CHAPTER 10

Agents in Approximate Environments

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ABSTRACT. The starting point of this research is the multimodal approach to modeling multiagent systems, especially Beliefs, Goals and Intention systems. Such an approach is suitable for specifying and verifying many subtle aspects of agents' informational and motivational attitudes.

However, in this chapter we make a shift in a perspective. More precisely, we propose the method of embedding multimodal approaches into a form of approximate reasoning suitable for modeling perception, namely a *similarity-based approximate reasoning*. We argue that this formalism allows one to both keep the intuitive semantics compatible with that of multimodal logics as well as to model and implement phenomena occurring at the perception level.

1. From Quantitative to Symbolic Modeling of Multiagent Systems

The overall goal of modeling reality is to create its adequate description. Especially in the initial phases of modeling, the proper choice of the underlying formalisms is essential, as it provides means for knowledge representation and reasoning, reflecting both the properties of an environment and the application in question. In order to adequately make the choice, one should take into account the quality of information available during the entire model life cycle including its development, use and maintenance. Clearly, many important issues are involved here. This chapter concentrates on the quality of available information such as possible incompleteness, uncertainty and imprecision. The underlying formalism cannot be adequately chosen without addressing these issues.

Formal approaches to multiagent systems are concerned with equipping software agents with functionalities for, first, reasoning and communicating, and then, acting. The majority of formal frameworks starts from the set of beliefs usually represented in a symbolic, qualitative, and somehow idealized, way. However, as illustrated in Figure 1, the basic layer of intelligent systems is perception.

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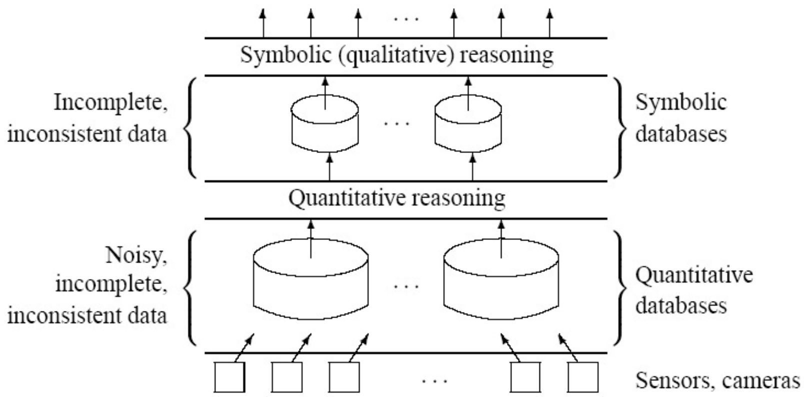

FIGURE 1. Layers of intelligent systems.

In our simplified view, perception captures the agent's ability to observe its environment. This observation functionality might be implemented on top of a sensor platform of a mobile robot designed to act in the physical world. The results of perception, e.g., sensors' measurements, are inherently quantitative. Therefore we naturally deal with a meta-level duality: sensors provide *quantitative* characteristics, while reasoning tasks require symbolic, i.e., qualitative, representations and inference mechanisms. As agents are equipped with different perceptual capabilities and typically perceive the environment from different perspectives (see Figure 2), the integration of perception with higher level, possibly approximate, knowledge structures is unavoidable.

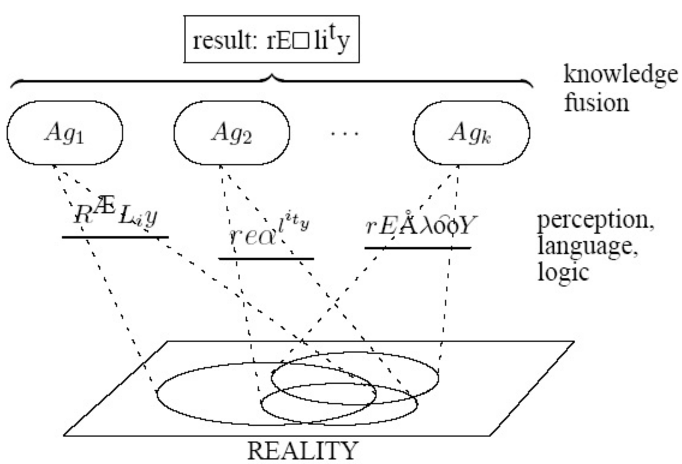

FIGURE 2. Perception in multiagent systems.

It is a well known phenomenon that the lack of precise information about the environment leads to consideration of its various variants, some of them being superfluous. Then, in the course of completing information, non-feasible variants become excluded. For example, if we do not know whether a specific object is a car, once we find it out, our knowledge becomes certain. Therefore there is no need to consider other variants anymore. On the other hand, in real-world applications the knowledge might be complete, but still uncertain. For example, no matter how many measurements of car's height we make using a short ruler, we can learn the result only up to a given precision which, at a certain point, cannot be further improved. While we cannot be sure about the actual result of measurement, we are able to approximate it. This observation constitutes the first underpinning principle of this chapter.

Another underpinning principle of this research is the multimodal approach to modeling BGI systems ([**35, 37, 38, ?, 42, 61**]).[3] BGI logics naturally serve as a high level specification of a multiagent system. Nevertheless, the commonly accepted methodological assumptions are subject to such substantial restrictions like:

- a high complexity of modal logics is known and not easy to overcome
- incomplete and uncertain information has not been adequately tackled.

On the other hand, this approach allows one to specify and verify various subtle properties concerning agents' cooperation, coordination, coalition formation, communication, etc. A typical property expressible in multimodal BGI logics could be

$$Bel_j(Int_i(keep\_low\_temperature)) \rightarrow Goal_j(keep\_low\_pressure), \tag{13}$$

meaning that if agent j believes that agent i intends to keep the temperature low, then agent j aims to keep the pressure low.

In this chapter we make a shift in the multimodal perspective. Assuming that approximate results of agents' perception and vague concepts referring to reality commonly appear in multiagent environments, any description of a reality is an approximation of its factual state. Accepting this point of view, our aim is to equip agents with a pragmatic and tractable machinery to verify whether a particular state of the world, actual or hypothetical, *surely* or *possibly* satisfies their beliefs, goals and intentions.

For example, we will be able to express properties like

$$[\sigma_t]low\_temperature \wedge \langle\sigma_p\rangle low\_pressure. \tag{14}$$

Assuming that the accuracy of temperature and pressure sensors is modeled respectively by similarity relations σ_t, σ_p, formula (14) expresses the property that temperature is surely low and the pressure might be low. Here $[\sigma_t]$ and $\langle\sigma_p\rangle$ are modal operators, precisely described in Section 6.

The immediate gain is that we deal with the first-order formalism allowing one to embed models of sensors and perception,[4] and to deal with vague concepts. For a discussion how to approach vague concepts within the similarity based framework, see also [**?**]. Observe

[3] BGI is an acronym for *Beliefs, Goals and Intentions* traditionally called *Beliefs, Desires and Intentions*.

[4] For the idea of embedding sensors' models within the similarity structures framework, see [**?**].

that specifications like (13), in fact, contain vague concepts which have to be further modeled.[5] Assuming that agents actually work over relational or deductive databases and that similarities themselves are tractable, we stay within a very pragmatic, tractable framework, allowing one to implement multimodal BGI specifications in an efficient manner.

The chapter is structured as follows. In Section 1 we discuss problems appearing in modeling of multiagent systems, especially in the context of perception. Next, in Section 2, we emphasize the rôle of approximate concepts and theories. In Sections 3 and 4 we recall the multimodal approach to modeling BGI systems and similarity structures, respectively. In Section 5 we discuss the perspectives of single agents, while Section 6 is devoted to calculus based on the Propositional Dynamic Logic allowing for modeling the perspectives of teams of agents. Section 7 introduces the language and semantics for expressing properties of approximate BGI systems. Finally, Section 8 concludes the chapter.

1.1. Agents with Sensors: An Example. To illustrate some points addressed in this chapter, consider the following example.

EXAMPLE 10.1. *Assume that the overall maintenance goal of the system is to keep the situation $safe$. For simplicity we assume that there is one agent only and omit in formulas indices referring to agents. The safety depends on the temperature and pressure as follows, where situations are characterized by temperature and pressure pairs:*

> *it is believed that a given situation is safe if and only if temperature t is not high or when temperature t is high while pressure p is moderate.*

Formally:

(15) $$Bel\Big(safe(t,p) \equiv \big(\neg high(t) \vee (high(t) \wedge moderate(p))\big)\Big).$$

Among the many possible goals of the system, some include the sufficient conditions to keep the situation safe:

(16) $$Goal(\neg high(t)),\; Goal(high(t) \wedge moderate(p)).$$

One of the system intentions, which create a consistent subsets of goals (see Section 3), is the one to keep the temperature high and the pressure moderate:

(17) $$Int(high(t) \wedge moderate(p)).$$

Note that (17), in suitable multimodal logics (see, e.g., [**35, 37, 38**] *and Section 3), is equivalent to the conjunction $Int(high(t)) \wedge Int(moderate(p))$. Therefore, one can consider two intentions:*

(18) $$Int(high(t)),\; Int(moderate(p)).$$

The above beliefs, goals and intentions, expressed by (15), (16) and (18), contains concepts $high(t)$ and $moderate(p)$ which, in a given situation, have to be further specified.

[5] Vagueness in the context of modal logics has been discussed, e.g., in [**?**], but this approach seems not very natural for modeling perception.

We provide this specification by the following beliefs:

(19) $$Bel\Big(high(t) \equiv 60 \leq t \leq 100\Big)$$

(20) $$Bel\Big(moderate(p) \equiv 0 \leq p \leq 4\Big).$$

Assuming that in the considered environment the temperature is in the interval $[-20, 100]$*, we also have that*

(21) $$Bel(\neg high(t) \equiv -20 \leq t < 60).$$

Thus we have:

(22) $$Bel\Big(safe(t, p) \equiv -20 \leq t < 60 \ \vee \ (60 \leq t \leq 100 \ \wedge \ 0 \leq p \leq 4)\Big).$$

Let us consider two agents ag_1 *and* ag_2 *cooperating in this system:*

- ag_1 *responsible for measuring temperature*
- ag_2 *responsible for measuring temperature and pressure.*

The agents are equipped with sensors:

- *for* $i \in \{1, 2\}$*, agent* ag_i *is equipped with sensor* τ_i *for measuring temperature with an associated predicate* $T_i(t)$ *(*$T_i(t)$ *is* TRUE *when the temperature is approximately* t*)*
- *agent* ag_2 *is additionally equipped with sensor* π *for measuring pressure with an associated predicate* $P(p)$ *(*$P(p)$ *is* TRUE *when the pressure is approximately* p*).*

Sensors τ_i *and* π *function up to a given accuracy. Therefore measurement errors inevitably appear. Here it is assumed that measurement errors are not greater than* ϵ_{τ_i} *for sensors* τ_i *and* ϵ_π *for sensor* π*. In this context some modeling questions naturally arise.*

(1) *What is the meaning of beliefs, goals and intentions when sensors' measurements deviate from reality?*
(2) *Can we use crisp definitions (as (19), (20) or (22)) in the presence of inaccurate perception? (For example, should it be believed that the situation is safe, when* $t = 100$ *and* $p = 4$*?) Measurement errors have to be taken into the consideration here (see the continuation of this discussion provided in Examples 10.3, 10.4, 10.10, 10.13).*
(3) *What calculus should be used to fuse results of observations of different, possibly heterogenous agents'? It is especially important when sensors accuracies are not the same (see Examples 10.10, 10.13).*■

In this chapter we make a step towards introducing approximate reasoning into multiagent context. More precisely, we propose the method of embedding multimodal approaches into a form of approximate reasoning suitable for modeling perception, namely a *similarity-based approximate reasoning*. We argue that this formalism allows one to both keep the intuitive semantics compatible with that of multimodal logics and to model and implement phenomena occurring at the perception level.

It is a position chapter, as we mainly address methodological issues and possible avenues to solve them. Typical logical issues like providing proof systems and investigating their meta-properties is left for further research. We substantially extend our approach of [**?**] (see Sections 2, 3, 4 and 5) and combine it with calculus proposed by P. Doherty and ourselves in [**?**] (see Section 6). The substantially new material is included in Sections 2 and 7 as well as in many discussions and a series of examples.

2. Approximate is Ubiquitous

Traditional modeling is usually centered around crisp (precise) definitions of the modeled reality. The prerequisite is that the objective reality is or can be crisply modeled. For example, when talking about speed of cars on a road, one uses a crisp definition stating that the speed of a car is the rate of change of its displacement from a given starting point. Then one uses quantitative representation of distance and time and calculates the speed, assuming that the result is precise. Despite the fact that numbers representing measurements might not be accurate, when using qualitative descriptions like *fast* or *slow*, it becomes difficult to provide their precise definitions. We might classify some speeds to surely be fast, some to surely not be fast, while having difficulties to classify some others.

According to the literature (see, e.g., [**?**]), a concept is *approximate* (vague) when it has borderline cases, i.e., some objects cannot be surely classified to the concept or to its complement. Approximate concepts and phenomena frequently appear in computer science, in particular in AI and knowledge representation.

McCarthy [**?**] argues that computer science logics should be able to deal with:

- *approximate objects* and not fully defined *concepts*, lacking "if-and-only-if" definitions
- *approximate relations* among approximate entities
- *approximate theories* comprising approximate concepts and relations.

The majority of the environments in which intelligent systems are situated are unpredictable, dynamic, and also very complex. Therefore, the relevant knowledge representation formalisms need to be partial and approximate in their very nature. Importantly, we expect that intelligent systems are capable to dynamically construct representations of observed objects and to integrate these representations with other static aspects.

EXAMPLE 10.2. *A team of robots can be given a goal "to remove snow from a road and leave the road safe enough for driving". In this description not only the concept "safe enough" is vague. Most probably robots deal with other approximate concepts and actions with approximate effects. Even the concept of "snow" becomes approximate, e.g., in the middle of the melting process or, on the contrary, when it is freezing.*■

Beyond doubt, any sensible description of physical world includes approximate concepts and relations. Consequently, they appear at all levels of intelligent systems dealing with practical applications:

(1) at the object level
(2) at the description (definition, specification) level

(3) at the reasoning level.

The lowest *object* level deals with approximate objects of an application in question. Although approximativeness results from various reasons, in our analysis we will focus on the agent-level, assuming that agent's activity starts more or less explicitly from a perception.

Whatever form of perception is considered, it is hardly assumed to be perfect. Most of the time, its results are imprecise or, in the worst case, even misleading. Therefore, it is increasingly important to build the bridge between low-level sensory data and its fusion with qualitative knowledge structures. Importantly, multiagent environments are often as complex as those faced by humans: partial and approximate in their very nature. These unpleasant properties need to be reflected in adequate knowledge structures and techniques.

Next, the intermediate *description* level consists of approximate predicates, approximate specifications (axioms) and approximate rules.

Finally, the *reasoning* level involves approximate predicates inherited from previous levels, in addition to approximations of precise predicates, resulting from incomplete reasoning processes. It happens that such processes cannot sometimes be fully carried out, due to their high complexity, the lack of sufficient resources or other reasons.

The above distinction leads to a more precise classification of approximate concepts appearing in complex knowledge-based systems:

- *perception layer* (readings of object and environment attributes)
  - limited accuracy of sensors and other devices
  - time and other restrictions placed on completing measurements
  - environmental conditions
  - noise, limited reliability and failure of physical devices
- *information fusion layer* (semantical structures resulting from measurements)
  - approximative nature of perception
  - possible inconsistences in measurements
  - a possible partiality of available information
- *knowledge representation layer* (including concepts definitions, and rules)
  - the use of approximate concepts while defining the crisp ones
  - the use of approximate definitions of new concepts starting from the crisp ones
  - approximations of definitions due to a complexity of the classification problem
  - the use of approximate theories on agency
- *planning and reasoning layer*
  - bounded resources, typically time limits for planning and reasoning
  - an incomplete, uncertain and imprecise knowledge
  - a cumulation of approximate knowledge fussed on earlier stages
- *communication layer*
  - strictly linguistic restrictions
    - linguistic quantifiers like ”usually”, ”most”, ...

    * linguistic modifiers like "more", "less", ...
  – technical limitations and failures of communication media.

In the sequel we shall focus on a special class of approximate concepts constructed by means of similarities and approximations.

3. Multimodal Models of BGI Systems

The BGI model of agency comprises *beliefs* referring to agent's informational attitudes, *goals* (or desires), *intentions* and then *commitments* defining together its motivational stance. In general, these attitudes refer to the multiagent environment that comprise both environment itself and agents acting in it. A "mental state" of an BGI-agent *ag* is characterized by:

- *beliefs*: $Bel_{ag}A$ expresses that agent *ag* beliefs that a condition *A* is satisfied;
- *goals*: $Goal_{ag}A$ expresses that agent *ag* aims to reach the state of the environment satisfying *A*
- *intentions*: $Int_{ag}A$ expresses that agent *ag* intends to reach the state of the environment satisfying *A*.

While goals, reflecting a variety of (long or short term) agent's perspectives, might be inconsistent, intentions create a consistent subset of goals the agent chooses to focus on. In [**35, 37, 38**] it is shown how intentions initiate a goal-directed activity, finally reflected in *commitments*.

EXAMPLE 10.3 (Example 10.1 continued). *Assume that agent ag_1 considered in Example 10.1 is capable to regulate temperature and agent ag_2 is capable to adjust pressure based on the temperature readings. In this case, intentions expressed as (18) can be distributed among these agents:*

$$Int_{ag_1}(high(t)),\ Int_{ag_2}(moderate(p)). \tag{23}$$

Observe that both agents can share belief (15), while (19) can naturally be associated with ag_1, and (20) with ag_2. Therefore the set of ag_1 beliefs is the following:

$$\begin{aligned} &Bel_{ag_1}\Big(safe(t,p) \equiv \big(\neg high(t) \vee (high(t) \wedge moderate(p))\big)\Big)\\ &Bel_{ag_1}\Big(high(t) \equiv 60 \leq t \leq 100\Big) \end{aligned} \tag{24}$$

Analogically, for ag_2 we have:

$$\begin{aligned} &Bel_{ag_2}\Big(safe(t,p) \equiv \big(\neg high(t) \vee (high(t) \wedge moderate(p))\big)\Big)\\ &Bel_{ag_2}\Big(moderate(p) \equiv 0 \leq p \leq 4\Big). \end{aligned} \tag{25}$$

Observe that belief (22) is obtained by gathering knowledge distributed between ag_1 and ag_2 (for the discussion of distributed knowledge *we refer the reader to* [**42, 61**]*).*■

Traditionally, BGI systems have been formalized in terms of multimodal logics tailored to model agents' informational and motivational attitudes. Beliefs, goals and intentions can naturally be expressed via modal operators (see, e.g., [**35, 37, 38, ?, ?**]). The underlying semantics is based on Kripke structures, providing a general modeling machinery. However, already an individual level of agent's specification, in the light of limited perception capabilities, uncertain and imprecise information is problematic.

The theory of informational attitudes has been formalized in terms of epistemic logic in [**42, 61**]. As regards motivational attitudes, the situation is much more complex. In Distributed Cooperative Problem Solving, a group as a whole needs to act in a coherent pre-planned way, presenting a unified *collective* motivational attitude staying in accordance with individual ones, but having a higher priority. All in all, in BGI systems agent's attitudes are considered on the three levels: individual, bilateral, and collective. A coherent and conceptually unified theory of motivational attitudes has been developed by Dunin-Kęplicz and Verbrugge (see, [**35, 37, 38, ?**]).

Despite many advantages, (multi)modal logics are often questioned when:

- *complexity* is an issue
- *perception* modeling is substantial
- *vague concepts* over (pseudo) continuous/dense domains are present.

Unfortunately, these problems become essential in multiagent environments where we usually deal with both agents' bounded resources and also limited precision of sensors, video cameras and other equipment. It is commonly agreed that approximative nature of real-world environments is either not adequately captured by modal theories or the constructed models become too complex.

It appears, however, as we show in this chapter, that the notion of similarity structures [**?, ?**] can successfully substitute or complement modal logics. These structures keep intuitive Kripke-like semantics and permit to use results from modal logics, while *substantially simplifying calculations and reduce the complexity of reasoning*. In the sequel, we propose a method that originates from the following observations:

(1) many agent theories are expressed in multimodal logics
(2) there is a natural correspondence between modal theories and similarity structures
(3) similarity structures can be used to define approximations of concepts and relations
(4) these approximations lead to approximate reasoning that is tractable over relational and deductive databases.

These issues inspired us to isolate and define *approximate* BGI *systems*, denoted by αBGI , where agents' beliefs, goals and intentions are expressed by means of approximate theories (see Section 7). In practice, such theories are usually represented by deductive databases. As the database technology can be adjusted to reflect the approximate phenomena, we deal with approximate databases [**?**]. Perhaps the most impressive advantage of such an approach, that is of using approximate databases, is a tractable querying machinery. It is achievable, when the similarity relation as well as all the underlying operations (like the arithmetical ones), are tractable once using traditional tractable querying machinery. (See [**?**] for an in-depth discussion).

4. Similarity Structures

4.1. Preliminaries. A natural generalization of crisp relations are rough sets and rough relations, as introduced by Pawlak in [**?**]. These can be further refined to approximate relations based on a notion of a similarity structure. More precisely, we use similarity-based neighborhoods (see, e.g., [**?, ?**]), covering the underlying domain. Thus, the lower and upper approximations of relations are defined in terms of neighborhoods rather than equivalence classes which is the case in rough sets. Approximate relations and similarity structures have been shown to be versatile in application requiring approximate knowledge structures [**?, ?, ?**].

To appropriately reflect the inherent properties of the application domain, proper constraints on similarity relations defining upper and lower approximations have to be identified. For example, sometimes the relation should not be transitive since similar objects do not naturally chain in a transitive manner. In order to represent arbitrary notions of similarity in a universe of individuals, similarity relations have no initial constraints. These issues are discussed in the context of rough sets (see, e.g., [**?, ?, ?**]).

DEFINITION 10.1. *By a* similarity structure *we mean any pair* $S = \langle U, \sigma \rangle$*, where* U *is a non-empty set and* $\sigma \subseteq U \times U$*. By a* neighborhood *of* u *w.r.t.* σ *we mean* $n^{\sigma}(u) \stackrel{\text{def}}{=} \{u' \in U \mid \sigma(u, u')\}$*. For* $A \subseteq U$*, the* lower and upper approximation of A w.r.t. S*, denoted by:* A_{S^+} *and* $A_{S^\oplus}$*, are defined by:*

$$A_{S^+} = \{u \in U \colon n^{\sigma}(u) \subseteq A\}$$
$$A_{S^\oplus} = \{u \in U \colon n^{\sigma}(u) \cap A \neq \emptyset\}. \qquad \blacksquare$$

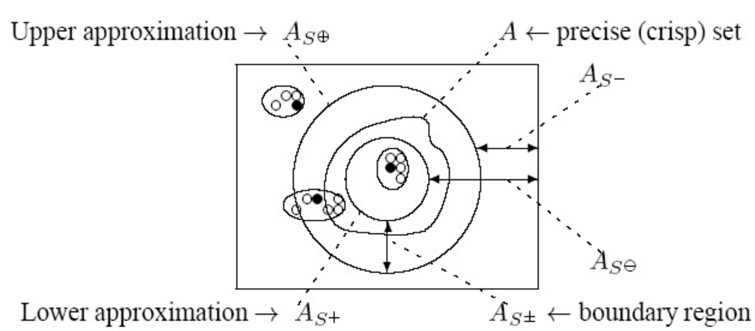

FIGURE 3. Approximations.

Let $S = \langle U, \sigma \rangle$ be a similarity structure and let $A \subseteq U$. Then (see also Figure 3):

- A_{S^+} contains elements which *surely belong* to A (since all elements similar to the considered one belong to A)
- $A_{S^\oplus}$ contains elements which *may or may not belong* to A and we are unable to judge what is the case (due to the indiscernibility of similar objects)[6]

[6] In addition to elements surely belonging to A.

- $A_{S^{\pm}} \stackrel{\text{def}}{=} A_{S^{\oplus}} - A_{S^{+}}$ contains elements which *may or may not belong* to A and we are unable to judge what is the case
- $A_{S^{-}} \stackrel{\text{def}}{=} -A_{S^{\oplus}}$ contains elements which *surely do not belong* to A (since all elements similar to the considered one do not belong to A)
- $A_{S^{\ominus}} \stackrel{\text{def}}{=} -A_{S^{+}}$ contains elements which *may or may not belong* to $-A$.

In the rest of the chapter, when similarity structure $S = \langle U, \sigma \rangle$ is known from the context, we often write $A_{\sigma^{+}}$ rather than $A_{S^{+}}$ and similarly for other approximation operators.

PROPOSITION 10.1. *Let $S = \langle U, \sigma \rangle$ be a similarity structure. Then:*

$$A_{S^{+}} = \{u \in A \mid \forall v\, (\sigma(u, v) \to v \in A)\}$$
$$A_{S^{\oplus}} = \{u \in A \mid \exists v\, (\sigma(u, v) \wedge v \in A)\}. \qquad \blacksquare$$

Observe the strong similarity between characterizations of lower and upper approximations provided in Proposition 10.1 and definitions of semantics of $\Box$ and $\Diamond$ in Kripke semantics of modal logics.

4.2. Kripke Structures for Similarity-Based Reasoning. Up to now we have shown, how to construct approximations starting from similarity structures. On the other hand, the semantics of logics for multiagent systems typically originates from Kripke structures. Therefore, we should address the question how to reflect a possible world semantics in our framework.

An obvious way to construct the accessibility relation R for similarity-based reasoning is to take the similarity relation σ and set

> $R(w, w') \stackrel{\text{def}}{\equiv}$ w differs from w' on the value of at most one variable (tuple of variables), say x, such that the value of x in w is similar to the value of x in w', i.e., when $\sigma(w(x), w'(x))$ holds, where $w(x)$ stands for the value of x in w.

Along this line of modeling, we do not really need Kripke structures, assuming that underlying similarity structures are known. When similarities are defined over continuous domains, we rather want to avoid the continuum of accessible worlds. Similarity relations include the whole information about such Kripke structures, so just one (current) world equipped with a suitable similarity structure (or a couple of structures) is sufficient.

REMARK 4.1. Observe that a substantial difference between multimodal BGI systems and our approach is that beliefs, goals and intentions are modeled separately via different accessibility relations, while similarity structures do not need to be specific for beliefs, goals or intentions. In a typical situation the same collection of similarity structures, reflecting perception, serves for modeling beliefs, goals and intentions.

However, other options are also justified. For example, during surveillance mission one team of agents, equipped with a specific sensor platform, may recognize the situation and construct a relevant database containing approximate beliefs. Then, another group of agents may plan and act along the previously done recognition. Both groups may differ with respect to their capabilities to perceive. ■

4.3. Correspondences between Approximations and Similarity. As observed in Proposition 10.1, there is a close relationship between correspondences among approximations and similarities and those considered in modal correspondence theory, when modal $\Box$ is considered as A_{S^+}, and $\Diamond$ as $A_{S^\oplus}$.

Let $S = \langle U, \sigma \rangle$ be a similarity structure. Then

(1) $A_{S^+} \subseteq A_{S^\oplus}$ is equivalent to the seriality of σ
(2) $A_{S^+} \subseteq A$ is equivalent to the reflexivity of σ
(3) $A \subseteq (A_{S^\oplus})_{S^+}$ is equivalent to the symmetry of σ
(4) $A_{S^+} \subseteq (A_{S^+})_{S^+}$ is equivalent to the transitivity of σ.

In paper [**?**] a technique analogous to those of modal correspondence theory has been considered. This technique supports automatizing the search for suitable correspondences.

5. A Single Agent's Perspective

It is commonly assumed that, despite the well known disadvantages of multimodal logics, like the omniscience problem, agents' individual beliefs, goals and intentions are characterized by modalities of the universal flavor ("boxes"). In αMAS we translate them into lower approximations w.r.t. suitable similarity structures. We then obtain characterization of situations that *surely* satisfy given beliefs, goals and intentions. This is a very natural approach for such properties, as $safe$. On the other hand, in the case of dual properties one is often interested in characterization of *possible* situations. For example, when considering dangers, it is essential to recognize when the situation *might be* dangerous before it *surely* is such. Therefore we additionally use upper approximations to express what *might be* believed, what *might be* a goal or what *might be* an intention.

The required characteristics of accessibility relations in BGI logics has to be ensured by the relevant semantical properties of the underlying similarity relations. If only approximation operators are given, without similarities themselves, one can use the approach outlined in [**?**] and apply the correspondences between approximations and similarities.

EXAMPLE 10.4 (Examples 10.1, 10.3 continued). *Recall that we have considered sensors τ_1, τ_2 for measuring temperature and π for measuring pressure with the measurement errors not greater than $\epsilon_{\tau_1}, \epsilon_{\tau_2}$ and ϵ_π (for τ_1, τ_2 and π, respectively). We then have three similarity structures, S_{τ_1}, S_{τ_2} and S_π reflecting the measurement errors of τ_1, τ_2 and π, respectively.*

More precisely, S_{τ_i} ($i \in \{1,2\}$) can be defined, e.g., to be $\langle T, \sigma_{\tau_i} \rangle$, where

$$T \stackrel{\text{def}}{=} [-20, 100] \textit{ and } \sigma_{\tau_i}(t, t') \stackrel{\text{def}}{\equiv} |t - t'| \le \epsilon_{\tau_i}.$$

Similarly S_π can be defined, e.g., to be $\langle P, \sigma_\pi \rangle$, where

$$P \stackrel{\text{def}}{=} [0, 10] \textit{ and } \sigma_\pi(p, p') \stackrel{\text{def}}{\equiv} |p - p'| \le \epsilon_\pi.$$

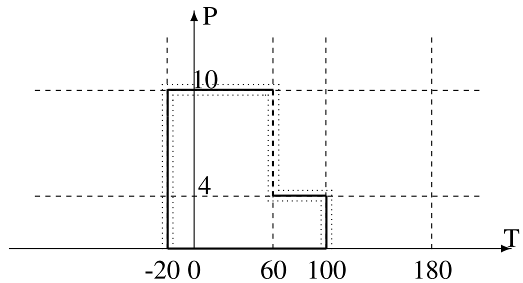

FIGURE 4. The area representing the crisp definition of $safe$ (marked by the thick line) and its approximations (marked by dotted lines).

In such a case,

(26) $[-20, 60)_{S^+_{\tau_i}} = [-20 + \epsilon_{\tau_i}, 60 - \epsilon_{\tau_i})$ $[-20, 60)_{S^\oplus_{\tau_i}} = [-20, 60 + \epsilon_{\tau_i})$

(27) $[60, 100]_{S^+_{\tau_i}} = [60 + \epsilon_{\tau_i}, 100 - \epsilon_{\tau_i}]$ $[60, 100]_{S^\oplus_{\tau_i}} = [60 - \epsilon_{\tau_i}, 100]$

(28) $[0, 4]_{S^+_\pi} = [\epsilon_\pi, 4 - \epsilon_\pi]$ $[0, 4]_{S^\oplus_\pi} = [0,\ 4 + \epsilon_\pi]$.

Note that the upper approximation of $[60, 100]$ *in (27) is* $[60 - \epsilon_{\tau_i}, 100]$ *rather than* $[60 - \epsilon_{\tau_i}, 100 + \epsilon_{\tau_i}]$ *since the universe* T *is* $[-20, 100]$.

Let us now approximate intentions to keep the temperature high and to keep pressure moderate like in Example 10.1. As calculated in (27), the first intention:

- *is surely satisfied when t is in the lower approximation of the interval* $[60, 100]$, *i.e., when* $t \in [60 + \epsilon_{\tau_i}, 100 - \epsilon_{\tau_i}]$, *also expressed as* $60 + \epsilon_{\tau_i} \leq t \leq 100 - \epsilon_{\tau_i}$
- *might be satisfied when t is in the upper approximation of the interval* $[60, 100]$ *i.e., when* $t \in [60 - \epsilon_{\tau_i}, 100]$, *which can be expressed as* $60 - \epsilon_{\tau_i} \leq t \leq 100$.

Similarly, according to (28), the second intention is surely satisfied when $\epsilon_\pi \leq p \leq 4 - \epsilon_\pi$ *and might be satisfied when* $0 \leq p \leq 4 + \epsilon_\pi$.

However, in real life the situation is not always that simple. For example, how can we deduce that the situation is (surely) safe? Using characterizations provided as (26) and (28), one could be tempted to define the lower approximation of $safe$ *to be*

(29) $-20 \leq t < 60 - \epsilon_{\tau_i} \ \vee\ [60 + \epsilon_{\tau_i} \leq t \leq 100 - \epsilon_{\tau_i} \ \wedge\ \epsilon_\pi \leq p \leq 4 - \epsilon_\pi]$.

Unfortunately, formula (29) does not properly reflect this approximation. In fact, Figure 4 shows the correct approximations. In particular, when the temperature is 60 and pressure is 2, formula (29) is not satisfied, while such a point is actually in the lower approximation of $safe$.■

In Example 10.4 we dealt with combining similarity relations and using suitable calculus on approximations. The problem of combining similarity relations and applying them within the framework of modal logics has been addressed, e.g., in [**?**, **?**]. However, reasoning based on these approaches is quite complex. In the sequel we will investigate whether using similarity structures and relational or deductive databases will improve the complexity of reasoning.

6. Team's Perspective

In multiagent settings paradigmatic activities, like coordination and negotiations naturally need agents' awareness about other and about their environment. Therefore, awareness becomes a vital ingredient of modeling. Sharing and/or fusing knowledge of agents calls for sometimes subtle methods of lifting individual beliefs to a group level. In this section we recall ideas of [**?**] needed for definition of the αBGI language and semantics (see Section 7), where Propositional Dynamic Logic (PDL) has been reinterpreted and applied to fuse similarities representing agents' perception.

6.1. Propositional Dynamic Logic. Originally, PDL has been introduced as a tool for expressing properties of programs and reasoning about them (see, e.g., [**?**]). Below we use it as a calculus on similarity relations. More precisely, programs in PDL are binary relations on states. We replace programs by (binary) similarity relations. Program connectives can then used to build up complex similarities for groups of agents. The resulting calculus has been discussed in [**?**].

6.1.1. *Syntax.* The language of PDL consists of formulas and programs.[7] In what follows we shall replace the term "programs" by "similarity relation symbols". Let $\mathcal{P}$ and $\mathcal{S}$ be countable sets of propositions and similarity relation symbols, respectively. Relational expressions and formulas are built from $\mathcal{P}$ and $\mathcal{S}$ using:

- *propositional connectives* $(\neg, \vee, \wedge, \rightarrow, \equiv)$;
- *modalities* indexed by similarity relation symbols $[\sigma], \langle\sigma\rangle$;
- *relational operators* $;\, , \cup\, , \cap\, ,^*\, ,^{-1}$ and *test operator* ?.

More precisely, *similarity expressions*, denoted by $\mathcal{E}$, are built inductively assuming that $\mathcal{S} \subseteq \mathcal{E}$ and whenever $\sigma_1, \sigma_2 \in \mathcal{E}$ and $A \in \mathcal{F}$ then

$$[\sigma_1; \sigma_2], [\sigma_1 \cup \sigma_2], [\sigma_1 \cap \sigma_2], [\sigma_1^*], [\sigma_1^{-1}], [A?] \in \mathcal{E}.$$

If A, B are PDL formulas and σ is a similarity expression then

$$\neg A, A \vee B, A \wedge B, A \rightarrow B, A \equiv B, [\sigma]A, \langle\sigma\rangle A$$

are also PDL formulas.

6.1.2. *Semantics.* The semantics of PDL is defined using the notion of Kripke frames of the form $\mathcal{K} = \langle U, \Pi, \Sigma\rangle$, where

- U is a set of objects
- $\Pi : \mathcal{P} \longrightarrow 2^U$ (for each proposition A, Π assigns a set of objects, for which A is TRUE)
- $\Sigma : \mathcal{S} \longrightarrow 2^{U\times U}$ (for each similarity relation symbol σ, Σ assigns a binary relation on U).

Let $\mathcal{K} = \langle U, \Pi, \Sigma\rangle$ be a Kripke structure, $a \in U$, A, B be formulas and σ_1, σ_2 be similarity relation symbols. The satisfiability relation is then defined as follows:

[7] We actually deal with the concurrent dynamic logic with converse, as we consider operators $\cap$ and $^{-1}$, too — see [**?, ?, ?**].

- $\mathcal{K}, a \models A$ iff $a \in \Pi(A)$, when $A \in \mathcal{P}$
- $\mathcal{K}, a \models \neg A$ iff $\mathcal{K}, a \not\models A$
- $\mathcal{K}, a \models A \vee B$ iff $\mathcal{K}, a \models A$ or $\mathcal{K}, a \models B$
- $\mathcal{K}, a \models [\sigma]A$ iff for any $b \in U$ such that $\sigma(a, b)$ we have $\mathcal{K}, b \models A$
- $\mathcal{K}, a \models \langle\sigma\rangle A$ iff there is $b \in U$ such that $\sigma(a, b)$ and $\mathcal{K}, b \models A$,

where Σ is extended to cover all expressions on similarity relations recursively:

- $\Sigma(\sigma_1; \sigma_2) \stackrel{\text{def}}{=} \Sigma(\sigma_1) \circ \Sigma(\sigma_2)$, where $\circ$ is the composition of relations
- $\Sigma(\sigma_1 \cup \sigma_2) \stackrel{\text{def}}{=} \Sigma(\sigma_1) \cup \Sigma(\sigma_2)$, where $\cup$ at the righthand side of equality is the union of relations
- $\Sigma(\sigma_1 \cap \sigma_2) \stackrel{\text{def}}{=} \Sigma(\sigma_1) \cap \Sigma(\sigma_2)$, where $\cap$ at the righthand side of equality is the intersection of relations
- $\Sigma(\sigma^*) \stackrel{\text{def}}{=} (\Sigma(\sigma))^*$, where * at the righthand side of equality is the transitive closure of a relation
- $\Sigma(\sigma^{-1}) \stackrel{\text{def}}{=} (\Sigma(\sigma))^{-1}$, where $^{-1}$ at the righthand side of equality is the converse of a relation
- $\Sigma(A?) \stackrel{\text{def}}{=} \{\langle a, a\rangle \mid \mathcal{K}, a \models A\}$.

6.2. Useful Properties of Approximations Expressible in PDL. One can easily observe that modalities of PDL can be used to express properties of approximations. Namely:

(1) $[\sigma]A$ expresses the lower approximation of A w.r.t.. σ, i.e., A_{σ^+}
(2) $\langle\sigma\rangle A$ expresses the upper approximation of A w.r.t.. σ, i.e., $A_{\sigma^\oplus}$.

EXAMPLE 10.5.

(1) $([\sigma_1]on\_road \wedge \langle\sigma_2\rangle fast) \rightarrow car$ — *if according to σ_1 an object is surely on road and according to σ_2 its speed might be fast, then conclude that it is a car.*
(2) $(\langle\sigma_1\rangle hot \vee \langle\sigma_2\rangle hot) \rightarrow dangerous$ — *if according to σ_1 or to σ_2 an object might be hot, then conclude that it is dangerous.*■

Expression σ^* defines the transitive closure of a relation σ, i.e., it makes an object o similar to an object o' if there is $k \geq 1$ and a chain of objects $o_1, \ldots, o_k$ such that $o_1 = o, o_k = o'$ and for all $1 \leq i \leq k - 1$, o_i is similar to o_{i+1}, i.e., $\sigma(o_i, o_{i+1})$ holds.

EXAMPLE 10.6. *In the rough set theory* [**?**]*, the underlying similarity relations are equivalence relations so rather than considering arbitrary relations σ, one should consider their reflexive, symmetric and transitive closures, $(\sigma \cup id \cup \sigma^{-1})^*$, where id is the identity relation (e.g., defined by the test statement* TRUE?*).*■

Other typical operations on programs can be interpreted in multiagent setting as follows, where σ_1 and σ_2 are similarity relations of the two agents ag_1 and ag_2:

- $\sigma_1; \sigma_2$ is the composition of relations, i.e., it makes an object o_1 similar to an object o_2, if agent ag_1 finds o_1 similar to some object o', and ag_2 considers object o' similar to o_2

- $\sigma_1 \cup \sigma_2$ is the set-theoretical union of relations, i.e., it makes an object o_1 similar to an object o_2, if at least one of agents ag_1, ag_2 considers objects o_1 and o_2 similar
- $\sigma_1 \cap \sigma_2$ is the set-theoretical intersection of relations, i.e., it makes an object o_1 similar to an object o_2, if both agents ag_1 and ag_2 consider objects o_1 and o_2 similar.

EXAMPLE 10.7. *Assume that agent ag_1 observes objects o_1, o_2 and finds that they are of a similar color ($\sigma_1(o_1, o_2)$ holds). Assume further that ag_2 observes objects o_2, o_3 and finds their color similar, too ($\sigma_2(o_2, o_3)$ holds). We are interested whether the color of o_1 is similar to the color of o_3, as expressed by*

$$((\sigma_1; \sigma_2) \cup (\sigma_2; \sigma_1))(o_1, o_3).$$

Therefore, e.g., $\langle(\sigma_1; \sigma_2) \cup (\sigma_2; \sigma_1)\rangle red$ expresses that a given object might be red according to the fused knowledge of ag_1 and ag_2. On the other hand, $[\sigma_1 \cap \sigma_2] red$ expresses the fact that both agents find a given object to be red.■

Test permits to create conditional definitions which can be viewed as a sort of meta-reasoning over similarities, as illustrated by the following example.

EXAMPLE 10.8. *In some circumstances the choice of similarity may depend on the state of the environment. For example, if the temperature is high, the observed process might be more sensitive on pressure (reflected by similarity σ_1) than when the temperature is not high (reflected by similarity σ_2). Then*

$$(high\_temp?; \sigma_1) \cup ((\neg high\_temp)?; \sigma_2))$$

expresses the guarded choice between σ_1 and σ_2 dependent on the temperature ("if the temperature is high then use σ_1 otherwise use σ_2").■

6.3. Reasoning over Concrete Similarity Structures. The version of dynamic logic we consider is highly undecidable (see, e.g., [**?**]). Removing the $\cap$ operator makes the logic decidable but still quite complex. When dealing with concrete similarity structures, the calculus is useful in practice.

EXAMPLE 10.9. *Consider a sensor measuring a given parameter, say ρ in the scale* $[0, 10]$. *Assume that the measurement error is not greater than 0.5. Thus we deal with a similarity structure $\langle U, \sigma\rangle$, where $U \stackrel{\text{def}}{=} [0, 10]$ and $\sigma(x, y) \stackrel{\text{def}}{\equiv} |x - y| \le 0.5$. Assume also that*

the value of ρ is acceptable *when $\rho \in [2.6,\ 6.8]$.*

Suppose that we are interested in evaluating formula $\langle\sigma\rangle acc$, where acc abbreviates "acceptable". Then it can easily be seen that

$$\langle\sigma\rangle acc \equiv \rho \in [2.1,\ 7.3] \ \text{ and } \ [\sigma] acc \equiv \rho \in [3.1,\ 6.3].$$

Therefore one can use these definitions instead of modal operators.■

EXAMPLE 10.10 (Examples 10.1, 10.3, 10.4 continued). *In Example 10.4 two similarity structures, S_{τ_1} and S_{τ_2}, reflecting perceptual limitations of agents ag_1 and ag_2, have been associated with temperature sensors. Different measurement results can lead the two* *agents to inconsistent points of view and mismatch in understanding the current situation. For example, assume that measurement errors considered in Example 10.4 are* $\epsilon_{\tau_1} = 0.1$ *and* $\epsilon_{\tau_2} = 0.15$. *Then the reading of* ag_1 *could be* 61.01 *while at the same time the reading of* ag_2 *could be* 58.88. *According to the crisp definition of high, given by (19),* ag_1 *considers the temperature high, while* ag_2 *considers it not high. Moreover, from (27) and further discussion it follows that the temperature is surely high for* ag_1, *while the temperature reported by the sensor of* ag_2 *is for* ag_1 *surely not high. This calls for fusing information from different sources. The problem of mutual understanding among communicating agents has been addressed in* [**?, ?**]. *Here we use more general calculus for information fusion, proposed in* [**?**] *and discussed in the current section.*

Importantly, a uniform principle fitting well to all situations is not to be expected here. For example, the similarity relation σ, *reflecting fused reading of temperatures could be*

- $\sigma \stackrel{\text{def}}{=} \sigma_{\tau_1}$, *when it is justified to trust the first sensor more than the second one*
- $\sigma \stackrel{\text{def}}{=} [moderate(p)?;\sigma_1] \cup [(\neg moderate(p))?;\sigma_2]$, *when the first sensor works within a "moderate" scale of pressures only*
- *etc.*

Observe that the resulting σ *can still be tuned. The intuition is that by adding successive iterations, the resulting similarity is more situation-sensitive. For example, safety may be increased by iterating* σ,

- σ *itself may be used for a normal safety level*
- $\sigma \cup (\sigma;\sigma)$ *can be used for an increased safety level*
- $\sigma \cup (\sigma;\sigma) \cup (\sigma;\sigma;\sigma)$ *can be used in circumstances, when a relatively higher safety is required*
- *etc.*■

REMARK 6.1. Even if the considered version of dynamic logic is highly undecidable (see, e.g., [**?**]), dealing with similarity structures and relational or deductive databases makes the situation tractable, assuming that the underlying similarity relation is tractable. Importantly, in practical reasoning systems one is usually interested in querying databases rather than verifying whether a given formula is a tautology or is satisfiable.

In order to compute the set of objects x satisfying $\langle\sigma\rangle A$ one simply queries the database using the first-order formula

$\exists y(\sigma(x,y) \wedge A(y))$, where y refers to objects (e.g., specified by rows in database tables).[8]

Similarly, computing $[\sigma]A$ depends on supplying to the database the query

$$\forall y(\sigma(x,y) \rightarrow A(y)).$$

Operators on similarity relations allowed in the proposed language are first-order definable, except for * which, as transitive closure, also leads to tractable queries (see, e.g., [**?**]).

[8] This query computes all values of x satisfying $\exists y(\sigma(x,y) \wedge A(y))$.

Note also that restricting full PDL to its fragments can improve complexity significantly. For example, its tractable Horn fragment suitable for approximate reasoning has been isolated in [**?**].■

7. The αBGI Language and Semantics

In what follows we assume that $\mathcal{L}$ is the *base language*, i.e., the set of *basic formulas*. It can be the language of the classical propositional logic, of the classical first-order logic, of the version of PDL discussed in Section 6.1 or any other logic suitable for a particular application domain. In applications, $\mathcal{L}$ is typically a language of underlying relational or deductive database.

It is assumed that the semantics of $\mathcal{L}$ is given by assigning to each formula the set of elements (objects) of the underlying domain, i.e., for a given domain U there is a mapping $\Pi : \mathcal{L} \longrightarrow 2^U$. Intuitively, for $A \in \mathcal{L}$, $\Pi(A)$ is the set of objects for which A is true.[9] In many applications Π can be defined by database queries.

EXAMPLE 10.11. *Assume we have two objects: the first one, o_1, large and red and the second one, o_2, small and red. In this case $U = \{o_1, o_2\}$.*

- *Let the base language be the classical propositional logic with two propositions large and red. Then, for example, it is natural (although not necessary, as we leave much freedom here) to set $\Pi(red) = \{o_1, o_2\}$ and $\Pi(large \wedge red) = \{o_1\}$.*
- *Let the base language be the classical first-order logic with two unary relations Large, Red and one binary relation Obj consisting of tuples describing objects in U,*
$$Obj \stackrel{\text{def}}{=} \{\langle large, red\rangle, \langle small, red\rangle\}.$$
Then, $\Pi(Red(x))$ is $\{o_1, o_2\}$, while $\Pi(\exists x(Obj(large, x)))$ is $\{o_1\}$.
*A bit more intriguing question could be related to the case when formulas have free variables, e.g., what should be $\Pi(Obj(large, x))$. One of natural solutions is to apply the standard querying machinery (e.g., the classical first-order queries, as described in [**?**]). The result would be $x = red$. Therefore Π should return all red objects, i.e., $\{o_1, o_2\}$.* ■

REMARK 7.1. It is worth emphasizing that objects can be relatively simple, belonging to concepts like "red", "large", etc. as well as arbitrarily complex, like "safe speed", "bad weather conditions", "criminal activity", etc. ■

7.1. Syntax.

We assume that $\mathcal{A}$ is the set of agents and $\mathcal{L}$ is the base language.

As in Section 6.1, we assume the set $\mathcal{S}$ of similarity relation symbols. Recall the $\mathcal{E}$ denotes the set of similarity expressions.

*The set of α*BGI *formulas*, denoted by $\mathcal{F}$, is the smallest set closed under classical propositional connectives, such that $\mathcal{L} \subseteq \mathcal{F}$ and for $a \in \mathcal{A}$, $A \in \mathcal{F}$ and $\sigma \in \mathcal{E}$,

$$Bel_a^{\langle\sigma\rangle}A, Bel_a^{[\sigma]}A, Goal_a^{\langle\sigma\rangle}A, Goal_a^{[\sigma]}A, Int_a^{\langle\sigma\rangle}A, Int_a^{[\sigma]}A \in \mathcal{F}.$$

Intuitively, assuming that the perception of agent a is modeled by similarity expression σ,

[9] Observe that this approach is compatible with semantics of dynamic logic, given in Section 6.1.2.

- $Bel_a^{\langle\sigma\rangle}A$ means that from the point of view of agent a the state of the world possibly satisfies belief A
- $Bel_a^{[\sigma]}A$ means that from the point of view of agent a the state of the world surely satisfies belief A,

and analogously for modalities expressing goals and intentions $Goal_a^{\langle\sigma\rangle}$, $Goal_a^{[\sigma]}$, $Int_a^{\langle\sigma\rangle}$, $Int_a^{[\sigma]}$.

7.2. Semantics. Let $\mathcal{L}$ be the base language. Assume we are given a Kripke structure $\mathcal{K} = \langle U, \Pi, \Sigma\rangle$, as in Section 6.1.2, except that the mapping Π is defined for all formulas in $\mathcal{L}$, $\Pi : \mathcal{L} \longrightarrow 2^U$. The semantics of αBGI formulas is given by extending Π to cover all αBGI formulas, where $X \in \{Bel, Goal, Int\}$:

$$\begin{array}{lcl}
\Pi(\neg A) & \stackrel{\text{def}}{=} & -\Pi(A) \\
\Pi(A \vee B) & \stackrel{\text{def}}{=} & \Pi(A) \cup \Pi(B) \\
\Pi(A \wedge B) & \stackrel{\text{def}}{=} & \Pi(A) \cap \Pi(B) \\
\Pi(A \rightarrow B) & \stackrel{\text{def}}{=} & -\Pi(A) \cup \Pi(B) \\
\Pi(A \equiv B) & \stackrel{\text{def}}{=} & (-\Pi(A) \cup \Pi(B)) \cap (-\Pi(B) \cup \Pi(A)) \\
\Pi(X_a^{\langle\sigma\rangle}A) & \stackrel{\text{def}}{=} & \{o \in U \mid \exists o'(\Sigma(\sigma)(o, o') \wedge o' \in \Pi(A))\} \\
\Pi(X_a^{[\sigma]}A) & \stackrel{\text{def}}{=} & \{o \in U \mid \forall o'(\Sigma(\sigma)(o, o') \rightarrow o' \in \Pi(A))\}.
\end{array}$$

For any αBGI formula $A \in \mathcal{F}$, Kripke structure $\mathcal{K} = \langle U, \Pi, \Sigma\rangle$ and any $o \in U$, we define the satisfiability relation $\models_{\alpha\text{BGI}}$ by setting:

$$\mathcal{K}, o \models_{\alpha\text{BGI}} A \text{ iff } o \in \Pi(A).$$

EXAMPLE 10.12. *Agent ag classified the objects from U according to their colors:*

$$Bel_{ag}(light \equiv (white \vee yellow)).$$

However, ag cannot distinguish between yellow and orange, which is modeled by the similarity relation

$$\sigma \stackrel{\text{def}}{=} \{\langle o_1, o_2\rangle \mid o_1, o_2 \in \Pi(yellow) \cup \Pi(orange)\} \cup \{\langle o, o\rangle \mid o \in U\}.$$

Assuming further that

$$\Pi(white) = \{o_1\}, \Pi(yellow) = \{o_2, o_3\}, \Pi(orange) = \{o_4\},$$

we have $\Pi(Bel_{ag}^{\langle\sigma\rangle} light) = \{o_1, o_2, o_3, o_4\}$, *while* $\Pi(Bel_{ag}^{[\sigma]} light) = \{o_1\}$. ■

EXAMPLE 10.13 (Examples 10.1, 10.3, 10.4, 10.10 continued). *In Example 10.3 (formula (23)) there have been two intentions,*

$$Int_{ag_1} high(t) \text{ and } Int_{ag_2} moderate(p).$$

According to calculations given in Example 10.4 (see formulas (27), (28) and the following discussion), assuming $\epsilon_{\tau_1} = 0.1$ *and* $\epsilon_\pi = 0.2$, *we have*

$$\begin{array}{l}
Int_{ag_1}^{[\sigma_{\tau_1}]} high(t) \;\equiv\; 60 + \epsilon_{\tau_1} \leq t \leq 100 - \epsilon_{\tau_1} \;\equiv\; 60.1 \leq t \leq 99.9 \\
Int_{ag_1}^{\langle\sigma_{\tau_1}\rangle} high(t) \;\equiv\; 60 - \epsilon_{\tau_1} \leq t \leq 100 + \epsilon_{\tau_1} \;\equiv\; 59.9 \leq t \leq 100.1 \\
Int_{ag_2}^{[\sigma_\pi]} moderate(p) \;\equiv\; \epsilon_\pi \leq p \leq 4 - \epsilon_\pi \;\equiv\; 0.2 \leq t \leq 3.8 \\
Int_{ag_2}^{\langle\sigma_\pi\rangle} moderate(p) \;\equiv\; 0 \leq p \leq 4 + \epsilon_\pi \;\equiv\; 0 \leq t \leq 4.2.
\end{array}$$

*Let us return to the question of proper approximations of beliefs in a specification of a $safe$ situation (see Example 10.10: formula (29) and later discussion). Safety condition (22) refers both to temperature and pressure. It is therefore necessary to create a new similarity structure on tuples $\langle t, p\rangle$ representing states described by temperature and pressure, on the basis of the two underlying similarity structures. Such a construction heavily depends on an application in question (see, e.g., [**?, ?**]). For example, a finer granularity for pressure might be required when temperature exceeds a certain threshold.*

Here, to keep things simple, let us assume that the adequate similarity structure is defined over the cartesian product $[-20, 180] \times [0, 10]$ of intervals representing domains of temperatures and pressures (as assumed in Example 10.4), where the similarity relation, denoted by $\sigma_\times$, is defined by

$$\sigma_\times(\langle t_1, p_1\rangle, \langle t_2, p_2\rangle) \stackrel{\text{def}}{\equiv} \sigma_{\tau_1}(t_1, t_2) \wedge \sigma_\pi(p_1, p_2).$$

In this case,

$$\begin{aligned} Bel_{ag_2}^{[\sigma_\times]} safe(t, p) \equiv \; & [(-20 + \epsilon_{\tau_1} \leq t \leq 60 - \epsilon_{\tau_1}) \wedge (0 \leq p \leq 10 - \epsilon_p)] \\ & \vee [(-20 + \epsilon_{\tau_1} \leq t \leq 100 - \epsilon_{\tau_1}) \wedge (0 \leq p \leq 4 - \epsilon_\pi)] \\ Bel_{ag_2}^{\langle\sigma_\times\rangle} safe(t, p) \equiv \; & [(-20 - \epsilon_{\tau_1} \leq t \leq 60 + \epsilon_{\tau_1}) \wedge (0 \leq p \leq 10 + \epsilon_p)] \\ & \vee [(-20 - \epsilon_{\tau_1} \leq t \leq 100 + \epsilon_{\tau_1}) \wedge (0 \leq p \leq 4 + \epsilon_\pi)], \end{aligned}$$

as indicated in Figure 4.■

7.3. The Complexity of Queries. Assume that a given Kripke structure $\mathcal{K} = \langle U, \Pi, \Sigma\rangle$ satisfies:

- U is a finite set such that $| U |= n$, for a natural number $n > 0$
- for any $A \in \mathcal{L}$ and $o \in U$, condition $o \in \Pi(A)$ can be verified deterministically in time polynomial w.r.t. n
- for any $\sigma \in \mathcal{S}$, and $o_1, o_2 \in U$, condition $\Sigma(\sigma)(o_1, o_2)$ can be verified deterministically in time polynomial w.r.t. n.

Under these assumptions, for any αBGI formula A and any $o \in U$, the satisfiability condition $\mathcal{K}, o \models_{\alpha\text{BGI}} A$ can also be checked deterministically in time polynomial w.r.t. n. This can be seen by treating formula A as a query to $\mathcal{K}$ considered as a deductive database.

8. Conclusions

Usually, modeling multiagent systems starts from an idealized level of agents' beliefs. Our approach is grounded where agent activities actually start: at the level of modeling results of agents' perception. This change of perspective enabled a shift from the multimodal modeling of BGI systems to approximate multiagent systems called αBGI. Taking into account a dynamic and unpredictable environment the agent acts in, it is justified to assume that information available to agents via their sensor platforms is incomplete, uncertain and imprecise. To reflect this unfavorable combination of properties, we have equipped agents with a powerful machinery to verify whether a particular state of the world, actual or hypothetical, surely or possibly satisfies their beliefs, goals and intentions.

The immediate gain is that the introduced first-order formalism permits both to embed typical models of sensors and perception and to deal with vague concepts. Assuming that agents actually work over relational or deductive databases and that similarities themselves are tractable, we have obtained a pragmatic, tractable framework, allowing one to implement αBGI specifications in an efficient manner.

We have also reinterpreted dynamic logic to serve as a calculus on similarities. This way some subtle methods of fusing information available from heterogenous sources are naturally expressible. We proposed the calculus to become a vital component of αBGI specifications. In summary, we focused on methodological issues and proposed how to deal with them. Typical logical issues like providing proof systems and investigating their meta-properties are left for further research.

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