On Mutual Understanding among Communicating Agents*

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Abstract. In this paper, we propose a framework that provides software agents with the ability to ask *approximate* questions to each other in the context of heterogeneous ontologies. The framework combines the use of logic-based techniques with ideas from rough set theory. Initial queries by an agent are transformed into approximate queries using weakest sufficient and strongest necessary conditions on the query and are interpreted as lower and upper approximations on the query. Once the base communication ability is provided, the framework is extended to situations where there is not only a mismatch between agent ontologies, but the agents have varying ability to perceive their environments. These limitations on perceptive capability are formalized using the idea of tolerance spaces.

1 Introduction

With the inception of the World-Wide Web (WWW), a distributed information infrastructure has been set up containing a vast repository of information resources. This infrastructure is designed primarily for human use, with little support for the deployment of software agents which can take advantage of these information resources to assist humans in accomplishing a number of different information processing and gathering tasks. The next stage in the evolution of the the WWW is to enhance the current infrastructure with support for explicit, machine accessible descriptions of information content on the Web. These machine accessible descriptions of information content should be usable and understandable by machines, in particular software agents. Tim Berners-Lee has used the term *Semantic Web* – a web of data that can be processed directly or indirectly by machines[3], to describe this next phase in the evolution of the Web.

The meaning or semantics of diverse information content has to be accessible to software agents for use and reasoning if sophisticated knowledge intensive tasks are to be automated in the Web context. Most importantly, just as humans cooperate and communicate in a common language and conceptual space in order to achieve complex tasks,

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so will software agents, both with other software agents and with humans. There is a great deal of research activity in this area, particularly in providing the necessary tools to support communication among agents and construction and use of shared *ontologies*.

Webster's dictionary defines *ontology* as,

> "the branch of metaphysics dealing with the nature of being or reality".

In artificial intelligence, more specifically knowledge representation, the term is used somewhat more pragmatically to describe how we choose to "slice up" reality and represent these choices in representational structures used for reasoning about agent environments at various levels of abstraction. One common way of "slicing" or conceptualizing is to specify a base set of individuals, properties, relations and dependencies between them. This choice is particularly amenable to a straightforward use of logic as a representational tool.

In knowledge engineering circles, the term "ontology" has taken on a related but somewhat different meaning as explicit computational structures which "provide a machine-processible semantics of information sources that can be communicated between different agents (software and human)"[9]. Ontologies are used to facilitate the construction of domain models and to provide the vocabulary and relations used in this process. Gruber [11] provides the following definition:

> "An ontology is a formal, explicit specification of a shared conceptualization."

The intention is that ontologies should facilitate the use of knowledge sharing and reuse among software agents and humans alike.

Just as the Web is currently a heterogeneous collection of information sources, it is reasonable to assume that the future semantic web will include a collection of heterogeneous domain-specific ontologies, sometimes with semantic or syntactic overlap and sometimes not. One particularly relevant issue demanding attention is how two or more software agents can communicate in a cooperative task when there is a mismatch between the particular ontologies each has access to. This is a difficult problem which demands a number of different solutions since the nature of the types of mismatch in ontologies will vary within both the syntactic and semantic spectrum.

A number of different approaches to resolving the problem of communication in the context of heterogeneous ontologies have been proposed in the literature. Bailin and Truszkowski [2] provide a useful classification along the following lines, each with their own strengths and weaknesses:

- **Standardization of ontologies** – Develop standard ontologies for specific domains and acquire agreement upon them.
- **Aggregation of ontologies** – Develop broader ontologies that include the multiplicity of smaller ontologies and provide the expert in one field with access to the vocabulary and definitions of the related fields.
- **Integration of ontologies –** Use a variety of alignment techniques and supplement the original ontologies with mappings that link corresponding or related concepts.

- **Mediation between ontologies** – Originally proposed in the context of heterogeneous databases, mediators are pieces of software that translate between different schemata (ontologies). A request for information arrives at a mediator in terms of one or more ontologies; the mediator translates this into an appropriate request using the ontologies at the information source; the output is then translated back into an ontological form understandable by the sender of the request.

In this paper, we will propose a number of logic-based techniques combined with ideas from rough set theory that can provide software agents with the ability to ask *approximate* questions to each other in the context of heterogeneous ontologies. The techniques assume that some integration of existing ontologies has been provided. The idea of a mediator is implicit in the approach but is transparent to the communicating agents since each agent has its own mediator which only generates queries another agent can answer given a particular context.

Once the base communication ability is provided, we will extend the idea to situations where there is not only a mismatch between agent ontologies, but the agents have varying ability to perceive their environments. Even though they may have concepts in common, their ability to perceive individuals as having specific properties or relations will be distinct. The question then is how this affects the questions that can be asked and the replies that can be generated by agents with perception functions limited to varying degrees.

Figure 1 provides a useful schematic of the basic problem and the assumptions made in the problem specification.

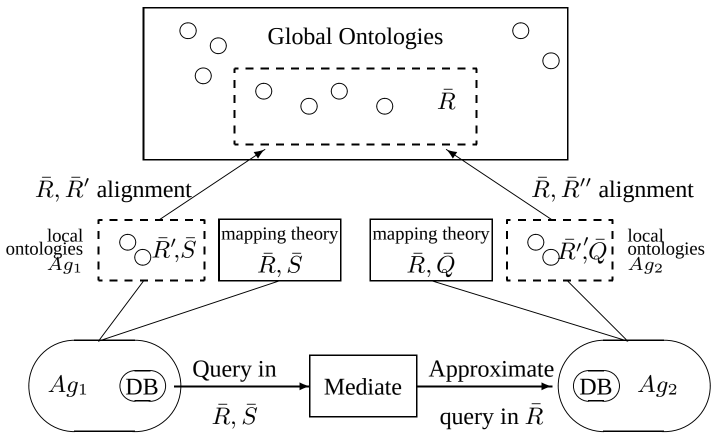

Fig. 1. Problem specification.

Assume agent Ag_1 has a local ontology consisting of concepts/relations in $\bar{R}'$ and $\bar{S}$ and that concepts/relations in $\bar{R}'$ have previously been aligned with those in $\bar{R}$, a subset from a global ontology repository assumed accessible to the agent community in question. In addition, Ag_1's knowledge base contains a mapping theory which represents dependencies between various concepts/relations in $\bar{R}$ and $\bar{S}$. These are assumed to be logical formulas in a fragment of first-order logic representing some sufficient and necessary conditions for concepts/relations in $\bar{R}$ and $\bar{S}$. Ag_1's database can contain rough set approximations of concepts/relations in $\bar{R}'$, $\bar{R}$ and $\bar{S}$.

Assume similarly, that agent Ag_2 has a local ontology consisting of concepts/relations in $\bar{R}''$ and $\bar{Q}$ and that concepts/relations in $\bar{R}''$ have previously been aligned with those in $\bar{R}$, the same subset that $Ag1$ has aligned R' with. In addition, Ag_2's knowledge base contains a mapping theory which represents dependencies between various concepts/relations in $\bar{R}$ and $\bar{Q}$ representing some sufficient and necessary conditions for concepts/relations in $\bar{R}$ and $\bar{Q}$. Ag_2's database can contain rough set approximations of concepts/relations in $\bar{R}''$, $\bar{R}$ and $\bar{Q}$.

From an external perspective, agents Ag_1 and Ag_2 have concepts/relations in $\bar{R}$ in common and therefore a common language to communicate, but at the same time, Ag_1 has the additional concepts/relations $\bar{S}$ disjoint from Ag_2, and Ag_2 has the additional concepts/relations $\bar{Q}$ disjoint from Ag_1. When reasoning about the world and in asking questions to other agents, it is only natural that Ag_1 would like to use concepts from $\bar{R}'$, $\bar{R}$ and $\bar{S}$. In a similar manner, Ag_2 would like to use concepts from $\bar{R}''$, $\bar{R}$ and $\bar{Q}$. Since we assume alignment of both $\bar{R}'$ and $\bar{R}''$ with $\bar{R}$, and that both agents know they have $\bar{R}$ in common, the communication issue reduces to that between the two sub-languages using vocabularies $\bar{R}$, $\bar{S}$ and $\bar{R}$,$\bar{Q}$.

Suppose agent Ag_1 wants to ask agent Ag_2 a question in Ag_1's own language. We will assume that any first-order or fixpoint formula using concepts/relations from $\bar{R}$, $\bar{S}$ can be used to represent the question. To do this, Ag_1 will supply the query α to its mediation function in addition to its mapping theory $T(\bar{R}, \bar{S})$. The mediation function will return a new approximate query consisting of

– the weakest sufficient condition of α under theory $T(\bar{R}, \bar{S})$ in the sub-language consisting of concepts/relations from $\bar{R}$ and
– the strongest necessary condition of α under theory $T(\bar{R}, \bar{S})$ in the sub-language consisting of concepts/relations from $\bar{R}$.

Both these formulas can be understood by agent Ag_2 because they are formulated using concepts/relations that Ag_2 can understand and that can be used to query its rough relational database for a reply to Ag_1. More importantly, it can be formally shown that agent Ag_1 can not ask a question more informative, under the assumptions we have made.

In the remainder of the paper, we will provide the details for this communicative functionality for software agents in the context of heterogeneous ontologies/schemata. We do this by first introducing rough set theory, weakest sufficient and strongest necessary conditions, and the connection to approximate queries. We then extend the results by introducing tolerance spaces. Tolerance spaces formalize limitations on an agent's perceptive capabilities. Such limitations influence the strength of the queries and replies generated by these agents. Some of these ideas were originally presented separately in [6] and [7]. This paper combines and extends the two.

2 Rough Sets

The methodology we propose in this paper uses a number of ideas associated with rough set theory which was introduced by Pawlak (see, e.g., [16]). In many AI applications one faces the problem of representing and processing incomplete, imprecise, and approximate data. Many of these applications require the use of approximate reasoning techniques. The assumption that objects can be observed only through the information available about them leads to the view that knowledge about objects in themselves, is insufficient for characterizing sets or relations precisely. We thus assume that any imprecise concept is replaced by a pair of precise concepts called the lower and the upper approximation of the imprecise concept, where

- the lower approximation consists of all objects which with certainty belong to the concept
- the upper approximation consists of all objects for which it is possible that they belong to the concept
- the complement of the upper approximation consists of all objects which with certainty do not belong to the concept
- the difference between the upper and the lower approximation constitutes a boundary region of an imprecise concept, i.e. the set of elements for which it is unknown whether they belong to the concept.

More precisely, by a rough set Z we shall understand a pair $Z = \langle X, Y\rangle$, where $X \subseteq Y$. The set X is interpreted as the *lower approximation* of Z and Y as the *upper approximation* of Z. We also use the notation Z^+ and $Z^\oplus$ to denote X and Y, respectively. By Z^- we denote the complement of $Z^\oplus$. The *boundary region* of Z, defined as $Z^\oplus - Z^+$, is denoted by $Z^\pm$.

By a *rough query* we shall understand it as a pair $\langle Q', Q''\rangle$, where Q' and Q'' are formulas of a given logic such that for any underlying database[3] D, $D \models Q' \to Q''$. By $\langle Q', Q''\rangle_D$ we denote the result of evaluating the query $\langle Q', Q''\rangle$ in the database D. In essence, a rough query provides an upper and lower approximation on the original crisp query.

3 Strongest Necessary and Weakest Sufficient Conditions

The strongest necessary and weakest sufficient conditions, as understood in this paper and defined below, have been introduced in [13] and further developed in [6].

[3] We deal with relational databases where queries are formulated as first-order or fixpoint formulas (for textbooks on this approach see, e.g., [1, 8, 12]).

Definition 3.1. *By* a necessary condition of a formula α on the set of relation symbols P under theory T *we shall understand any formula ϕ containing only symbols in P such that $T \models \alpha \rightarrow \phi$. It is the* strongest necessary condition, *denoted by* $\mathsf{SNC}(\alpha; T; P)$ *if, additionally, for any necessary condition ψ of α on P under T, $T \models \phi \rightarrow \psi$ holds.* ■

Definition 3.2. *By* a sufficient condition of a formula α on the set of relation symbols P under theory T *we shall understand any formula ϕ containing only symbols in P such that $T \models \phi \rightarrow \alpha$. It is the* weakest sufficient condition, *denoted by* $\mathsf{WSC}(\alpha; T; P)$ *if, additionally, for any sufficient condition ψ of α on P under T, $T \models \psi \rightarrow \phi$ holds.* ■

The set P in Definitions 3.1 and 3.2 is referred to as the *target language*.

The following lemma has been proven in [6].

Lemma 3.3. *For any formula α, any set of relation symbols P and theory T such that the set of free variables of T is disjoint with the set of free variables of α:*

- *the strongest necessary condition* $\mathsf{SNC}(\alpha; T; P)$ *is defined by* $\exists \bar{\Phi}.[T \wedge \alpha]$,
- *the weakest sufficient condition* $\mathsf{WSC}(\alpha; T; P)$ *is defined by* $\forall \bar{\Phi}.[T \rightarrow \alpha]$,

where $\bar{\Phi}$ consists of all relation symbols appearing in T and α but not in P. ■

The above characterizations are second-order. However, for a large class of formulas, one can obtain logically equivalent first-order formulas (see, e.g., [4, 10]) or fixpoint formulas (see, e.g., [14, 15]) by applying techniques for eliminating second-order quantifiers, Below we quote the result of [15] (Theorem 3.4), which allows one to eliminate second-order quantifiers for formulas of a certain form.

Let e, t be any expressions and s any subexpression of e. By $e(s := t)$ we shall mean the expression obtained from e by substituting each occurrence of s by t. Let $\alpha(\bar{x})$ be a formula with free variables $\bar{x}$. Then by $\alpha(\bar{x})[\bar{a}]$ we shall mean the application of $\alpha(\bar{x})$ to arguments $\bar{a}$. In what follows $\mathsf{lfp}\,\Phi.\alpha(\Phi)$ and $\mathsf{gfp}\,\Phi.\alpha(\Phi)$ denote the least and greatest fixpoint operators, respectively. A formula α is *positive* (respectively *negative*) wrt relation symbol Φ if it appears in α under an even (respectively odd) number of negations only.[4]

Theorem 3.4. Assume that all occurrences of the predicate variable Φ in the formula β bind only variables and that formula α is positive w.r.t. Φ.

- if β is negative w.r.t. Φ then

$$\exists \Phi \forall \bar{y}\ [\alpha(\Phi) \rightarrow \Phi(\bar{y})] \wedge [\beta(\neg \Phi)] \ \equiv\ \beta[\Phi(\bar{t}) := \mathsf{lfp}\,\Phi(\bar{y}).\alpha(\Phi)[\bar{t}]] \tag{1}$$

[4] It is assumed here that all implications of the form $p \rightarrow q$ are substituted by $\neg p \vee q$ and all equivalences of the form $p \equiv q$ are substituted by $(\neg p \vee q) \wedge (\neg q \vee p)$.

– if β is positive w.r.t. Φ then

$$\exists\Phi\forall\bar{y}[\Phi(\bar{y}) \to \alpha(\Phi)] \wedge [\beta(\Phi)] \;\equiv\; \beta[\Phi(\bar{t}) := \mathsf{gfp}\,\Phi(\bar{y}).\alpha(\Phi)[\bar{t}]]. \tag{2}$$

■

The resulting formula provided by Theorem 3.4 is a fixpoint formula. If the input formula is non-recursive wrt relations that are to be eliminated, then the resulting formula is a first-order formula[5]. The input formula can also be a conjunction of the form (1) or a conjunction of formulas of the form (2) since those conjunctions can be transformed equivalently to a form required in Theorem 3.4.

4 Agent Communication with Heterogeneous Ontologies

The original proposal for developing a communicative functionality for agents in the context of heterogeneous ontologies/schemata was initiated in [6]. In this case, only strongest necessary conditions replaced the original query and no appeal was made to approximate queries or rough set database. Let us now further develop the idea by using the proposal described in Section 1.

In this case, we assume an agent Ag_1 wants to ask a question Q to an agent Ag_2. Agent Ag_1 can use any of the terms in $\bar{R}, \bar{S}$, where the terms in $\bar{S}$ are unknown to agent Ag_2, while both have the terms in $\bar{R}$ in common. Let $T(\bar{R}, \bar{S})$ be a mapping theory in agent Ag_1's knowledge base describing some relationships between $\bar{R}$ and $\bar{S}$. It is then natural for agent Ag_1 to use its mediation function to first compute the weakest sufficient condition $\mathsf{WSC}(Q; T(\bar{R}, \bar{S}); \bar{R})$ and the strongest necessary condition $\mathsf{SNC}(Q; T(\bar{R}, \bar{S}); \bar{R})$, with the target language restricted to the common agent vocabulary $\bar{R}$ and then to replace the original query by the computed conditions.

The new query is generally not as precise as the original one, but is the best that can be asked. Namely,

- the weakest sufficient condition provides one with tuples satisfying the query with certainty
- the strongest necessary condition provides one with tuples that might satisfy the query
- the complement of the strongest necessary condition provides one with tuples that with certainty do not satisfy the query.

Observe that the difference between the strongest necessary and the weakest sufficient conditions contains tuples for which it is unknown whether they do or do not satisfy the query.

[5] In such a case fixpoint operators appearing on the righthand sides of formulas (1) and (2) are simply to be removed.

In summary, instead of asking the original query Q which can be an arbitrary first-order or fixpoint formula, agent Ag_1 will ask a pair of queries

$$\langle \mathsf{WSC}(Q; T(\bar{R}, \bar{S}); \bar{R}), \mathsf{SNC}(Q; T(\bar{R}, \bar{S}); \bar{R}) \rangle$$

which represent the the lower and upper approximation of Q. The following example illustrates the idea.

Example 4.1. Consider a situation where a ground operator (agent Ag_G) is communicating with a UAV[6] (agent Ag_V), which is flying over a road segment. Assume Ag_V can provide information about the following rough relations, $\bar{R}$, and that Ag_V has these in common with Ag_G:

- $V(x, y)$ – there is a visible connection between objects x and y
- $S(x, y)$ – the distance between objects x and y is small
- $E(x, y)$ – objects x and y have equal speed
- $C(x, z)$ – object x has color z.

We can assume that the concepts "visible connection", "small distance" and "color" were acquired via machine learning techniques with sample data generated from video logs provided by a UAV on previous flights while flying over similar road systems with traffic.

Assume also that agent Ag_G has a vocabulary consisting of $\bar{R}$, in addition to other relations $\bar{S}$, not known by Ag_V. $\bar{S}$ also includes a relation $Con(x, y)$, denoting that objects x and y are connected. Suppose that Ag_G knows the following facts about Con which are included in Ag_G's knowledge base:

$$\forall x, y.[V(x, y) \rightarrow Con(x, y)] \tag{3}$$
$$\forall x, y.[Con(x, y) \rightarrow (S(x, y) \wedge E(x, y))] \tag{4}$$

and that (3) and (4) are consistent (checking the consistency of such formulas with the contents of Ag_G's database can be done efficiently - see [5]).

Suppose Ag_G wants to ask Ag_V for information about all connected brown objects currently perceived by Ag_V. This can be represented as the following query,

$$Con(x, y) \wedge C(x, \mathsf{b}) \wedge C(y, \mathsf{b}), \tag{5}$$

where b stands for *brown*.

Since Ag_V can not understand queries with the term Con, Ag_G has to reformulate query (5) using only terms in $\bar{R}$ which are also understood by Ag_V. The most informative query it can then ask is:

$$\langle \mathsf{WSC}((5); (3) \wedge (4); \{V, S, E, C\}), \mathsf{SNC}((5); (3) \wedge (4); \{V, S, E, C\}) \rangle. \tag{6}$$

[6] Unmanned Aerial Vehicle.

By applying Lemma 3.3 and Theorem 3.4 one obtains[7] the following equivalent formulation of (6):

$$\langle V(x,y) \wedge C(x,\mathsf{b}) \wedge C(y,\mathsf{b}), \tag{7}$$
$$S(x,y) \wedge E(x,y) \wedge C(x,\mathsf{b}) \wedge C(y,\mathsf{b})\rangle. \tag{8}$$

Observe that objects perceived by Ag_V satisfying (7) belong to the lower approximation of the set of objects satisfying the original query (5) and objects perceived by Ag_V satisfying (8) belong to the upper approximation of the set of objects satisfying the original query (5). Thus:

- all objects satisfying formula (7) satisfy the original query (5)
- all objects not satisfying formula (8) do not satisfy the original query (5)
- on the basis of the available information and the capabilities of Ag_V, it remains unknown to Ag_G whether objects satisfying formula $((8) \wedge \neg(7))$ do or do not satisfy the original query (5).

Suppose Table 1 represents the actual situation on the road segment as sensed by Ag_V, where $\mathsf{b}, \mathsf{dr}, \mathsf{r}$ stand for "brown", "dark red" and "red", respectively. Table 1 represents

Object	V	S	E	C
1	2	2, 5	2, 5	b
2	1	1, 3, 4	1, 3, 4	b
3	-	2	2	b
4	-	2	2	r
5	-	1	1	dr

Table 1. Actual situation on the road segment considered in Example 4.1.

these relations by indicating, for each perceived object, with which entities a given relation holds. For example, the first row indicates that $V(1,2)$, $S(1,2)$, $S(1,5)$, $E(1,2)$, $E(1,5)$ and $C(1,\mathsf{b})$ hold.

Query (6), approximating the original query (5), computed over the database shown in Table 1, results in the following

$$\langle \{\langle 1,2\rangle, \langle 2,1\rangle\}, \{\langle 1,2\rangle, \langle 2,1\rangle, \langle 2,3\rangle, \langle 3,2\rangle\}\rangle,$$

which will be returned as an answer to Ag_G's original query. In consequence, Ag_G will know that tuples $\langle 1,2\rangle$, $\langle 2,1\rangle$ satisfy the query (5), tuples $\langle 2,3\rangle$, $\langle 3,2\rangle$ might satisfy the query and, for example, the tuple $\langle 1,5\rangle$ does not satisfy the query (in fact, object 5 is not brown). ■

[7] These steps can be computed automatically.

5 Tolerance Spaces

Tolerance spaces have been introduced in [7]. Technically, they allow us to partition a universe of individuals into indiscernibility or tolerance classes based on a parameterized tolerance relation. They provide a basis for dealing with the inaccuracy of agent perception capabilities.

Definition 5.1. *By a* tolerance function *on a set* U *we mean any function* $\tau : U \times U \longrightarrow [0, 1]$ *such that for all* $x, y \in U$,

$$\tau(x, x) = 1 \quad \textit{and} \quad \tau(x, y) = \tau(y, x).$$ ∎

Definition 5.2. *For* $p \in [0, 1]$ *by a* tolerance relation to a degree at least p based on τ, *we mean the relation* τ^p *given by*

$$\tau^p \stackrel{\text{def}}{=} \{\langle x, y\rangle \mid \tau(x, y) \geq p\}.$$

The relation τ^p *is also called the* parameterized tolerance relation. ∎

In what follows, $\tau^p(x, y)$ is used to denote the characteristic function for the relation τ^p. For a tuple $\bar{u} = \langle u_1, \ldots, u_k\rangle$ of elements of the domain, by $\tau^p(\bar{u})$ we denote the tuple of neighborhoods $\langle \tau^p(u_1), \ldots, \tau^p(u_k)\rangle$.

A parameterized tolerance relation is used to construct tolerance neighborhoods for individuals.

Definition 5.3. *By a* neighborhood function wrt τ^p *we mean a function given by*

$$n^{\tau^p}(u) \stackrel{\text{def}}{=} \{u' \in U \mid \tau^p(u, u') \textit{ holds}\}.$$

By a neighborhood of u wrt τ^p *we mean the value* $n^{\tau^p}(u)$. ∎

The concept of tolerance spaces plays a fundamental role in our approach.

Definition 5.4. *A* tolerance space *is defined as the tuple* $TS = \langle U, \tau, p\rangle$, *consisting of*

– *a nonempty set* U, *called the* domain *of* TS
– *a* tolerance function τ
– *a* tolerance parameter $p \in [0, 1]$. ∎

Consider a tolerance space $TS = \langle U, \tau, p\rangle$, and a relational database with universe U.[8] When an agent does not perceive a difference between similar (wrt a given tolerance function) objects, it instead perceives a difference between neighborhoods of elements rather than with elements themselves. In this case, a granulation of a database can be generated based on neighborhoods of individuals, as defined below.

[8] Here we focus on relational databases only. The extension to arbitrary relational structures is presented in [7].

Definition 5.5. *Let $M = \langle U, \{r_j\}_{j\in J}\rangle$ be a relational database and $TS = \langle U, \tau, p\rangle$ be a tolerance space. By a* granulation of M wrt TS, *we mean the structure*

$$M^{TS} = \langle U^{TS}, \{r_j^{TS}\}_{j\in J}\rangle$$

in which:

– $U^{TS} \stackrel{\text{def}}{=} \{n^{\tau^p}(u) : u \in U\}$ *is the set of all neighborhoods of elements in* U
– *for* $j \in J$, *if* r_j *is a* k*-ary relation, then* $r_j^{TS} \subseteq \underbrace{U^{TS} \times \ldots \times U^{TS}}_{k \text{ times}}$ *is defined by*

$$r_j^{TS}(n^{\tau^p}(x_1), \ldots, n^{\tau^p}(x_k)) \stackrel{\text{def}}{\equiv} n^{\tau^p}(r_j(x_1, \ldots, x_k)) \stackrel{\text{def}}{\equiv}$$
$$\exists . x'_1, \ldots, \exists x'_k . [x'_1 \in n^{\tau^p}(x_1) \wedge \ldots \wedge x'_k \in n^{\tau^p}(x_k) \wedge r_j(x'_1, \ldots, x'_k)].$$

For any formula α, *by* α^{TS} *we understand it to be the formula* α *in which any reference to a relation symbol, say* R, *is replaced by* R^{TS}. ■

Object	V	S	E	C
{1}	{2}	{2}, {5}	{2}, {5}	{b, dr}
{2}	{1}	{1}, {3}, {4}	{1}, {3}, {4}	{b, dr}
{3}	-	{2}	{2}	{b, dr}
{4}	-	{2}	{2}	{r}
{5}	-	{1}	{1}	{b, dr}

Table 2. Granulation of the relational database in Example 5.6, Table 1 wrt the perception capabilities of agent Ag_V.

Example 5.6. Consider the granulation of the relational database used in Example 4.1 (see Table 1) wrt the tolerance space $TS_V = \langle U, \tau_V, p_V\rangle$, where $\tau_V^{p_V}$ identifies equal elements and additionally dr with b. The resulting granulation is presented in Table 2. Observe that the arguments to relations are now neighborhoods induced by the associated tolerance space. Note that several tolerance spaces could be associated with each type of data in a table if desired. ■

6 Agent Communication with Heterogeneous Perceptive Capabilities

Consider a multi-agent application in a complex environment such as the Web where software agents reside, or a natural disaster in an urban area where physical robots reside. Each agent will generally have its own view of its environment due to a number of factors such as the use of different sensor suites, knowledge structures, reasoning processes, etc. Agents may also have different understandings of the underlying concepts which are used in their respective representational structures and will measure objects and phenomena with different accuracy. How then can agents with different knowledge structures and perceptive accuracies understand each other and effect meaningful communication and how can this be modeled? In this section, both tolerance spaces and upper and lower approximations on agent concepts and relations are used to define a means for agents to communicate when different sensor capabilities and different levels of accuracy in knowledge structures are assumed.

In Section 4, we showed how agents could communicate with each other in the context of heterogeneous ontologies. In this section, we will extend the approach by assuming that agents also have different perceptive capabilities. This will be done by associating with each agent, one or more tolerance spaces representing perceptive limitations. The net result will be that answers to queries will be represented in terms of neighborhoods of individuals, where the agent will be unable to determine which of the individuals in a neighborhood have been perceived. Initial work with these ideas may be found in [7].

We begin with a general definition of a *tolerance agent* also provided in [7].

Definition 6.1. *By a* tolerance agent *we shall understand any pair $\langle Ag, TS\rangle$, where Ag is an agent and TS is a tolerance space.* ∎

Here we do not define what an agent is specifically, as the framework we propose is independent of the particular details. The assumption is that the Ag part of a tolerance agent consists of common functionalities normally associated with agents such as planners, reactive and other methods, knowledge bases or structures, etc. The knowledge bases or structures are also assumed to have a relational component consisting of approximate relations which are derived and viewed through the agents limited sensor capabilities. When the agent introspects and queries its own knowledge base these limited perceptive capabilities should be reflected in any answer to a query.

Let us start with the simpler case when communicating tolerance agents have the same perception capabilities, i.e., the same tolerance spaces.

Definition 6.2. *Let $TS = \langle U, \tau, p\rangle$ be a tolerance space, $TA_1 = \langle Ag_1, TS\rangle$, $TA_2 = \langle Ag_2, TS\rangle$ be tolerance agents and let $Q = \langle Q_1, Q_2\rangle$ be a rough query asked by TA_1 and answered by TA_2. Let $M = \langle U, \{r_j\}_{j\in J}\rangle$ be a relational database. Then the meaning of Q wrt TS and M is defined as $\langle Q_1^{TS}, Q_2^{TS}\rangle_M$.* ∎

Remark 6.3. It is important to note that formulas Q_1^{TS} and Q_2^{TS} in Definition 6.2 refer to neighborhoods. Thus neighborhoods are to be encoded in databases as first-class citizens. It can easily be done since the number of neighborhoods is not greater than the number of elements of the underlying domain,[9] thus any neighborhood can be encoded by an element chosen from the neighborhoods. However, in what follows, for the clarity of presentation we use neighborhoods themselves rather than their encodings. ∎

Example 6.4. Consider a tolerance agent $\langle Ag_V, TS_V\rangle$, where Ag_V is as described in Example 4.1 and the tolerance space TS_V is as provided in Example 5.6 (i.e., Ag_V does

[9] In fact, it usually is much less than the number of elements of the domain.

not recognize the difference between colors dr and b). According to Definition 6.2, the approximation (wrt TS_V) of the query (5), given by $\langle(7),(8)\rangle$, is expressed by

$$\langle V^{TS_V}(x,y) \wedge C^{TS_V}(x,\{\mathsf{b},\mathsf{dr}\}) \wedge C^{TS_V}(y,\{\mathsf{b},\mathsf{dr}\}), \tag{9}$$
$$S^{TS_V}(x,y) \wedge E^{TS_V}(x,y) \wedge C^{TS_V}(x,\{\mathsf{b},\mathsf{dr}\}) \wedge C^{TS_V}(y,\{\mathsf{b},\mathsf{dr}\})\rangle. \tag{10}$$

Using the granulations of V^{TS_V}, S^{TS_V}, E^{TS_V} and C^{TS_V} wrt TS_V, from Table 2, $\langle(9),(10)\rangle$ evaluates to:

$$\langle\{\langle\{1\},\{2\}\rangle,\langle\{2\},\{1\}\rangle\},$$
$$\{\langle\{1\},\{2\}\rangle,\langle\{2\},\{1\}\rangle,\langle\{2\},\{3\}\rangle,\langle\{3\},\{2\}\rangle,\langle\{1\},\{5\}\rangle,\langle\{5\},\{1\}\rangle\}\rangle. \blacksquare$$

Suppose that two tolerance agents have different perceptive capabilities and consequently different tolerance spaces. It will then be necessary to define the meaning of queries and answers relative to the two tolerance agents. As previously advocated, a tolerance agent, when asked about a relation, answers by using the approximations of the relation wrt its tolerance space. On the other hand, the agent that asked the query has to understand the answer provided by the other agent wrt to its own tolerance space.

The dialog between two agents, say TA_1 (query agent) and TA_2 (answer agent), will then conform to the following schema:

1. TA_1 asks a question of TA_2 using a rough query $Q = \langle Q_1, Q_2\rangle$
2. TA_2 computes the answer approximating it according to its tolerance space and returns as an answer the approximations $QA = \langle Q_1^{TA_2}, Q_2^{TA_2}\rangle$
3. TA_1 receives QA as input and interprets it according to its own tolerance space. The resulting interpretation provides the answer to the query, as understood by TA_1 and taking into account the perceptive limitations of both agents.

This schema will only work properly under the assumption of a common vocabulary which has also been assumed in previous sections. The definitions describing this interaction now follow.

Definition 6.5. *Let $TS_1 = \langle U, \tau_1, p_1\rangle$, $TS_2 = \langle U, \tau_2, p_2\rangle$ be tolerance spaces defined over the same domain U and let R be a relation. Then the lower and upper approximations of R^{TS_2} wrt TS_1 are defined as*

$$R^{TS_2}{}_{TS_1^+} \stackrel{\text{def}}{=} \{n^{\tau_1^{p_1}}(\bar{u}) : R^{TS_2}(n^{\tau_2{}^{p_2}}(\bar{u})) \textit{ and } n^{\tau_2^{p_2}}(\bar{u}) \subseteq n^{\tau_1^{p_1}}(\bar{u})\}$$
$$R^{TS_2}{}_{TS_1^\oplus} \stackrel{\text{def}}{=} \{n^{\tau_1^{p_1}}(\bar{u}) : R^{TS_2}(n^{\tau_2{}^{p_2}}(\bar{u})) \textit{ and } n^{\tau_1^{p_1}}(\bar{u}) \cap n^{\tau_2^{p_2}}(\bar{u}) \neq \emptyset\}. \qquad \blacksquare$$

Remark 6.6. The intuition behind Definition 6.5 is that neighborhoods correspond to disjunctions. Namely, if an agent returns a neighborhood as a result, it means that due to limitations in its perception capabilities, it cannot distinguish between values in the neighborhood and, in consequence, it cannot verify which of the values from the neighborhood is actually perceived. For example, if the neighborhood is $\{\mathsf{brown}, \mathsf{red}\}$, it means that a perceived object is brown or red.

In consequence, the accepted notion of satisfiability reflects the intuitions of modal possibility. ■

Definition 6.7. *Let $TA_1 = \langle Ag_1, TS_1\rangle$, $TA_2 = \langle Ag_2, TS_2\rangle$ be tolerance agents with tolerance spaces as defined in Definition 6.5. Let $\langle Q_1, Q_2\rangle$ be a rough query, which is asked by TA_1 and answered by TA_2. Then the* meaning of the query *is given by approximations $\langle {Q_1^{TS_2}}_{TS_1^+}, {Q_2^{TS_2}}_{TS_1^\oplus}\rangle$.* ■

Example 6.8. Consider the tolerance agents $\langle Ag_V, TS_V\rangle$ and $\langle Ag_G, TS_G\rangle$ where:

– Ag_V and TS_V are as described in Examples 4.1 and 6.4
– $TS_G = \langle U, \tau_G, p_G\rangle$ such that $\tau_G^{p_G}$ identifies equal elements and additionally dr with r.

Suppose Ag_G wants to ask Ag_V for information about the colors of connected objects. A suitable query expressed in the language of Ag_G is:

$$\exists x, y.[Con(x, y) \wedge C(x, z_1) \wedge C(y, z_2)]. \tag{11}$$

Since Con is not in Ag_V's vocabulary, agent Ag_G has to approximate query (11) in a manner similar to that done in Section 4

$$\langle \mathsf{WSC}((11); (3) \wedge (4); \{V, S, E, C\}), \mathsf{SNC}((11); (3) \wedge (4); \{V, S, E, C\})\rangle. \tag{12}$$

By applying Lemma 3.3 and Theorem 3.4, Ag_G will obtain the following equivalent formulation of (12):

$$\begin{aligned}&\langle \exists x, y.[V(x, y) \wedge C(x, z_1) \wedge C(y, z_2)],\\ &\qquad \exists x, y.[S(x, y) \wedge E(x, y) \wedge C(x, z_1) \wedge C(y, z_2)]\rangle.\end{aligned}$$

Using Definition 6.7, agent Ag_V will then evaluate this rough query in the context of its perception capabilities, i.e., according to the database granulation given in Table 2. The answer returned by Ag_V, $QA = \langle Q_1^{TS_V}, Q_2^{TS_V}\rangle$ is,

$$\begin{aligned}&\langle \exists x, y.[V^{TS_V}(x, y) \wedge C^{TS_V}(x, z_1) \wedge C^{TS_V}(y, z_2)],\\ &\qquad \exists x, y.[S^{TS_V}(x, y) \wedge E^{TS_V}(x, y) \wedge C^{TS_V}(x, z_1) \wedge C^{TS_V}(y, z_2)]\rangle.\end{aligned}$$

Thus Ag_V will return the following answer to Ag_G:

$$\langle \{\langle \{\mathsf{b}, \mathsf{dr}\}, \{\mathsf{b}, \mathsf{dr}\}\rangle\}, \tag{13}$$

$$\{\langle \{\mathsf{b}, \mathsf{dr}\}, \{\mathsf{b}, \mathsf{dr}\}\rangle, \langle \{\mathsf{b}, \mathsf{dr}\}, \{\mathsf{r}\}\rangle, \langle \{\mathsf{r}\}, \{\mathsf{b}, \mathsf{dr}\}\rangle\}\rangle. \tag{14}$$

Ag_G will then compute the final answer by interpreting (13)-(14) relative to its tolerance space using Definition 6.7 and the database granulation shown in Table 3. The final answer, $\langle {Q_1^{TS_V}}_{TS_G^+}, {Q_2^{TS_V}}_{TS_G^\oplus}\rangle$, is

$$\langle \emptyset,\ \ \{\langle \{\mathsf{b}\}, \{\mathsf{b}\}\rangle, \langle \{\mathsf{b}\}, \{\mathsf{r}, \mathsf{dr}\}\rangle, \langle \{\mathsf{r}, \mathsf{dr}\}, \{\mathsf{b}\}\rangle, \langle \{\mathsf{r}, \mathsf{dr}\}, \{\mathsf{r}, \mathsf{dr}\}\rangle\}\rangle.$$ ■

Object	V	S	E	C
$\{1\}$	$\{2\}$	$\{2\}$, $\{5\}$	$\{2\}$, $\{5\}$	{b}
$\{2\}$	$\{1\}$	$\{1\}$, $\{3\}$, $\{4\}$	$\{1\}$, $\{3\}$, $\{4\}$	{b}
$\{3\}$	-	$\{2\}$	$\{2\}$	{b}
$\{4\}$	-	$\{2\}$	$\{2\}$	{r, dr}
$\{5\}$	-	$\{1\}$	$\{1\}$	{r, dr}

Table 3. Granulation of the relational database given in Table 1 wrt perception capabilities of agent Ag_G as defined in Example 6.8.

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