Database Technology

Topic 6: Functional Dependencies and Normalization

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Make sure you have a sheet of paper and a pen.

You will need them for the exercises today.

Quiz

- Constraint between two sets of attributes from a relation

> Let R be a relational schema with the attributes $A_1, A_2, \ldots, A_n$ and let X and Y be subsets of $\{A_1, A_2, \ldots, A_n\}$.
>
> Then, the functional dependency $X \rightarrow Y$ specifies the following constraint on *any* valid relation state r of R.
>
> For *any* two tuples t_1 and t_2 in state r we have that:
>
> if $t_1[X] = t_2[X]$, then $t_1[Y] = t_2[Y]$.

- Consider the relation **R(A,B,C,D,E,F)** with the following FDs:

  FD1: **A** → **BC**

  FD2: **C** → **AD**

  FD3: **DE** → **F**

- Is a state that contains the tuples (3,8,1,2,3,4) and (3,7,1,2,3,4) valid? 1) Yes 2) No

Reasoning About FDs

- Logical implications can be derived by using inference rules called **Armstrong's rules**:
  - *Reflexivity:* If Y is a subset of X, then $X \rightarrow Y$
  - *Augmentation:* If $X \rightarrow Y$, then $XZ \rightarrow YZ$
    (we use XY as a short form for $X \cup Y$)
  - *Transitivity:* If $X \rightarrow Y$ and $Y \rightarrow Z$, then $X \rightarrow Z$

- Additional rules can be derived:
  - *Decomposition:* If $X \rightarrow YZ$, then $X \rightarrow Y$
  - *Union:* If $X \rightarrow Y$ and $X \rightarrow Z$, then $X \rightarrow YZ$
  - *Pseudo-transitivity:* If $X \rightarrow Y$ and $WY \rightarrow Z$, then $WX \rightarrow Z$

Exercises: Armstrong Rules

Reflexivity: If Y is a subset of X, then $X \rightarrow Y$

Augmentation: If $X \rightarrow Y$, then $XZ \rightarrow YZ$

Transitivity: If $X \rightarrow Y$ and $Y \rightarrow Z$, then $X \rightarrow Z$

Decomposition: If $X \rightarrow YZ$, then $X \rightarrow Y$

Union: If $X \rightarrow Y$ and $X \rightarrow Z$, then $X \rightarrow YZ$

Pseudo-transitivity: If $X \rightarrow Y$ and $WY \rightarrow Z$, then $WX \rightarrow Z$

- Consider the relation **R(A,B,C,D,E,F)** with the following FDs:
  - FD1: **A → BC**
  - FD2: **C → AD**
  - FD3: **DE → F**
- Use the Armstrong rules to derive the following FD: **AC → D**

Exercises: Armstrong Rules

Reflexivity: If *Y* is a subset of *X*, then *X* → *Y*

Augmentation: If *X* → *Y*, then *XZ* → *YZ*

Transitivity: If *X* → *Y* and *Y* → *Z*, then *X* → *Z*

Decomposition: If *X* → *YZ*, then *X* → *Y*

Union: If *X* → *Y* and *X* → *Z*, then *X* → *YZ*

Pseudo-transitivity: If *X* → *Y* and *WY* → *Z*, then *WX* → *Z*

- Consider the relation **R(A,B,C,D,E,F)** with the following FDs:

  FD1: **A** → **BC**

  FD2: **C** → **AD**

  FD3: **DE** → **F**

- Use the Armstrong rules to derive the following FD: **AC** → **D**
  - FD4: **AC** → **AD** (Augmentation of FD2 with A)
  - FD5: **AC** → **D** (Decomposition of FD4)

Exercises: Armstrong Rules

Reflexivity: If *Y* is a subset of *X*, then *X* → *Y*

Augmentation: If *X* → *Y*, then *XZ* → *YZ*

Transitivity: If *X* → *Y* and *Y* → *Z*, then *X* → *Z*

Decomposition: If *X* → *YZ*, then *X* → *Y*

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Pseudo-transitivity: If *X* → *Y* and *WY* → *Z*, then *WX* → *Z*

- Consider the relation **R(A,B,C,D,E,F)** with the following FDs:
  FD1: **A → BC**
  FD2: **C → AD**
  FD3: **DE → F**

- Use the Armstrong rules to derive the following FD: **AC → D**
  - FD4: **AC → AD** (Augmentation of FD2 with A)
  - FD5: **AC → D** (Decomposition of FD4)

- Use the Armstrong rules to derive the following FD: **A → D**

Exercises: Armstrong Rules

Reflexivity: If *Y* is a subset of *X*, then *X* → *Y*

Augmentation: If *X* → *Y*, then *XZ* → *YZ*

Transitivity: If *X* → *Y* and *Y* → *Z*, then *X* → *Z*

Decomposition: If *X* → *YZ*, then *X* → *Y*

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Pseudo-transitivity: If *X* → *Y* and *WY* → *Z*, then *WX* → *Z*

- Consider the relation **R(A,B,C,D,E,F)** with the following FDs:
  FD1: **A** → **BC**
  FD2: **C** → **AD**
  FD3: **DE** → **F**
- Use the Armstrong rules to derive the following FD: **AC** → **D**
  - FD4: **AC** → **AD** (Augmentation of FD2 with A)
  - FD5: **AC** → **D** (Decomposition of FD4)
- Use the Armstrong rules to derive the following FD: **A** → **D**
  - FD6: **A** → **C** (Decomposition of FD1)
  - FD7: **A** → **AD** (Transitivity of FD6 and FD2)
  - FD8: **A** → **D** (Decomposition of FD7)

Exercises: Armstrong Rules

Reflexivity: If *Y* is a subset of *X*, then *X* → *Y*

Augmentation: If *X* → *Y*, then *XZ* → *YZ*

Transitivity: If *X* → *Y* and *Y* → *Z*, then *X* → *Z*

Decomposition: If *X* → *YZ*, then *X* → *Y*

Union: If *X* → *Y* and *X* → *Z*, then *X* → *YZ*

Pseudo-transitivity: If *X* → *Y* and *WY* → *Z*, then *WX* → *Z*

- Consider the relation **R(A,B,C,D,E,F)** with the following FDs:
  FD1: **A** → **BC**
  FD2: **C** → **AD**
  FD3: **DE** → **F**
- Use the Armstrong rules to derive the following FD: **AE** → **ABCDEF**

Exercises: Armstrong Rules

Reflexivity: If Y is a subset of X, then $X \rightarrow Y$

Augmentation: If $X \rightarrow Y$, then $XZ \rightarrow YZ$

Transitivity: If $X \rightarrow Y$ and $Y \rightarrow Z$, then $X \rightarrow Z$

Decomposition: If $X \rightarrow YZ$, then $X \rightarrow Y$

Union: If $X \rightarrow Y$ and $X \rightarrow Z$, then $X \rightarrow YZ$

Pseudo-transitivity: If $X \rightarrow Y$ and $WY \rightarrow Z$, then $WX \rightarrow Z$

- Consider the relation **R(A,B,C,D,E,F)** with the following FDs:

  FD1: **A → BC**

  FD2: **C → AD**

  FD3: **DE → F**

- Use the Armstrong rules to derive the following FD: **AE → ABCDEF**
  - FD9: **AE → BCE** (Augmentation of FD1 with E)
  - FD10: **AE → C** (Decomposition of FD9)
  - FD11: **AE → AD** (Transitivity of FD10 and FD2)
  - FD12: **AE → ADE** (Augmentation of FD11 with E)
  - FD13: **AE → DE** (Decomposition of FD12)
  - FD14: **AE → F** (Transitivity of FD13 and FD3)
  - FD15: **AE → ABCDEF** (Union of FD9, FD11, and FD14)

Computing (Super)Keys

```
function ComputeAttrClosure( X, F )
begin
    X+ := X;
    while F contains an FD Y → Z such that
            (i) Y is a subset of X+, and
            (ii) Z is not a subset of X+  do
        X+ := X+ U Z;
    end while
    return X+;
end
```

Warmup (cont'd)

```
function ComputeAttrClosure( X, F )
begin
    X+ := X;
    while F contains an FD Y → Z such that
        (i) Y is a subset of X+, and
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      X+ := X+ U Z;
    end while
    return X+;
end
```

- Consider the relation **R(A,B,C,D,E,F)** with the following FDs:

  FD1: **A** → **BC**

  FD2: **C** → **AD**

  FD3: **DE** → **F**

- Compute the **attribute closure** of *X* = { **A,E** } w.r.t. *F* = {FD1, FD2, FD3} to show that we have: **AE** → **ABCDEF**
  - Initially: X^+ = { **A,E** }
  - By using FD1: X^+ = { **A,E,B,C** }
  - By using FD2: X^+ = { **A,E,B,C,D** }
  - By using FD3: X^+ = { **A,E,B,C,D,F** }

Your Turn

```
function ComputeAttrClosure( X, F )
begin
        X+ := X;
        while F contains an FD Y → Z such that
                (i) Y is a subset of X+, and
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            X+ := X+ U Z;
        end while
        return X+;
end
```

- Consider the relation **R(A,B,C,D)** with the following set *F* of FDs:
  FD1: **AB** → **C**
  FD2: **BC** → **D**
  FD3: **D** → **B**
- Compute the following attribute closures w.r.t. *F* = {FD1, FD2, FD3}
  - $\{ B,C,D \}^+ =$?
  - $\{ A,C,D \}^+ =$?
  - $\{ A,B,D \}^+ =$?
  - $\{ A,B,C \}^+ =$?

Your Turn

```
function ComputeAttrClosure( X, F )
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    X+ := X;
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- Consider the relation **R(A,B,C,D)** with the following set *F* of FDs:
  - FD1: **AB** → **C**
  - FD2: **BC** → **D**
  - FD3: **D** → **B**
- Compute the following attribute closures w.r.t. *F* = {FD1, FD2, FD3}
  - $\{ B,C,D \}^+ = \{ B,C,D \}$
  - $\{ A,C,D \}^+ = \{ A,C,D,B \}$
  - $\{ A,B,D \}^+ = \{ A,B,D,C \}$
  - $\{ A,B,C \}^+ = \{ A,B,C,D \}$

Superkeys and Candidate Keys

- Consider the relation **R(A,B,C,D)** with the following set *F* of FDs:

  FD1: **AB** → **C**

  FD2: **BC** → **D**

  FD3: **D** → **B**

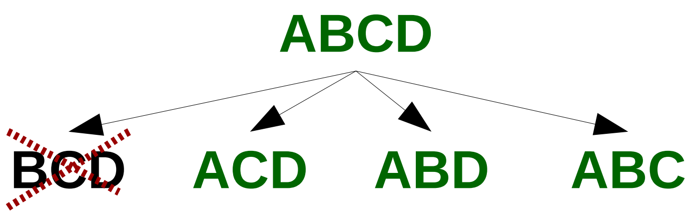
- A set *X* of attributes of *R* is a superkey if X^+ contains all the attributes of *R*

  – $\{ B,C,D \}^+ = \{ B,C,D \}$

  – $\{ A,C,D \}^+ = \{ A,C,D,B \}$

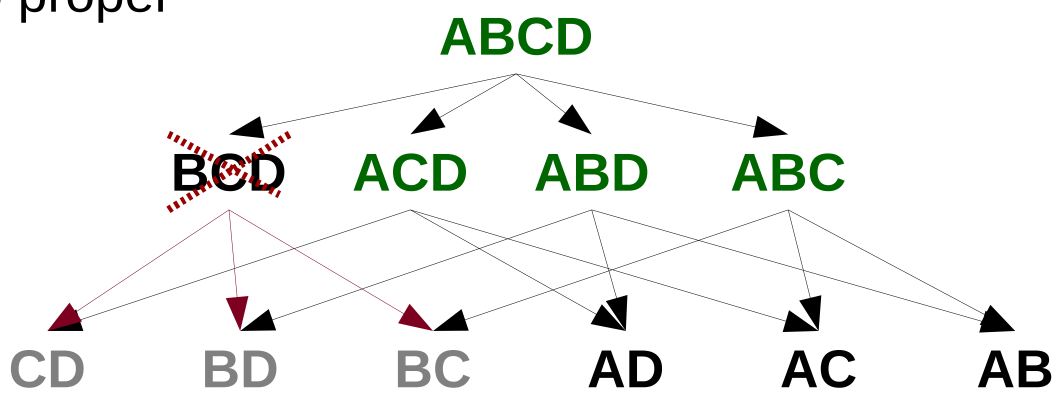
  – $\{ A,B,D \}^+ = \{ A,B,D,C \}$

  – $\{ A,B,C \}^+ = \{ A,B,C,D \}$



Superkeys and Candidate Keys

- Consider the relation **R(A,B,C,D)** with the following set *F* of FDs:
  FD1: **AB** → **C**
  FD2: **BC** → **D**
  FD3: **D** → **B**
- A set *X* of attributes of *R* is a superkey if X^+ contains all the attributes of *R*
- *X* is a candidate key (CK) if no proper subset *X'* of *X* is a superkey
  - $\{A,D\}^+ =$?
  - $\{A,C\}^+ =$?
  - $\{A,B\}^+ =$?



Superkeys and Candidate Keys

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- A set *X* of attributes of *R* is a superkey if X^+ contains all the attributes of *R*
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  - $\{A,D\}^+ = \{A,D,B,C\}$
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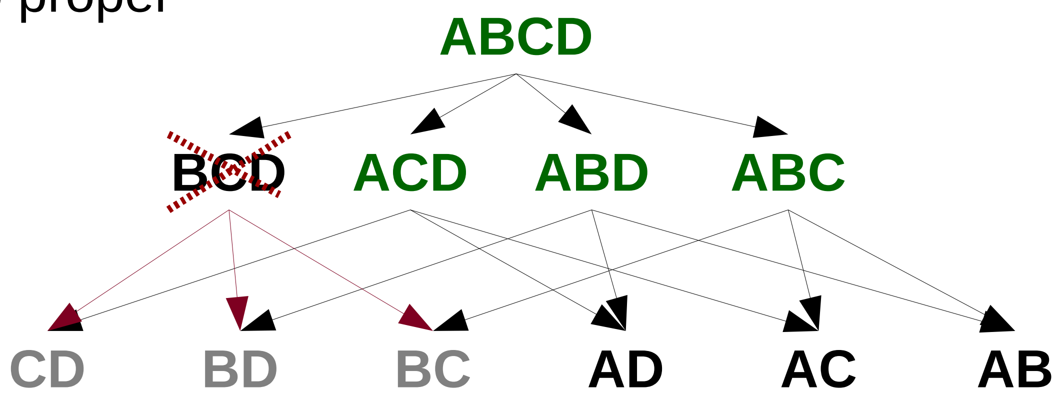

Superkeys and Candidate Keys

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  - $\{ A,D \}^+ = \{ A,D,B,C \}$
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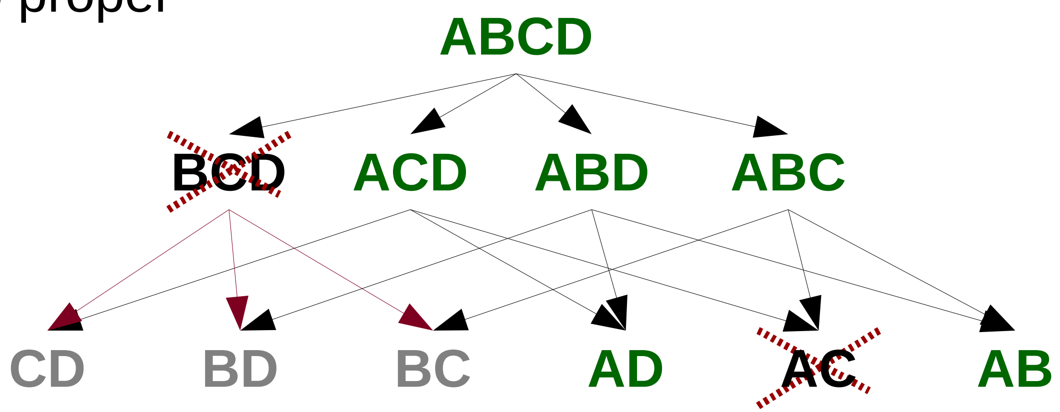

Superkeys and Candidate Keys

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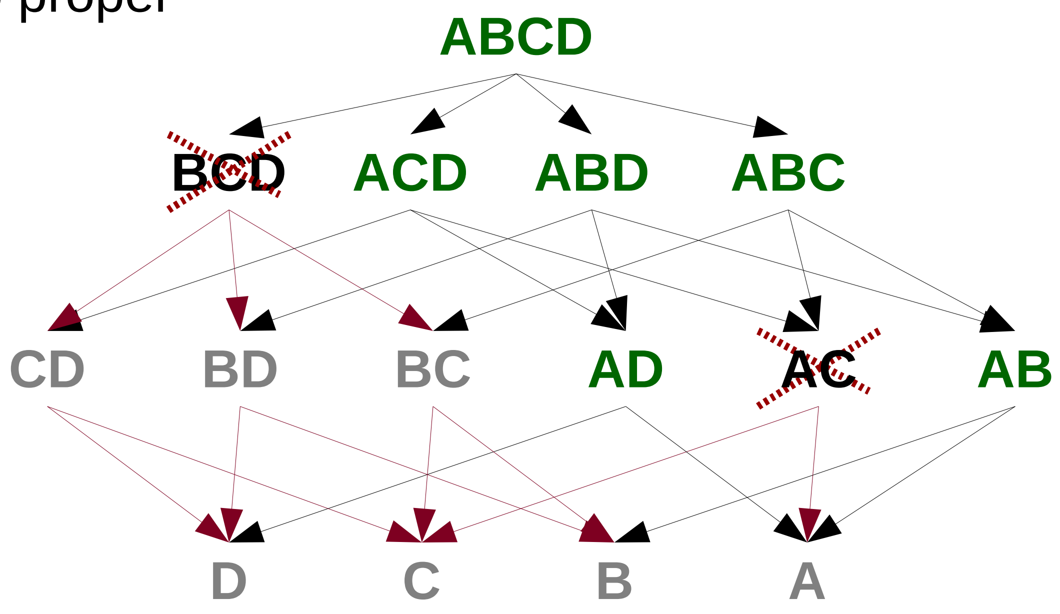

Superkeys and Candidate Keys

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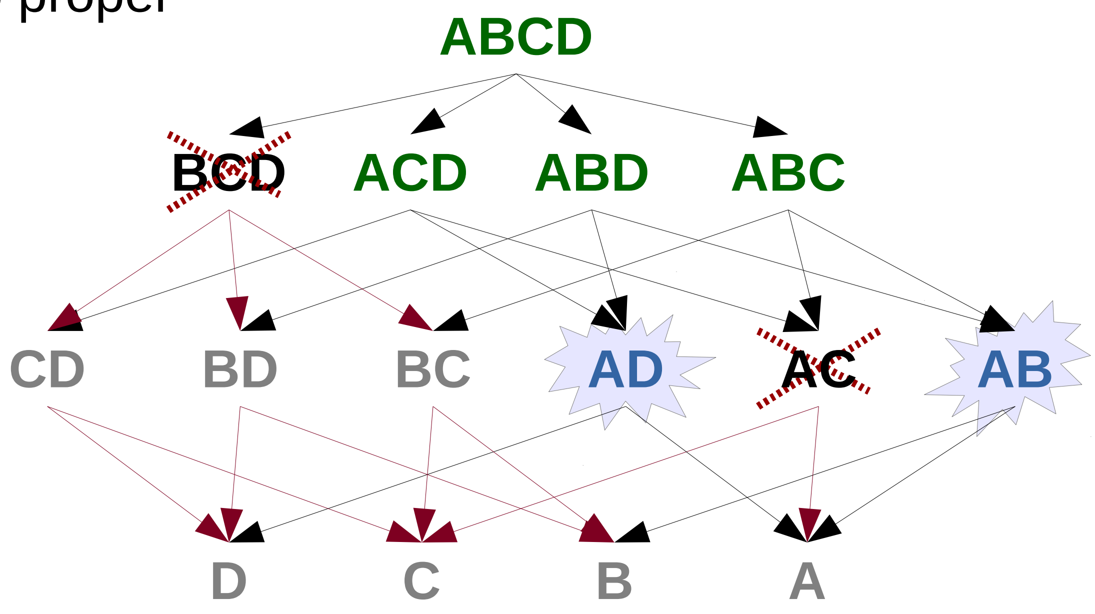

Superkeys and Candidate Keys

- Consider the relation **R(A,B,C,D)** with the following set *F* of FDs:
  FD1: **AB** → **C**
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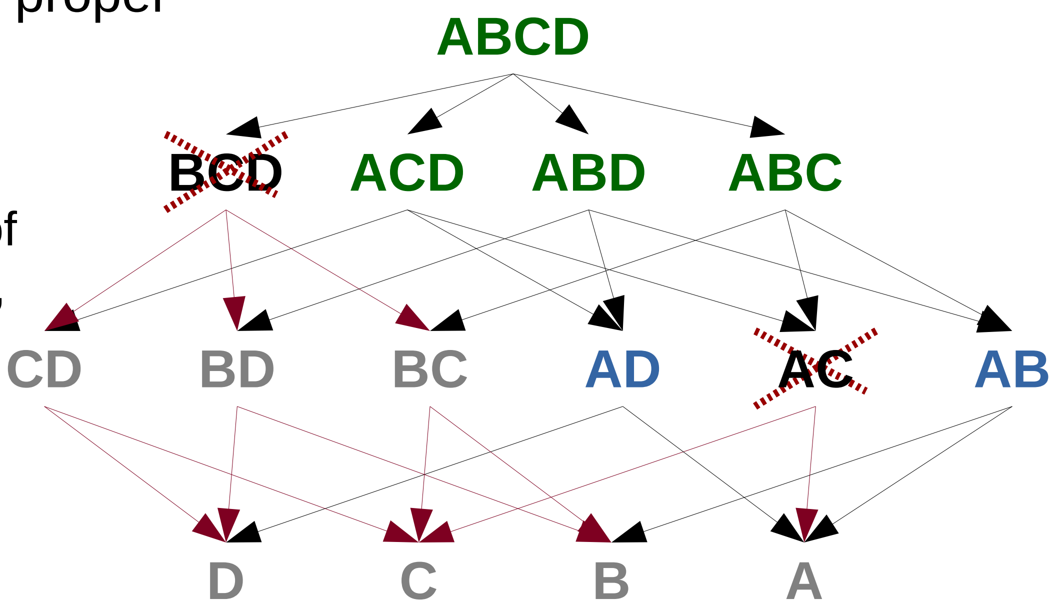
- A set *X* of attributes of *R* is a superkey if X^+ contains all the attributes of *R*
- *X* is a candidate key (CK) if no proper subset *X'* of *X* is a superkey
- Some steps may be skipped
  - If an attribute is in the RHS of FDs but nowhere in the LHS, it cannot be part of any CK
  - If an attribute is nowhere in the RHS, it must be part of any candidate key (such as *A* in the example)



Superkeys and Candidate Keys

- Consider the relation **R(A,B,C,D)** with the following set *F* of FDs:

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  ~~FD3: **D** → **B**~~

- A set *X* of attributes of *R* is a superkey if X^+ contains all the attributes of *R*
- *X* is a candidate key (CK) if no proper subset *X'* of *X* is a superkey
- Some steps may be skipped
  - If an attribute is in the RHS of FDs but nowhere in the LHS, it cannot be part of any CK
  - If an attribute is nowhere in the RHS, it must be part of every candidate key (such as *A* in the example)
- For instance, if we had only FD1 and FD2 (i.e., not FD3), we could immediately say that:
  - *A* and *B* must be in every CK
  - and, since $\{A,B\}^+ = \{A,B,C,D\}$, {A,B} is a superkey and thus the only CK

Your Turn

- Consider the relation **R(A,B,C,D,E,F)** with the following FDs:

  FD1: **A** → **BC**

  FD2: **C** → **AD**

  FD3: **DE** → **F**

- What are the candidate keys?

Your Turn

- Consider the relation **R(A,B,C,D,E,F)** with the following FDs:
  FD1: **A** → **BC**
  FD2: **C** → **AD**
  FD3: **DE** → **F**
- What are the candidate keys?
  - {A,E} and {C,E}
  - $\{ E \}^+ = \{ E \}$
  - $\{ A,E \}^+ = \{ A,E,B,C,D,F \}$
  - $\{ C,E \}^+ = \{ C,E,A,D,B,F \}$
  - $\{ D,E \}^+ = \{ D,E,F \}$