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Autonomous Helicopter Control Using Fuzzy Gain Scheduling

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Abstract

The work reported in this paper is aimed at achieving aggressive manoeuvrability for an unmanned helicopter APID MK-III by Scandicraft AB in Sweden. The manoeuvrability problem is treated at the level of attitude (pitch, roll, yaw) and the aim is to achieve stabilization of the attitude angles within much larger ranges than currently available. We present a novel fuzzy gain scheduling control approach based on two different types of linearization of the original nonlinear APID MK-III model. The performance of the fuzzy gain scheduled controllers is evaluated in simulation and shows that they are effective means for achieving the desired robust manoeuvrability.

1 Introduction

The overall objective of the Wallenberg Laboratory for Information Technology and Autonomous Systems (WITAS) at Linköping University is the development of an intelligent command and control system, containing active-vision sensors, which supports the operation of a unmanned air vehicle (UAV) in both semi- and full-autonomy modes. One of the UAV platforms of choice is the APID MK-III unmanned helicopter, by Scandicraft Systems AB (www.scandicraft.se). The intended operational environment is over widely varying geographical terrain with traffic networks and vehicle interaction of variable complexity, speed, and density. The present version of APID MK-III is capable of autonomous take-off, landing, and hovering as well as of autonomously executing pre-defined, point-to-point flight pattern. This is enough for performing missions like site mapping and surveillance, and electronic warfare and communications, but for the above mentioned operational environment a higher degree of manoeuvrability is desired. In this context, our goal is to explore the possibilities for achieving robust, "aggressive" manoeuvrability at the level of attitude control (pitch, roll, and yaw) and test a variety of control solutions in the APID MK-III simulation environment.

In this work we present attitude nonlinear controllers whose design and analysis are based on a fuzzy gain scheduling approach (FGS) using a mathematical model of APID MK-III. Both type of controllers achieve stabilization within much larger ranges for the attitude angles than currently available and their performance is evaluated in simulation. The design of gain scheduled controllers has, for a very long time, followed a two-step approach: first, the nonlinear system under control is linearized at a number of different operating points – normally, equilibrium points in the state space – and then linear controllers are designed for each operating point; second, a gain scheduler is designed by usually, an ad-hoc interpolation of the already designed linear controllers. Stability and robustness of the closed loop system are then evaluated through extensive simulation. In contrast, FGS is a one-step approach (see [3, 4]) – simultaneous synthesis of linear controllers and a gain scheduler with guaranteed global stability and robustness properties, avoiding the need for linearization at equilibrium points. The FGS approach, as shown in simulation, shows to be an effective control strategy enabling a robust manoeuvrability.

In Sect. 2 we introduce the model of APID MK-III used for attitude control and the basic underlying assumptions used in its derivation. We also point out the differences between this model and the ones used by the Berkeley AeRobot team (BEAR) and the Georgia Tech ASRT system. In Sect. 3 we present the FGS approach and apply it to the design and analysis of an attitude controller. In Sect. 4 we provide results from simulation that illustrate the performance of the proposed FGS controllers in terms of "extreme" roll, pitch, and yaw manoeuvres. Section 5 presents conclusions and directions for future work.

2 The APID MK-III model

The mathematical model used for the attitude control of APID MK-III, defined in the body frame (Eqs. 2-4) and inertial frame (Eq. 1), is of the form:

$$\ddot{z} = \frac{1}{m}(F_D + F_g - 1703.4\cos(\phi)\cos(\theta)\theta_0), \qquad (1)$$

$$\ddot{\phi} = -38.7072\dot{\phi} + 223.5824\phi_c\theta_0, \quad (2)$$
$$\ddot{\theta} = -10.1815\dot{\theta} - 58.3258\theta_c\theta_0, \quad (3)$$
$$\ddot{\psi} = -0.434\dot{\psi} + 31.9065(\alpha_{tr} + 0.09), \quad (4)$$

where the state vector is $(z, \phi, \theta, \psi, \dot{z}, \dot{\phi}, \dot{\theta}, \dot{\psi})$, i.e. altitude, attitude angles and their velocities. The control inputs are $(\phi_c, \theta_c, \theta_0, \alpha_{tr})$, i.e., these are the usual control inputs in terms of lateral and longitudinal cyclic pitches, and collective pitches for the main and tail rotors. The first equation describes the dynamics of altitude motion where F_D is a wind force, and F_g is the gravity force on the cabin. It is used here for the purpose of illustrating the major disadvantage of fuzzy gain scheduling when done via Taylor series linearization. Though Eq. (1) only increases the dimension of the state vector with just two additional states, z and $\dot{z}$, this leads to a large number of linear matrix inequalities (LMIs) used in the FGS design. This large number of LMIs is impossible to deal with using the existing LMI tools.

The assumptions underlying the above model are: (i) the variation of the rotor speed ω is constant as a consequence of maintaining constant throttle control at the nominal part of the power curve – the constant value of ω is implicit in the gain 1703.4; and (ii) the variations of the main rotor angles are small enough so that the magnitude of the main rotor force can be considered equal to the thrust force.

The uncertainty or unmodeled dynamics of the above model can be categorized as follows: (i) unmodeled aerodynamics – only the wind action, e.g., F_N, F_E, F_D on the body is considered; (i) the action of the tail rotor force on the angular accelerations is neglected; (ii) higher order dynamics such as rotor flapping dynamics is not considered at all, while the usually highly nonlinear link between the control inputs and servos of the main and tail rotors and governing equations are linearized and are implicit in the constant gains 1703.4, 223.5824, 58.3258, and 31.9065; and (iii) servo actuators are linked to the control inputs and are modeled by first-order transfer functions of the form $\dot{\delta} = -300\delta + 300u$ where u is any one of the control inputs and δ is a pseudo state variable.

The current control system for APID MK-III does not utilize the full range of the rotor attitude angles. As a consequence this produces lower rate-of-change of the attitude angles ϕ, θ and ψ, and consequently the control is done on rather small ranges for these – all this reduces manoeuvrability w.r.t. these angles. In this context, the objective of our study is to design an attitude controller which acts on much larger ranges of the attitude angles, i.e., $-\pi/4 \leq \phi \leq +\pi/4, -\pi/4 \leq \theta \leq +\pi/4, -\pi \leq \psi \leq +\pi$, by utilizing the full range of the rotor attitude angles. The latter, for the purpose of this study, are in the interval $[-0.7, +0.7]$ rad.

As already mentioned above, the contribution of the tail rotor force in terms of the tail rotor torque is not reflected in Eq. (2), while in the BEAR model [1] this contribution is accounted for. With regard to the Georgia Tech ASRT model [2] the difference is that, their dynamic inversion control is based on dynamics linearized about a nominal operating point. Then a direct NN based adaptive control architecture is used to adapt to errors caused by the linearized inverted model. The FGS approach uses an approximation of the above nonlinear system by a convex nonlinear combination of linear sub-models, each obtained at a different operating point/region.

3 Controller design: The FGS approach

The FGS approach used in this work consists of the following steps:

- Decoupling the nonlinearities in the control inputs by adding first-order actuator transfer functions – as a result, the nonlinearities are moved into the state;
- The new model is linearized either using Taylor series expansion around appropriately chosen points in the state space, or by bounding the nonlinearities in the state by linear functions – in this way the nonlinear model is approximated by a Takagi-Sugeno (TS) fuzzy model, which boils down to convex combination of linear sub-models;
- A gain scheduled output feedback $\mathcal{H}_\infty$ controller for the so-obtained approximated model is designed.

In what follows we will describe in more detail the above three steps of the design. After the first step the complete model becomes as follows:

$$\begin{aligned}
\dot{x}_1 &= x_5, \\
\dot{x}_2 &= x_6, \\
\dot{x}_3 &= x_7, \\
\dot{x}_4 &= x_8, \\
\dot{x}_5 &= \frac{1}{m}(F_D + F_g - 1703.4\cos(x_6)\cos(x_7)x_{11}), \\
\dot{x}_6 &= -38.7072\,x_6 + 223.5824\,x_9x_{11}, \\
\dot{x}_7 &= -10.1815\,x_7 - 58.3258\,x_{10}x_{11}, \\
\dot{x}_8 &= -0.434\,x_8 + 31.9065\,x_{12} + 0.09, \\
\dot{x}_9 &= -300\,x_9 + 300\,u_1, \qquad (5)\\
\dot{x}_{10} &= -300\,x_{10} + 300\,u_2, \\
\dot{x}_{11} &= -300\,x_{11} + 300\,u_3, \\
\dot{x}_{12} &= -300\,x_{12} + 300\,u_4,
\end{aligned}$$

where x_1 and x_5 are the altitude and its velocity; $x_2, \ldots, x_4$ are the attitude angles, and $x_6, \ldots, x_8$ are their angular rates; $x_9, \ldots, x_{12}$ are the servo states. Furthermore, u_1, u_2, u_3, and u_4 are the commanded cyclic pitch and roll together with the main and tail rotor collective pitches. Note that ϕ_c, θ_c, θ_0 and α_{tr} now are state variables.

At the next step there are two possible ways to represent the above model as a fuzzy system: Taylor series expansion in the fuzzy state space, and bounding the nonlinearities in the state by appropriate linear functions (linear bounding).

3.1 Taylor series linearization

This method is applied only to the attitude-part of the model, i.e. $\dot{x}_1$ and $\dot{x}_5$ are excluded. Here the nonlinearities are confined to the servo state variables x_9, x_{10} and x_{11} First we choose representative points in the domains of these variables – these points represent the centers for the fuzzy set partitioning of these domains. A combination of particular membership functions – one membership function for each variable – defines a fuzzy region. In this case, having three membership functions for each variable and three input variables we obtain 27 fuzzy regions. The center of a given fuzzy region is represented by the centers of the membership functions that define it, and this center is not an equilibrium point. Then a Taylor series expansion is performed at the center of each fuzzy region. For example, for the fuzzy region defined by $x_9 = Small$, $x_{10} = Large$ and $x_{11} = Small$, the Taylor series expansion at its center produces an affine linear subsystem of the form:

IF x_9 is $Small$ and x_{10} is $Large$ and x_{11} is $Small$
THEN $\dot{x} = A_i x + Bu + d_i$.

The dynamics of the overall fuzzy system – with 27 rules in the form from above – is then given as:

$$\dot{x} = \sum_{i=1}^{27} w_i(x)(A_i x + Bu + d_i), \tag{6}$$

where $\sum_{i=1}^{27} w_i(x) = 1$ and $w_i(x)$ are weights computed from the membership functions in the IF-part of the rules given particular values of the variables x_9, x_{10}, and x_{11}.

The design is performed using the methodology for self-scheduled output feedback controllers proposed in [5]. Although this technique only is applicable to linear subsystems we have extended it to affine linear subsystems, see [4] for details.

The drawback of the Taylor series expansion approach is the combinatorial explosion in the number of fuzzy regions. In the case of considering also equations $\dot{x}_1$ and $\dot{x}_5$, the number of fuzzy regions (two additional state variables) is raised to 243, which is not tractable with the available LMI tools. For this reason we will consider next a type of linearization that reduces drastically the number of fuzzy regions and thus the number of affine linear subsystems.

3.2 Linear-bounding of nonlinearities

Consider again the model described in (5). For the nonlinear terms in this model we choose a linear bounding such that the fuzzy system obtained represents exactly the nonlinear system (5).

We consider $x_9 x_{11}$, $x_{10} x_{11}$, and $\cos(x_6)\cos(x_7)x_{11}$, to be the nonlinear terms subject to linear bounding – these reside in the attitude equations ($\dot{x}_6$ and $\dot{x}_7$), and $\dot{x}_5$ respectively. The state variables involved in these nonlinear terms satisfy:

$$\begin{aligned} x_6, x_7 &\in [-\pi/4,\ \pi/4], \\ x_{11} &\in [\pi/18,\ 5\pi/18]. \end{aligned} \tag{7}$$

The state variable x_{11} is trivially bounded by

$$.1745 < x_{11} < .8727. \tag{8}$$

$\cos(x_6)$ and $\cos(x_7)$, taking into account the bounds from (7), can be bounded by the two constant functions:

$$0.7071 < \cos(x_6) < 1\ , \quad 0.7071 < \cos(x_7) < 1. \tag{9}$$

The above bounds result in

$$0.5 < \cos(x_6)\cos(x_7) < 1. \tag{10}$$

Then the above three nonlinear terms can be represented via the use of the derived upper and lower bounds in the following manner:

$$\begin{aligned} x_9 x_{11} &= F_1^1 0.8727 x_9 + F_1^2 0.1745 x_9, \\ x_{10} x_{11} &= F_1^1 0.8727 x_{10} + F_1^2 0.1745 x_{10}, \\ \cos(x_6)\cos(x_7)x_{11} &= F_2^1 x_{11} + F_2^2 0.5 x_{11}, \end{aligned}$$

where F_1^1, $F_2^1 \in [0,1]$, $F_1^2 = 1 - F_1^1$ and $F_2^2 = 1 - F_2^1$. By solving the above equations for F_1^1, F_1^2, F_2^1 and F_2^2, we obtain the following membership functions:

$$F_1^1(x_{11}) = (x_{11} - 0.1745)/0.6981, \tag{11}$$
$$F_1^2(x_{11}) = (0.8727 - x_{11})/0.6981, \tag{12}$$
$$F_2^1(x_6, x_7) = 2\cos(x_6)\cos(x_7) - 1, \tag{13}$$
$$F_2^2(x_6, x_7) = 2 - 2\cos(x_6)\cos(x_7). \tag{14}$$

The graphs of the membership functions F_1^1 and F_1^2 are shown in Fig. 1, and the graphs of F_2^1 and F_2^2 are shown in Fig. 2. The fuzzy model is then expressed

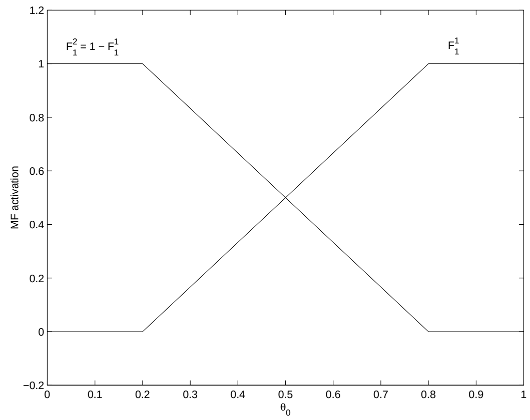

Figure 1: Membership functions x_{11}.

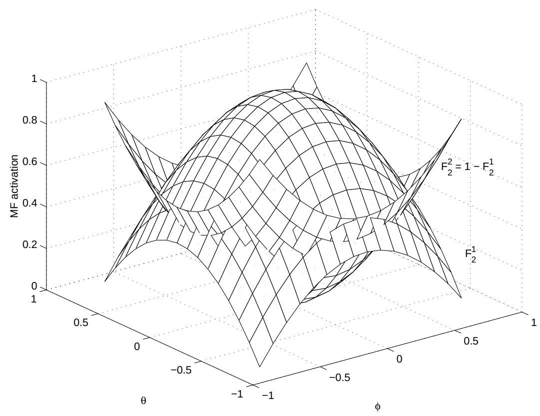

Figure 2: Membership functions for $[x_6, x_7]$.

as the following set of only four rules:

$$\begin{aligned} 1 &: \text{IF } x_{11} \text{ is } F_1^1 \text{ and } [x_6, x_7] \text{ is } F_2^1 \\ &\quad \text{THEN} \quad \dot{x} = A_1 x + Bu, \\ 2 &: \text{IF } x_{11} \text{ is } F_1^1 \text{ and } [x_6, x_7] \text{ is } F_2^2 \\ &\quad \text{THEN} \quad \dot{x} = A_2 x + Bu, \\ 3 &: \text{IF } x_{11} \text{ is } F_1^2 \text{ and } [x_6, x_7] \text{ is } F_2^1 \\ &\quad \text{THEN} \quad \dot{x} = A_3 x + Bu, \\ 4 &: \text{IF } x_{11} \text{ IS } F_1^2 \text{ and } [x_6, x_7] \text{ is } F_2^2 \\ &\quad \text{THEN} \quad \dot{x} = A_4 x + Bu. \end{aligned}$$

In the above rules the matrix A_1 is the Jacobian obtained by Taylor series expansion of (5) for values of x_6, x_7, and x_{11} such that $F_1^1(x_{11}) = 1$, and $F_2^1(x_6, x_7) = 1$. The rest of A_2, A_3, and A_4 are obtained in the same manner. The B matrix is identical for all rules and contains the gains for the servo actuators connected to the control inputs.

The global model is then represented as:

$$\dot{x} = \sum_{i=1}^{4} w_i(x_6, x_7, x_{11})(A_i x + Bu). \tag{15}$$

In the above, w_i is the degree to which a rule is activated given some values for x_6, x_7,and x_{11}. Furthermore, $w_1 = F_1^1(x_{11}) \cdot F_2^1(x_6, x_7)$, $w_2 = F_1^1(x_{11}) \cdot F_2^2(x_6, x_7)$, $w_3 = F_1^2(x_{11}) \cdot F_2^1(x_6, x_7)$ and $w_4 = F_1^2(x_{11}) \cdot F_2^2(x_6, x_7)$, with $\sum_{i=1}^{4} w_i = 1$.

Given the TS fuzzy model, we obtain a fuzzy gain scheduled dynamic output feedback $\mathcal{H}_\infty$ controller of the form:

$$\begin{bmatrix} \dot{x}_c \\ u \end{bmatrix} = \sum_{i=1}^{4} w_i \begin{bmatrix} A_c^i & B_c^i \\ C_c^i & D_c^i \end{bmatrix} \begin{bmatrix} x_c \\ y \end{bmatrix}, \tag{16}$$

using the results in [5]. The controller was designed to track desired values in altitude and attitude angles. Integral action was introduced to avoid steady state errors. The actuator states are limited to a certain range and this is accounted for in the controller design. The outputs from the system that are fed into the controller are taken to be $x_1, \ldots, x_8$. The servo state x_{11} must of course also be measured because of its use in the scheduling.

4 Simulation results

The numerical experiment is performed with the fuzzy gain scheduler designed in Sect. 3.2. The experiment shows simultaneously altitude trajectory following and attitude set-point tracking at the limits of the attitude angles – Figures 3-6. Fig. 7 depicts the control signals. The diagrams show a time delay of approximately 1 sec. between the reference and the response for the altitude (z). The settling time for the attitude angles are respectively 6 sec. for the pitch and the roll, and 3 sec. for the yaw.

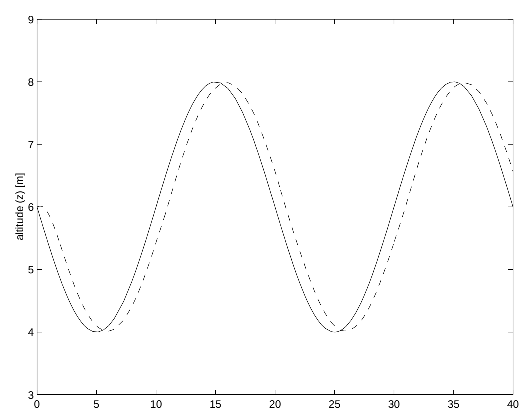

Figure 3: Altitude trajectory following, desired - solid and actual - dashed.

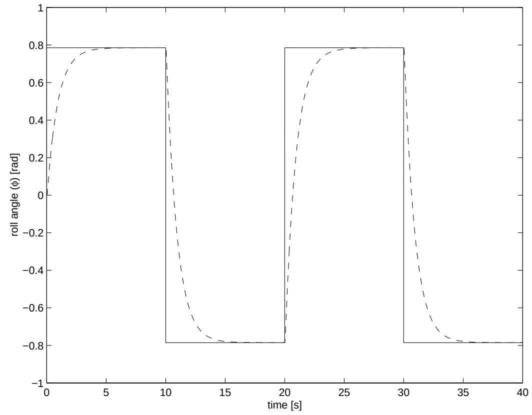

Figure 4: Roll set-point tracking, desired - solid and actual - dashed.

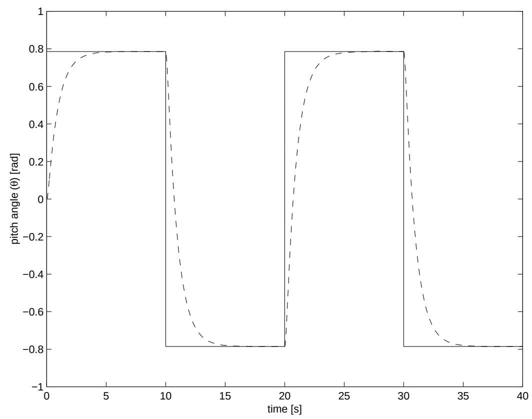

Figure 5: Pitch set-point tracking, desired - solid and actual - dashed.

5 Conclusions

This work has shown the applicability of FGS, using linear-bounding type of linearization, to the altitude and attitude control of an unmanned helicopter. The performance of the fuzzy gain scheduled controller when evaluated in simulation achieves stabilization of the attitude angles within much larger ranges than the ones currently available on the APID MK-III platform.

Future work will address the use of FGS presented here for robust position/velocity control and trajectory tracking.

6 Acknowledgements

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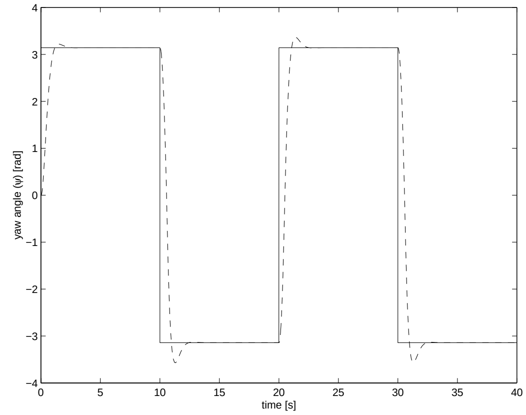

Figure 6: Yaw set-point tracking, desired - solid and actual - dashed.

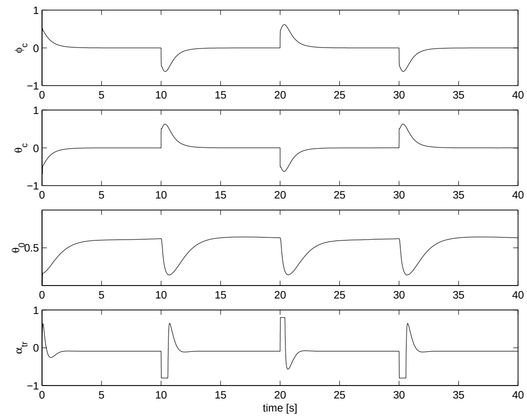

Figure 7: Control signals, u_1 - u_4, top to bottom respectively.

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