in [-\pi/4,\ \pi/4], \\
x_{11} &\in [\pi/18,\ 5\pi/18].
\end{aligned} \tag{3}
$$

Furthermore, x_{11} is trivially bounded by

$$.1745 < x_{11} < .8727. \tag{4}$$

$\cos(x_6)\cos(x_7)$, taking into account the bounds from (3), can be bounded by two constant functions:

$$0.7071 < \cos(x_6) < 1\ , \quad 0.7071 < \cos(x_7) < 1, \tag{5}$$

which gives

$$0.5 < \cos(x_6)\cos(x_7) < 1. \tag{6}$$

Now the three nonlinear terms can be fully described by the upper and lower bounds derived above in the following manner:

$$
\begin{aligned}
x_9 x_{11} &= F_1^1 0.8727 x_9 + F_1^2 0.1745 x_9 \\
x_{10} x_{11} &= F_1^1 0.8727 x_{10} + F_1^2 0.1745 x_{10} \\
\cos(x_6)\cos(x_7)x_{11} &= F_2^1 x_{11} + F_2^2 0.5 x_{11}
\end{aligned}
$$

where F_1^1, $F_2^1 \in [01]$, $F_1^2 = 1 - F_1^1$ and $F_2^2 = 1 - F_2^1$. By solving the above equations for F_1^1, F_1^2, F_2^1 and F_2^2 we obtain the following membership functions

$$\begin{aligned} F_1^1 &= (x_{11} - 0.1745)/0.6981 && (7)\\ F_1^2 &= (0.8727 - x_{11})/0.6981 && (8)\\ F_2^1 &= 2\cos(x_6)\cos(x_7) - 1 && (9)\\ F_2^2 &= 2 - 2\cos(x_6)\cos(x_7) && (10) \end{aligned}$$

The graphs of the membership functions F_1^1 and F_1^2 are shown in Fig. 1 and the graphs of F_2^1 and F_2^2 in Fig 2.

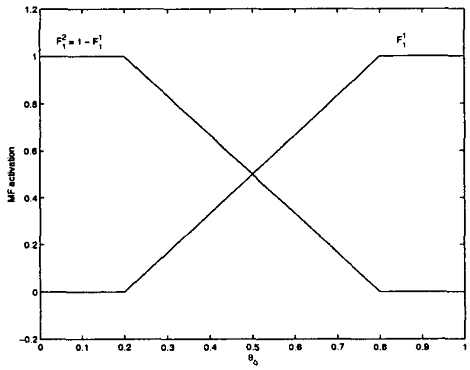

Fig. 1. Membership functions for x_{11}.

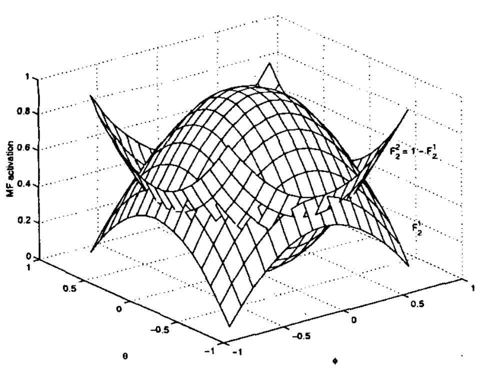

Fig. 2. Membership functions for $[x_6, x_7]$.

The TS rule base is then expressed as follows:

1 : IF x_{11} IS F_1^1 and $[x_6, x_7]$ IS F_2^1
THEN $\dot{x} = A_1 x + Bu$

2 : IF x_{11} IS F_1^1 and $[x_6, x_7]$ IS F_2^2
THEN $\dot{x} = A_2 x + Bu$

3 : IF x_{11} IS F_1^2 and $[x_6, x_7]$ IS F_2^1
THEN $\dot{x} = A_3 x + Bu$

4 : IF x_{11} IS F_1^2 and $[x_6, x_7]$ IS F_2^2
THEN $\dot{x} = A_4 x + Bu$

In the above rules the matrix A_1 is the Jacobian obtained by Taylor series expansion of (2) for values of x_6, x_7, and x_{11} such that $F_1^1(x_{11}) = 1$, and $F_2^1(x_6, x_7) = 1$. The rest of A_2, A_3, and A_4 are obtained in the same manner. The B matrix is identical for all rules and contains the gains for the servo actuators connected to the control inputs. The global model is then represented as:

$$\dot{x} = \sum_{i=1}^{4} w_i(x_6, x_7, x_{11})(A_i x + Bu), \qquad (11)$$

where w_i is the total activation for each rule: $w_1 = F_1^1 \cdot F_2^1$, $w_2 = F_1^1 \cdot F_2^2$, $w_3 = F_1^2 \cdot F_2^1$ and $w_4 = F_1^2 \cdot F_2^2$, with $\sum_{i=1}^{4} w_i = 1$. Given the TS fuzzy model, we obtain a fuzzy gain scheduled dynamic output feedback $\mathcal{H}_\infty$ controller of the form:

$$\begin{bmatrix} \dot{x}_c \\ u \end{bmatrix} = \sum_{i=1}^{4} w_i \begin{bmatrix} A_c^i & B_c^i \\ C_c^i & D_c^i \end{bmatrix} \begin{bmatrix} x_c \\ y \end{bmatrix} \qquad (12)$$

using the results in [1] for self-scheduled output feedback controllers. Although this technique only is applicable to linear subsystems we have extended it to affine linear subsystems, see [3] for more details. The controller was designed to track desired values in altitude and attitude angles. Integral action was introduced to avoid steady state errors. The actuator states are limited to a certain range and this is accounted for in the controller design. The outputs from the system which are fed into the controller are taken to be $x_1, \ldots, x_8$. The servo state x_{11} must of course also be measured because of its use in the scheduling.

IV. The Mamdani-type controller

As already mentioned, the Mamdani-type of linguistic controller is used to generate desired values for attitude angles given desired horizontal velocities at a given altitude. This type of controller has a heuristic nature which reflects the experience of a human "pilot" who is an expert in remotely controling the vehicle. The motivation for resorting to such a heuristic approach is as follows:

- The available equations describing the dynamics of horizontal motion do not take into account aerodynamic effects related to the main rotor.
- Also the contributions of the tail rotor torque and force are neglected.

Thus using these equations to derive desired attitude angles, given desired horizontal velocities, is not a reliable option. Instead, the Mamdani-type of linguistic controller uses the magnitude of acceleration and velocity-error to infer attitude angles that if achieved will reduce the velocity error to zero. Thus they "mimic " a human "pilot's" behavior when trying to achieve certain desired velocities via remote control.

In this context, the rules used to compute desired values for pitch are of the form:

IF e_{v_x} *is Neg and* $\dot{e}_{v_x}$ *is Neg* **THEN** *desired pitch is Pos,*

where e_{v_x} is the longitudinal velocity-error and $\dot{e}_{v_x}$ is the longitudinal acceleration. The "heuristic" interpretation of this particular rule is as follows: if the current longitudinal velocity is higher than the desired one and we are accelerating, i.e., we are moving further away from the desired velocity which is caused by a negative pitch angle. In order to bring the current velocity back to the desired one we have to slow down the longitudinal motion and reverse the acceleration. This is done by bringing the pitch from a negative to a positive angle. Furthermore, Neg and Pos are linguistic labels for the magnitudes of e_{v_x}, $\dot{e}_{v_x}$, and the pitch. The meaning of these linguistic labels is given by fuzzy sets defined on the physical domains of e_{v_x}, $\dot{e}_{v_x}$, and the pitch. Figure 3 illustrates the above rule in terms of these membership functions. All in all there are 9 rules describing the relationship between e_{v_x}, $\dot{e}_{v_x}$ and the pitch.

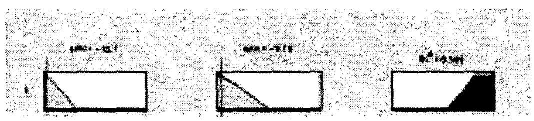

Fig. 3. Rule for longitudinal speed with membership functions.

The rules used to compute desired values for roll are of the form:

IF e_{v_y} *is Neg and* $\dot{e}_{v_y}$ *is Neg* **THEN** *desired roll is Neg,*

where e_{v_y} is the lateral velocity-error and $\dot{e}_{v_x}$ is the lateral acceleration. The "heuristic" interpretation of this particular rule is as follows: if the current lateral velocity is higher than the desired one and we are accelerating, i.e., we are moving further away from the desired velocity which is caused by a positive roll angle. In order to bring the current velocity back to the desired one we have to slow down the lateral motion and reverse the acceleration. This is done by bringing the roll from a positive to a negative angle. Furthermore, Neg and Pos are linguistic labels for the magnitudes of e_{v_y}, $\dot{e}_{v_y}$, and the roll. Figure 4 illustrates the above rule in terms of membership functions corresponding to these linguistic labels. All in all there are 9 rules describing the relationship between e_{v_y}, $\dot{e}_{v_y}$ and the roll.

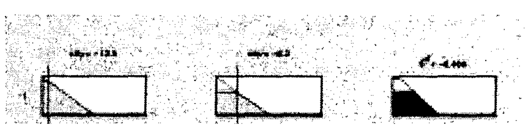

Fig. 4. Rule for lateral speed with membership functions.

The desired value for the yaw is computed by rules as:

IF e_χ *is Pos and* $\dot{e}_\chi$ *is Neg* **THEN** *desired yaw is Zero.*

where e_χ is the heading-error and $\dot{e}_\chi$ is its rate of change. The "heuristic" interpretation of this particular rule is as follows: if the current heading is higher than the desired one and we are reducing it, i.e., we are moving closer to the desired heading which is caused by certain orientation of the horizontal velocity. In this case we maintain the current yaw. Furthermore, Neg, Pos, and Zero are linguistic labels for the magnitudes of e_χ, $\dot{e}_\chi$, and the current yaw. Figure 5 illustrates the above rule in terms of membership functions corresponding to these linguistic labels. All in all there are 9 rules describing the relationship between e_χ, $\dot{e}_\chi$ and the yaw.

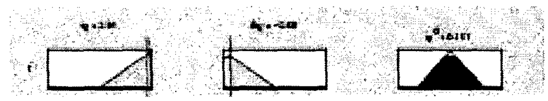

Fig. 5. Rule for heading with membership functions.

The first two types of rules neglect the cross-couplings between pitch and roll angles in the dynamics of longitudinal and lateral motions. However, these couplings are taken care by the heading rules that in addition also prevent side-slip by restricting the yaw to be always equal to the heading. Furthermore, the pitch and roll angles affect the dynamics of vertical motion so that they cause a drop in altitude. Preventing this is taken care of at the level of the TS controller. The control scheme computing desired attitude angles given desired horizontal velocities at a given altitude is presented in Fig. 6.

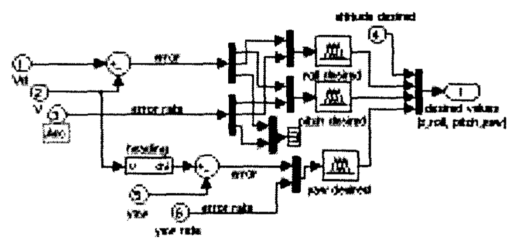

Fig. 6. The Mamdani controller.

V. Simulation results

The integration between the Mamdani and TS controllers is illustrated in Fig. 7.

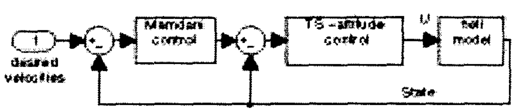

Fig. 7. The integrated controller.

The numerical experiments are performed with the controllers designed in the previous sections and acting on the nonlinear model from Sect. 3.

The first experiment, depicted in Fig. 8, shows the results from set-point regulation around a desired low and high longitudinal velocities.

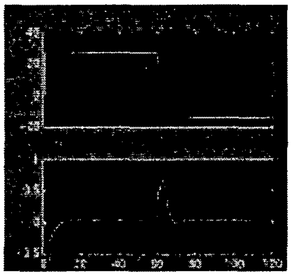

Fig. 8. Upper-part: Low and high $\dot{x}$-velocity set-point regulation. Lower-part: Corresponding desired pitch values.

The second experiment, depicted in Fig. 9, shows the results from set-point regulation around a desired low and high lateral velocities.

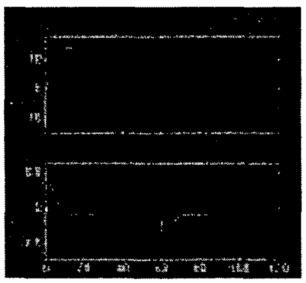

Fig. 9. Upper-part: Low and high $\dot{y}$-velocity set-point regulation. Lower-part: Corresponding desired roll values.

The last experiment, depicted in Fig. 10, shows the results from tracking a desired heading computed from desired horizontal velocities.

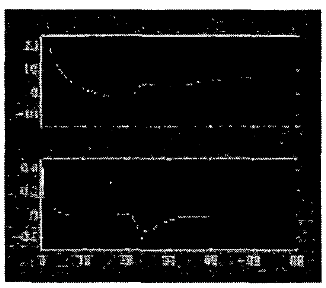

Fig. 10. Upper-part: Tracking error for velocities. Lower-part: Corresponding tracking error for heading and yaw.

VI. Conclusions

This work has shown the applicability of our approach, using a combination of linguistic and model-based fuzzy control of an unmanned helicopter. The performance of the controller when evaluated in simulation achieves stabilization of horizontal high-speed velocities and altitude using attitude angles within much larger ranges than the ones currently available on the APID MK-III platform.

Future work will address the use of the approach presented here for position control and for the purpose of behavior-based helicopter control.

VII. Acknowledgements

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