Database Technology

Functional Dependencies and Normalization

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Make sure you have a sheet of paper and a pen.

You will need them for the exercises today.

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Quiz

- Constraint between two sets of attributes from a relation

> Let *R* be a relational schema with the attributes $A_1, A_2, \ldots, A_n$ and let *X* and *Y* be subsets of $\{A_1, A_2, \ldots, A_n\}$.
>
> Then, the functional dependency $X \rightarrow Y$ specifies the following constraint on *any* valid relation state *r* of *R*.
>
> For *any* two tuples t_1 and t_2 in state *r* we have that:
>
> if $t_1[X] = t_2[X]$, then $t_1[Y] = t_2[Y]$.

- Consider the relation **R(A,B,C,D,E,F)** with the following FDs:

  FD1: **A** → **BC** FD2: **C** → **AD** FD3: **DE** → **F**

- Is a state of R that contains the two tuples (3, 8, 1, 2, 3, 4) and (3, 8, 1, 7, 3, 9) valid?

  A) Yes B) No

A	B	C	D	E	F
3	8	1	2	3	4
3	8	1	7	3	9

Reasoning About FDs

- Logical implications can be derived by using inference rules called **Armstrong's rules**:
  - *Reflexivity:* If Y is a subset of X, then $X \rightarrow Y$
  - *Augmentation:* If $X \rightarrow Y$, then $XZ \rightarrow YZ$

    (we use XY as a short form for $X \cup Y$)
  - *Transitivity:* If $X \rightarrow Y$ and $Y \rightarrow Z$, then $X \rightarrow Z$

- Additional rules can be derived:
  - *Decomposition:* If $X \rightarrow YZ$, then $X \rightarrow Y$
  - *Union:* If $X \rightarrow Y$ and $X \rightarrow Z$, then $X \rightarrow YZ$
  - *Pseudo-transitivity:* If $X \rightarrow Y$ and $WY \rightarrow Z$, then $WX \rightarrow Z$

Exercises: Armstrong Rules

Reflexivity: If Y is a subset of X, then $X \rightarrow Y$

Augmentation: If $X \rightarrow Y$, then $XZ \rightarrow YZ$

Transitivity: If $X \rightarrow Y$ and $Y \rightarrow Z$, then $X \rightarrow Z$

Decomposition: If $X \rightarrow YZ$, then $X \rightarrow Y$

Union: If $X \rightarrow Y$ and $X \rightarrow Z$, then $X \rightarrow YZ$

Pseudo-transitivity: If $X \rightarrow Y$ and $WY \rightarrow Z$, then $WX \rightarrow Z$

- Consider the relation **R(A,B,C,D,E,F)** with the following FDs:
  - FD1: **A → BC**
  - FD2: **C → AD**
  - FD3: **DE → F**
- Use the Armstrong rules to derive the following FD: **AC → D**

Exercises: Armstrong Rules

Reflexivity: If Y is a subset of X, then $X \rightarrow Y$

Augmentation: If $X \rightarrow Y$, then $XZ \rightarrow YZ$

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Union: If $X \rightarrow Y$ and $X \rightarrow Z$, then $X \rightarrow YZ$

Pseudo-transitivity: If $X \rightarrow Y$ and $WY \rightarrow Z$, then $WX \rightarrow Z$

- Consider the relation **R(A,B,C,D,E,F)** with the following FDs:
  FD1: **A** → **BC**
  FD2: **C** → **AD**
  FD3: **DE** → **F**
- Use the Armstrong rules to derive the following FD: **AC** → **D**
  - FD4: **AC** → **AD** (Augmentation of FD2 with A)
  - FD5: **AC** → **D** (Decomposition of FD4)

Exercises: Armstrong Rules

Reflexivity: If Y is a subset of X, then $X \rightarrow Y$

Augmentation: If $X \rightarrow Y$, then $XZ \rightarrow YZ$

Transitivity: If $X \rightarrow Y$ and $Y \rightarrow Z$, then $X \rightarrow Z$

Decomposition: If $X \rightarrow YZ$, then $X \rightarrow Y$

Union: If $X \rightarrow Y$ and $X \rightarrow Z$, then $X \rightarrow YZ$

Pseudo-transitivity: If $X \rightarrow Y$ and $WY \rightarrow Z$, then $WX \rightarrow Z$

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- Use the Armstrong rules to derive the following FD: **AC → D**
  - FD4: **AC → AD** (Augmentation of FD2 with A)
  - FD5: **AC → D** (Decomposition of FD4)
- Use the Armstrong rules to derive the following FD: **A → D**

Exercises: Armstrong Rules

Reflexivity: If Y is a subset of X, then $X \rightarrow Y$

Augmentation: If $X \rightarrow Y$, then $XZ \rightarrow YZ$

Transitivity: If $X \rightarrow Y$ and $Y \rightarrow Z$, then $X \rightarrow Z$

Decomposition: If $X \rightarrow YZ$, then $X \rightarrow Y$

Union: If $X \rightarrow Y$ and $X \rightarrow Z$, then $X \rightarrow YZ$

Pseudo-transitivity: If $X \rightarrow Y$ and $WY \rightarrow Z$, then $WX \rightarrow Z$

- Consider the relation **R(A,B,C,D,E,F)** with the following FDs:
  - FD1: **A → BC**
  - FD2: **C → AD**
  - FD3: **DE → F**
- Use the Armstrong rules to derive the following FD: **AC → D**
  - FD4: **AC → AD** (Augmentation of FD2 with A)
  - FD5: **AC → D** (Decomposition of FD4)
- Use the Armstrong rules to derive the following FD: **A → D**
  - FD6: **A → C** (Decomposition of FD1)
  - FD7: **A → AD** (Transitivity of FD6 and FD2)
  - FD8: **A → D** (Decomposition of FD7)

Exercises: Armstrong Rules

Reflexivity: If *Y* is a subset of *X*, then $X \rightarrow Y$

Augmentation: If $X \rightarrow Y$, then $XZ \rightarrow YZ$

Transitivity: If $X \rightarrow Y$ and $Y \rightarrow Z$, then $X \rightarrow Z$

Decomposition: If $X \rightarrow YZ$, then $X \rightarrow Y$

Union: If $X \rightarrow Y$ and $X \rightarrow Z$, then $X \rightarrow YZ$

Pseudo-transitivity: If $X \rightarrow Y$ and $WY \rightarrow Z$, then $WX \rightarrow Z$

- Consider the relation **R(A,B,C,D,E,F)** with the following FDs:
  FD1: **A** → **BC**
  FD2: **C** → **AD**
  FD3: **DE** → **F**
- Use the Armstrong rules to derive the following FD: **AE** → **ABCDEF**

Exercises: Armstrong Rules

Reflexivity: If Y is a subset of X, then $X \to Y$

Augmentation: If $X \to Y$, then $XZ \to YZ$

Transitivity: If $X \to Y$ and $Y \to Z$, then $X \to Z$

Decomposition: If $X \to YZ$, then $X \to Y$

Union: If $X \to Y$ and $X \to Z$, then $X \to YZ$

Pseudo-transitivity: If $X \to Y$ and $WY \to Z$, then $WX \to Z$

- Consider the relation **R(A,B,C,D,E,F)** with the following FDs:
  - FD1: **A → BC**
  - FD2: **C → AD**
  - FD3: **DE → F**
- Use the Armstrong rules to derive the following FD: **AE → ABCDEF**
  - FD9: **AE → BCE** (Augmentation of FD1 with E)
  - FD10: **AE → C** (Decomposition of FD9)
  - FD11: **AE → AD** (Transitivity of FD10 and FD2)
  - FD12: **AE → ADE** (Augmentation of FD11 with E)
  - FD13: **AE → DE** (Decomposition of FD12)
  - FD14: **AE → F** (Transitivity of FD13 and FD3)
  - FD15: **AE → ABCDEF** (Union of FD9, FD11, and FD14)

Exercise

- Consider the relation **R(A,B,C,D,E,F)** with the following FDs:
  - FD1: **A** → **BC**
  - FD2: **C** → **AD**
  - FD3: **DE** → **F**
- Use the Armstrong rules to derive the following FD: **AE → ABCDEF**
- Consider the following state of relation R:

A	B	C	D	E	F
3	8	1	2	3	4

- Add another tuple to this state such that the resulting extended state is valid with respect to the new FD (i.e., does not violate the FD)

Exercise

- Consider the relation **R(A,B,C,D,E,F)** with the following FDs:

  FD1: **A** → **BC**

  FD2: **C** → **AD**

  FD3: **DE** → **F**

- Use the Armstrong rules to derive the following FD: **AE → ABCDEF**
- Consider the following state of relation R:

A	B	C	D	E	F
3	8	1	2	3	4
4	..	..	..	3	..
~~3~~	~~..~~	~~..~~	~~..~~	~~3~~	~~..~~

- Add another tuple to this state such that the resulting extended state is valid with respect to the new FD (i.e., does not violate the FD)
  - any tuple in which the value for A or E (or both) is different from the corresponding values of the given tuple

Computing (Super)Keys

function *ComputeAttrClosure*(*X*, *F*)
begin
    $X^+ := X$;
    while *F* contains an FD $Y \rightarrow Z$ such that
        (i) *Y* is a subset of X^+, and
        (ii) *Z* is not a subset of X^+ **do**
      $X^+ := X^+ \cup Z$;
    end while
    return X^+;
end

Warmup (cont'd)

```
function ComputeAttrClosure( X, F )
begin
    X⁺ := X;
    while F contains an FD Y → Z such that
        (i) Y is a subset of X⁺, and
        (ii) Z is not a subset of X⁺  do
      X⁺ := X⁺ U Z;
    end while
    return X⁺;
end
```

- Consider the relation **R(A,B,C,D,E,F)** with the following FDs:
  - FD1: **A** → **BC**
  - FD2: **C** → **AD**
  - FD3: **DE** → **F**
- Compute the **attribute closure** of *X* = { **A,E** } w.r.t. *F* = {FD1, FD2, FD3} to show that we have: **AE** → **ABCDEF**
  - Initially: X^+ = { **A,E** }
  - By using FD1: X^+ = { **A,E,B,C** }
  - By using FD2: X^+ = { **A,E,B,C,D** }
  - By using FD3: X^+ = { **A,E,B,C,D,F** }

Your Turn

function *ComputeAttrClosure*(*X*, *F*)
begin
    $X^+ := X$;
    while *F* contains an FD $Y \rightarrow Z$ such that
        (i) *Y* is a subset of X^+, and
        (ii) *Z* is not a subset of X^+ **do**
        $X^+ := X^+ \cup Z$;
    end while
    return X^+;
end

- Consider the relation **R(A,B,C,D)** with the following set *F* of FDs:
  FD1: **AB** → **C**
  FD2: **BC** → **D**
  FD3: **D** → **B**
- Compute the following attribute closures w.r.t. *F* = {FD1, FD2, FD3}
  - $\{B,C,D\}^+ =$?
  - $\{A,C,D\}^+ =$?
  - $\{A,B,D\}^+ =$?
  - $\{A,B,C\}^+ =$?

Your Turn

```
function ComputeAttrClosure( X, F )
begin
    X+ := X;
    while F contains an FD Y → Z such that
        (i) Y is a subset of X+, and
        (ii) Z is not a subset of X+ do
      X+ := X+ U Z;
    end while
    return X+;
end
```

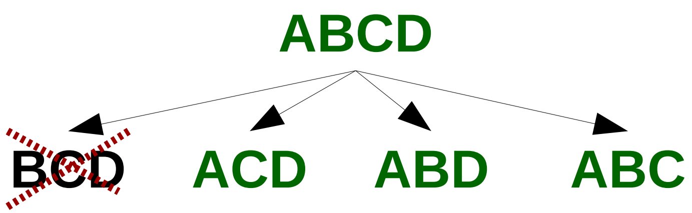
- Consider the relation **R(A,B,C,D)** with the following set *F* of FDs:
  - FD1: **AB** → **C**
  - FD2: **BC** → **D**
  - FD3: **D** → **B**
- Compute the following attribute closures w.r.t. *F* = {FD1, FD2, FD3}
  - $\{ B,C,D \}^+ = \{ B,C,D \}$
  - $\{ A,C,D \}^+ = \{ A,C,D,B \}$
  - $\{ A,B,D \}^+ = \{ A,B,D,C \}$
  - $\{ A,B,C \}^+ = \{ A,B,C,D \}$

Superkeys and Candidate Keys

- Consider the relation **R(A,B,C,D)** with the following set *F* of FDs:
  FD1: **AB** → **C**
  FD2: **BC** → **D**
  FD3: **D** → **B**

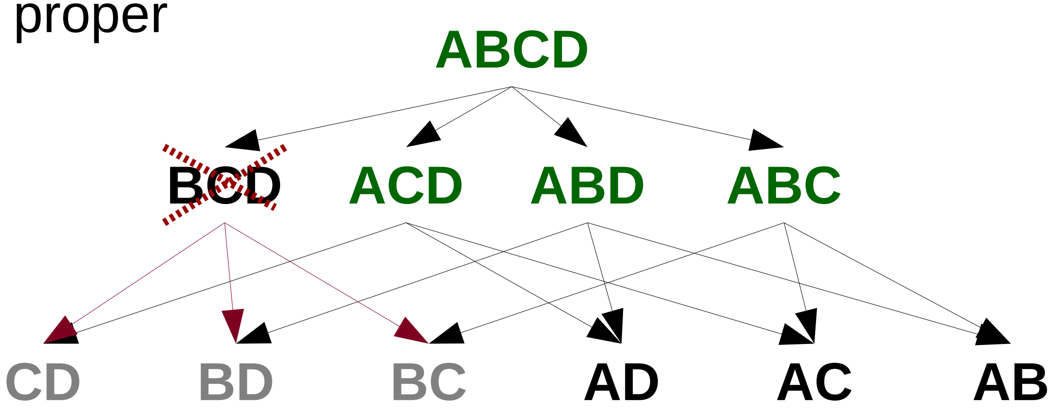
- A set *X* of attributes of *R* is a superkey if X^+ contains all the attributes of *R*

  - $\{ B,C,D \}^+ = \{ B,C,D \}$
  - $\{ A,C,D \}^+ = \{ A,C,D,B \}$
  - $\{ A,B,D \}^+ = \{ A,B,D,C \}$
  - $\{ A,B,C \}^+ = \{ A,B,C,D \}$



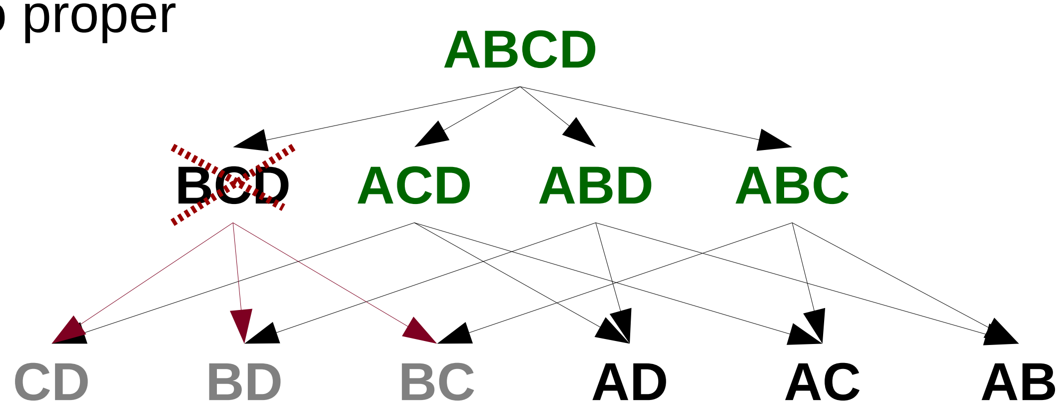
Superkeys and Candidate Keys

- Consider the relation **R(A,B,C,D)** with the following set *F* of FDs:
  FD1: **AB** → **C**
  FD2: **BC** → **D**
  FD3: **D** → **B**
- A set *X* of attributes of *R* is a superkey if X^+ contains all the attributes of *R*
- *X* is a candidate key (CK) if no proper subset *X'* of *X* is a superkey
  - $\{A,D\}^+ =$?
  - $\{A,C\}^+ =$?
  - $\{A,B\}^+ =$?



Superkeys and Candidate Keys

- Consider the relation **R(A,B,C,D)** with the following set *F* of FDs:
  FD1: **AB** → **C**
  FD2: **BC** → **D**
  FD3: **D** → **B**
- A set *X* of attributes of *R* is a superkey if X^+ contains all the attributes of *R*
- *X* is a candidate key (CK) if no proper subset *X'* of *X* is a superkey
  - $\{A,D\}^+ = \{A,D,B,C\}$
  - $\{A,C\}^+ = \{A,C\}$
  - $\{A,B\}^+ = \{A,B,C,D\}$



Superkeys and Candidate Keys

- Consider the relation **R(A,B,C,D)** with the following set *F* of FDs:
  FD1: **AB** → **C**
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- A set *X* of attributes of *R* is a superkey if X^+ contains all the attributes of *R*
- *X* is a candidate key (CK) if no proper subset *X'* of *X* is a superkey
  - $\{A,D\}^+ = \{A,D,B,C\}$
  - $\{A,C\}^+ = \{A,C\}$
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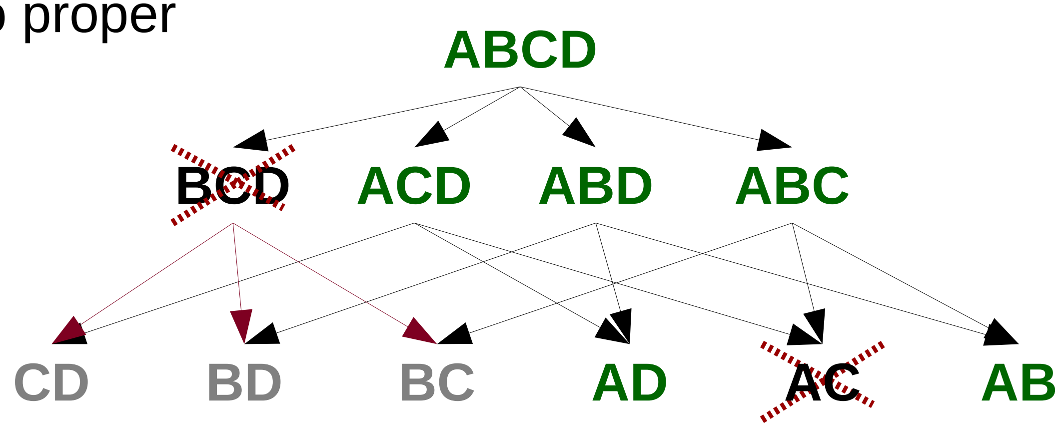

Superkeys and Candidate Keys

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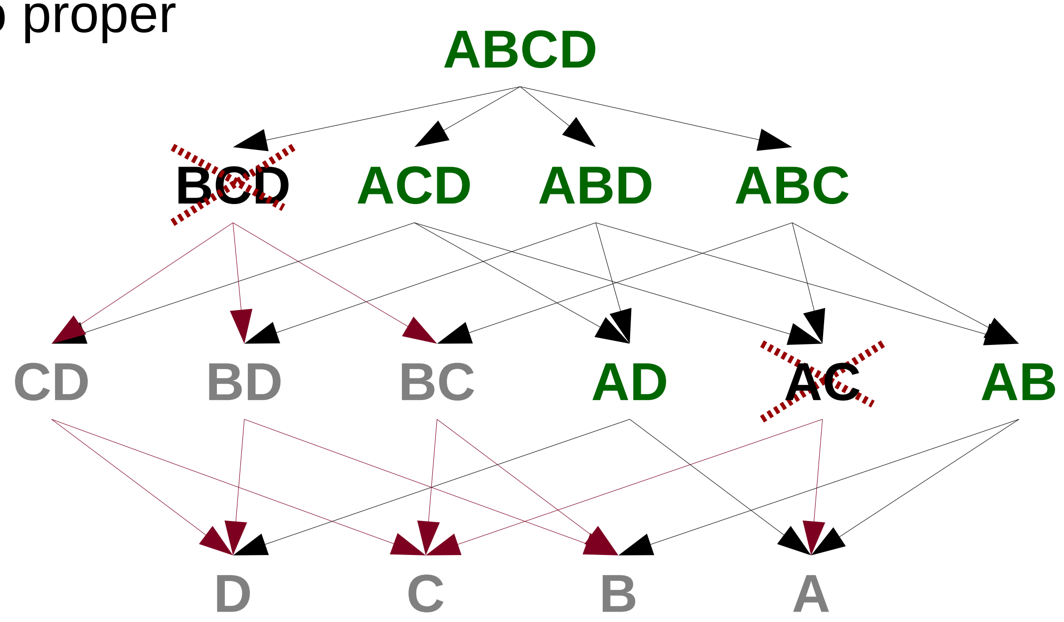

Superkeys and Candidate Keys

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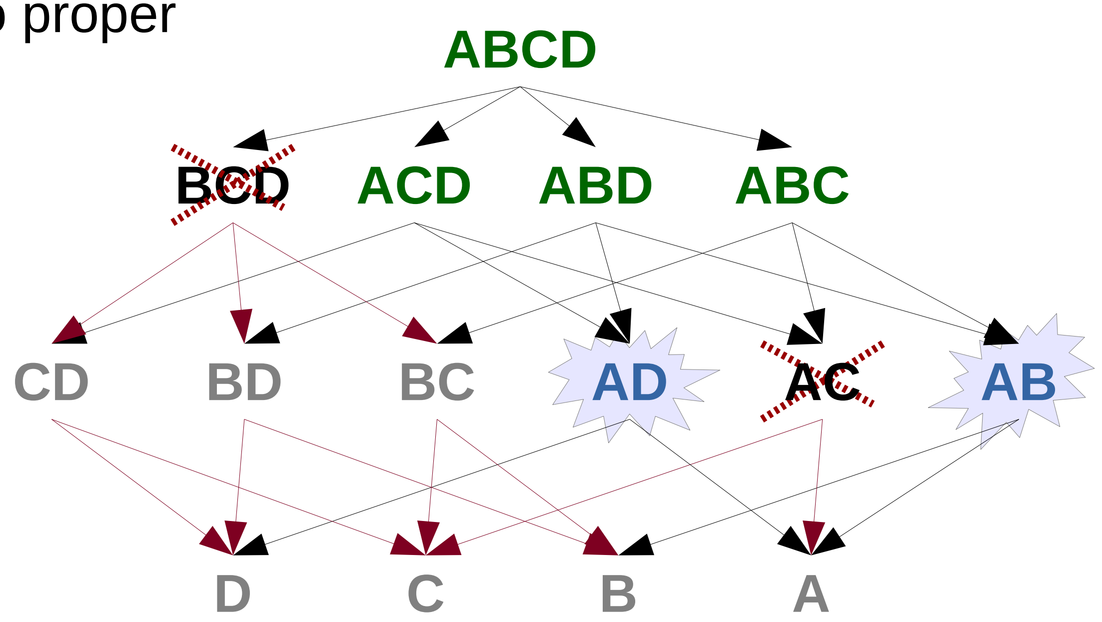

Superkeys and Candidate Keys

- Consider the relation **R(A,B,C,D)** with the following set *F* of FDs:
  FD1: **AB** → **C**
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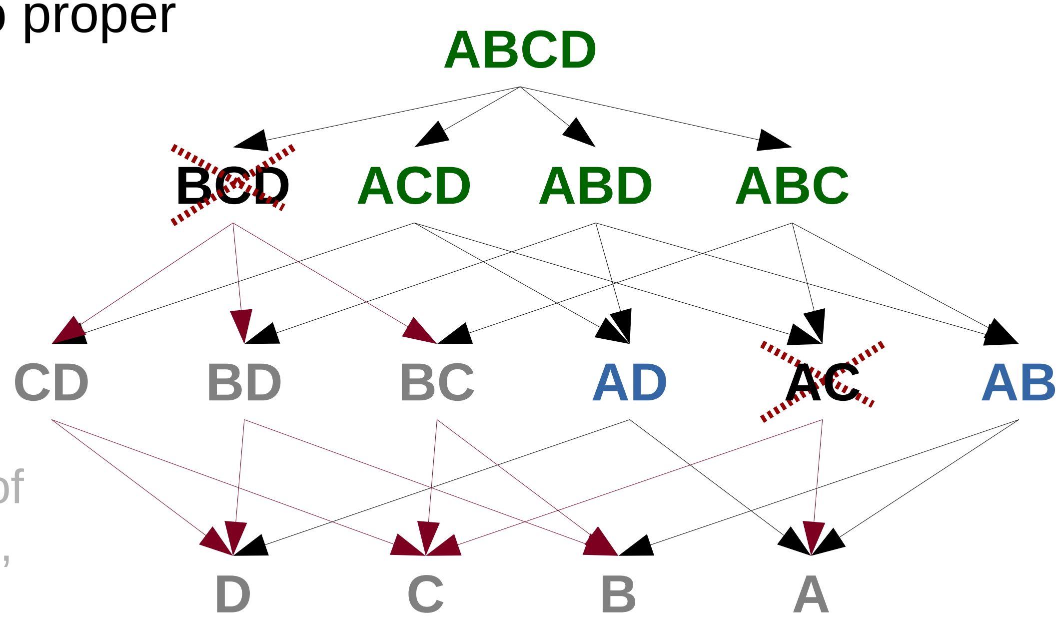
- A set *X* of attributes of *R* is a superkey if X^+ contains all the attributes of *R*
- *X* is a candidate key (CK) if no proper subset *X'* of *X* is a superkey
- Some steps may be skipped
  - If an attribute is nowhere in the RHS, it must be part of *every* candidate key (such as *A* in the example)
  - If an attribute is in the RHS of FDs but nowhere in the LHS, it cannot be part of any CK



Superkeys and Candidate Keys

- Consider the relation **R(A,B,C,D)** with the following set *F* of FDs:
  FD1: **AB** → **C**
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- A set *X* of attributes of *R* is a superkey if X^+ contains all the attributes of *R*
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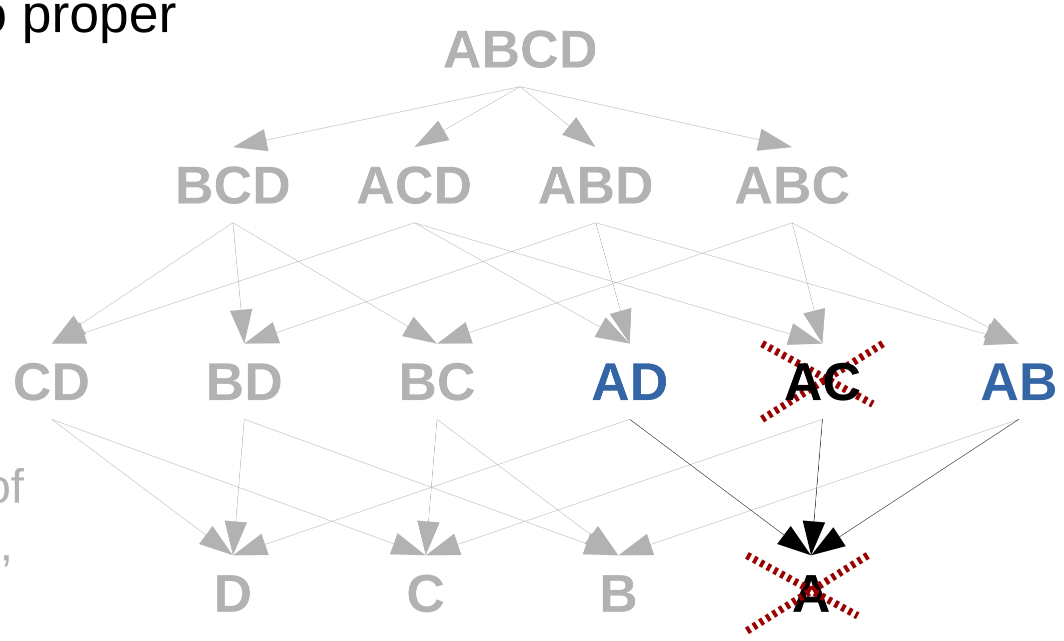


Superkeys and Candidate Keys

- Consider the relation **R(A,B,C,D)** with the following set *F* of FDs:

  FD1: **AB** → **C**

  FD2: **BC** → **D**

  ~~FD3: **D** → **B**~~

- A set *X* of attributes of *R* is a superkey if X^+ contains all the attributes of *R*

- *X* is a candidate key (CK) if no proper subset *X'* of *X* is a superkey

- Some steps may be skipped
  - If an attribute is nowhere in the RHS, it must be part of *every* candidate key (such as *A* in the example)
  - If an attribute is in the RHS of FDs but nowhere in the LHS, it cannot be part of any CK

- For instance, if we had only FD1 and FD2 (i.e., not FD3), we could immediately say that:
  - *D* cannot be in any CK
  - *A* and *B* must be in every CK
  - and, since $\{A,B\}^+ = \{A,B,C,D\}$, we see that {A,B} is a superkey and, thus, the only CK

Your Turn

- Consider the relation **R(A,B,C,D,E,F)** with the following FDs:

  FD1: **A** → **BC**

  FD2: **C** → **AD**

  FD3: **DE** → **F**

- What are the candidate keys?

Your Turn

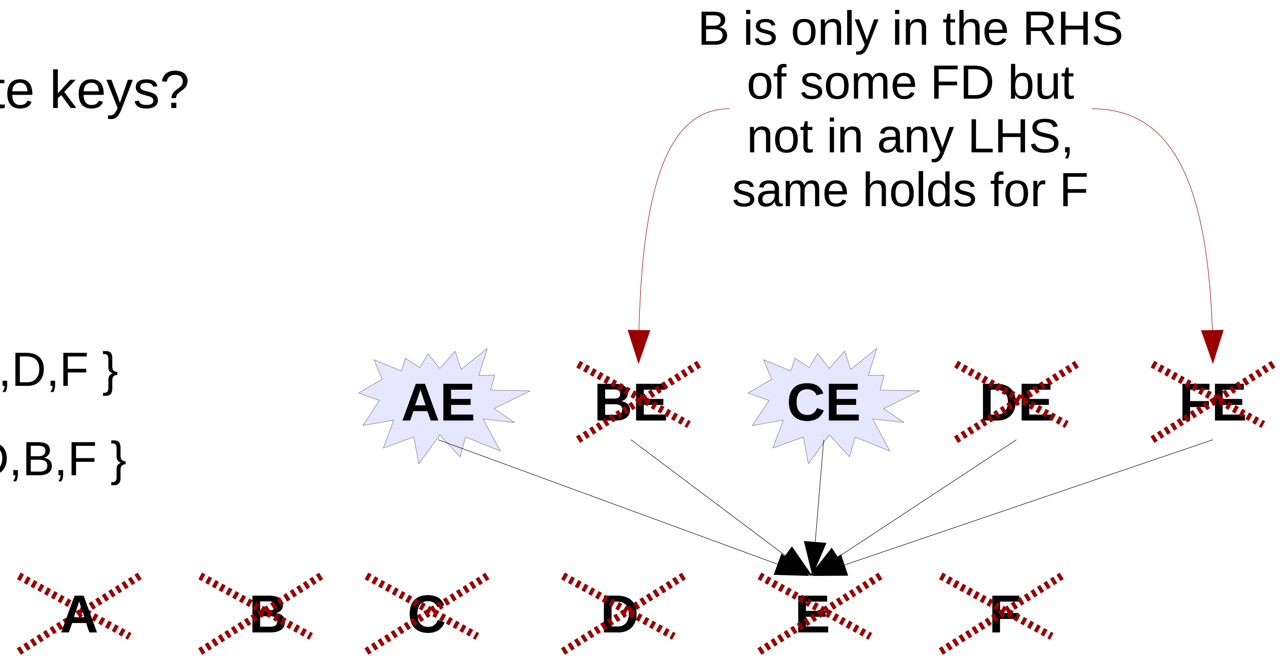
- Consider the relation **R(A,B,C,D,E,F)** with the following FDs:

  FD1: **A** → **BC**

  FD2: **C** → **AD**

  FD3: **DE** → **F**

- What are the candidate keys?
  - {A,E} and {C,E}
  - $\{ E \}^+ = \{ E \}$
  - $\{ A,E \}^+ = \{ A,E,B,C,D,F \}$
  - $\{ C,E \}^+ = \{ C,E,A,D,B,F \}$
  - $\{ D,E \}^+ = \{ D,E,F \}$



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