Improving Gaussian Elimination on Multi-Core Systems

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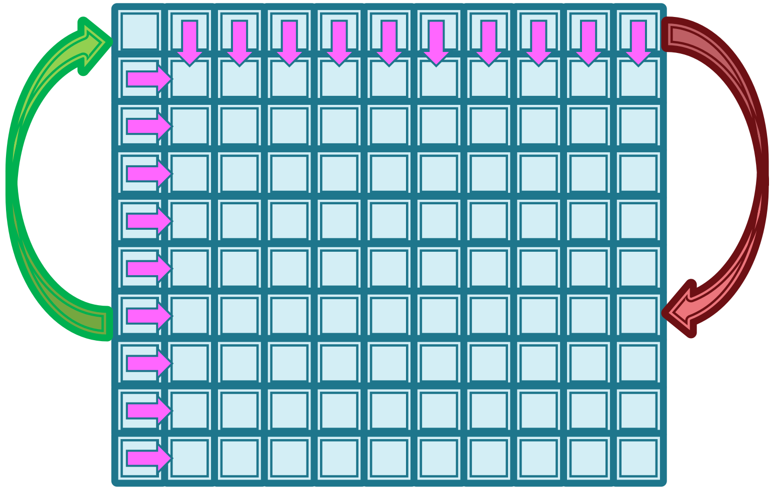
Agenda

- Algorithm overview.
- Trivial parallelization.
- Problems.
- Sequential optimization
- Proposed solutions.
- Experimental results.
- Conclusions and future work.

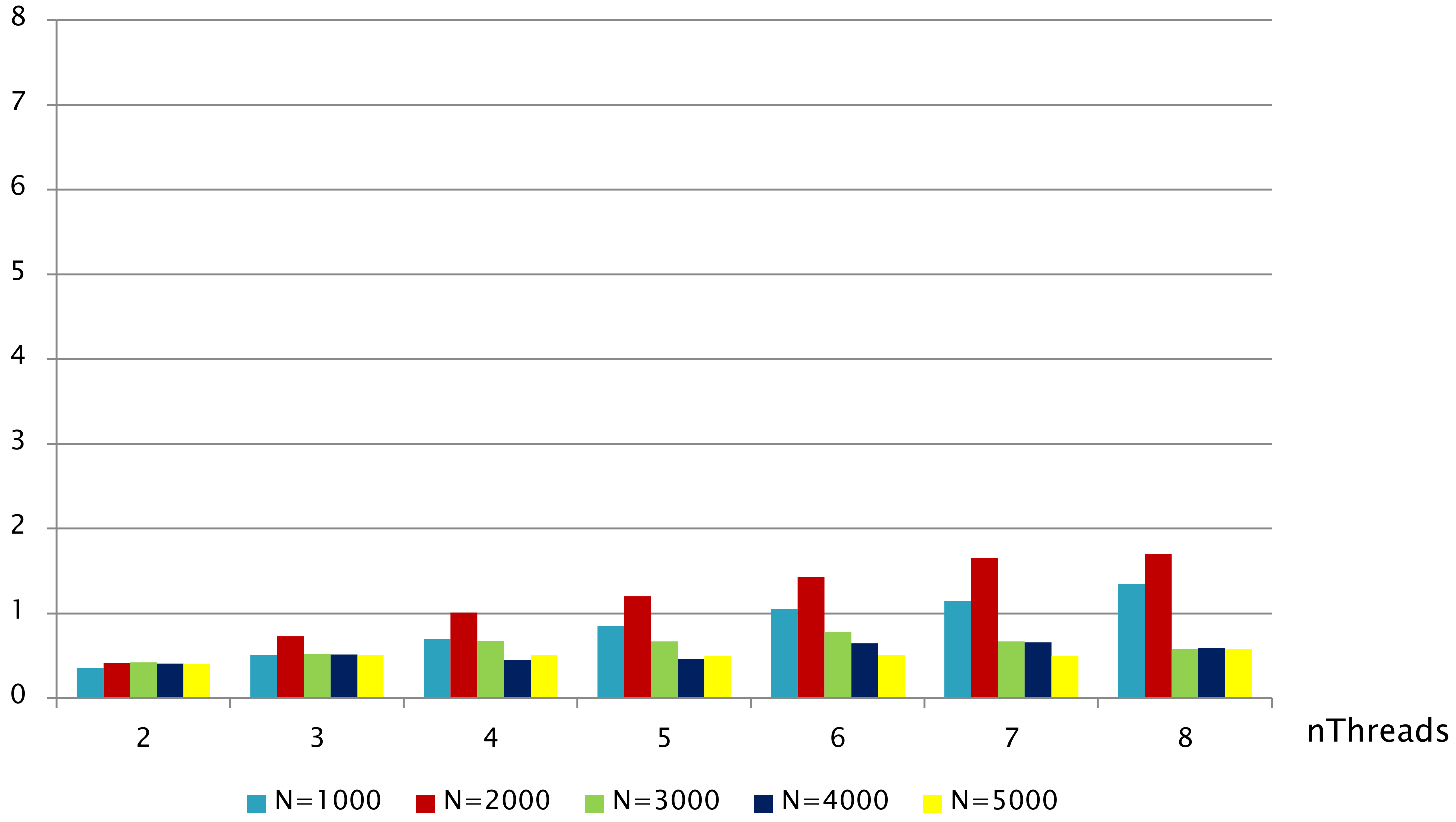
Overview of forward elimination

```
for i=1 to n−1
  find pivotPos in column i
  if pivotPos ≠ i
    exchange rows(pivotPos,i)
  end if

  for j=i+1 to n
    A(i,j) = A(i,j)/A(i,i)
  end for j
 !$omp parallel do private ( i ,j )
  for j=i+1 to n+1
    for k=i+1 to n
      A(k,j)=A(k,j)−A(k,i)×A(i,j)
    end for k
  end for j
end for i
```



Trivial OMP Speed up



Problems

- Poor data locality
- Pivoting is done by master thread
- Overheads of creating and destroying threads at each iteration
- Sequential optimization

Sequential optimization

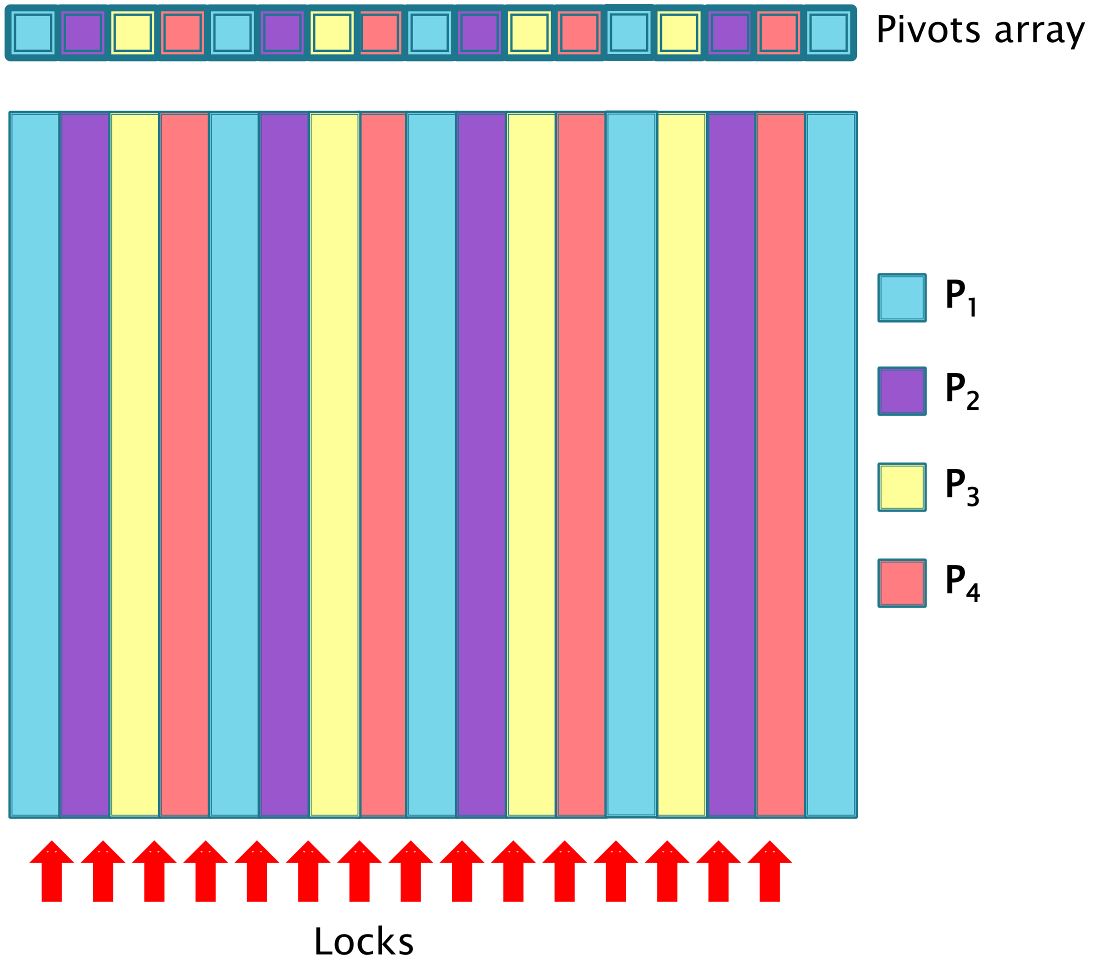
- Replace division by the constant pivot
- Avoid loop invariant access in the inner most loop
- Eliminate the check for pivot changing position
- Make use of fortran array notation

```
Do k=j+1,n
        A(k,j)=A(k,j)/A(j,j)
End do
```

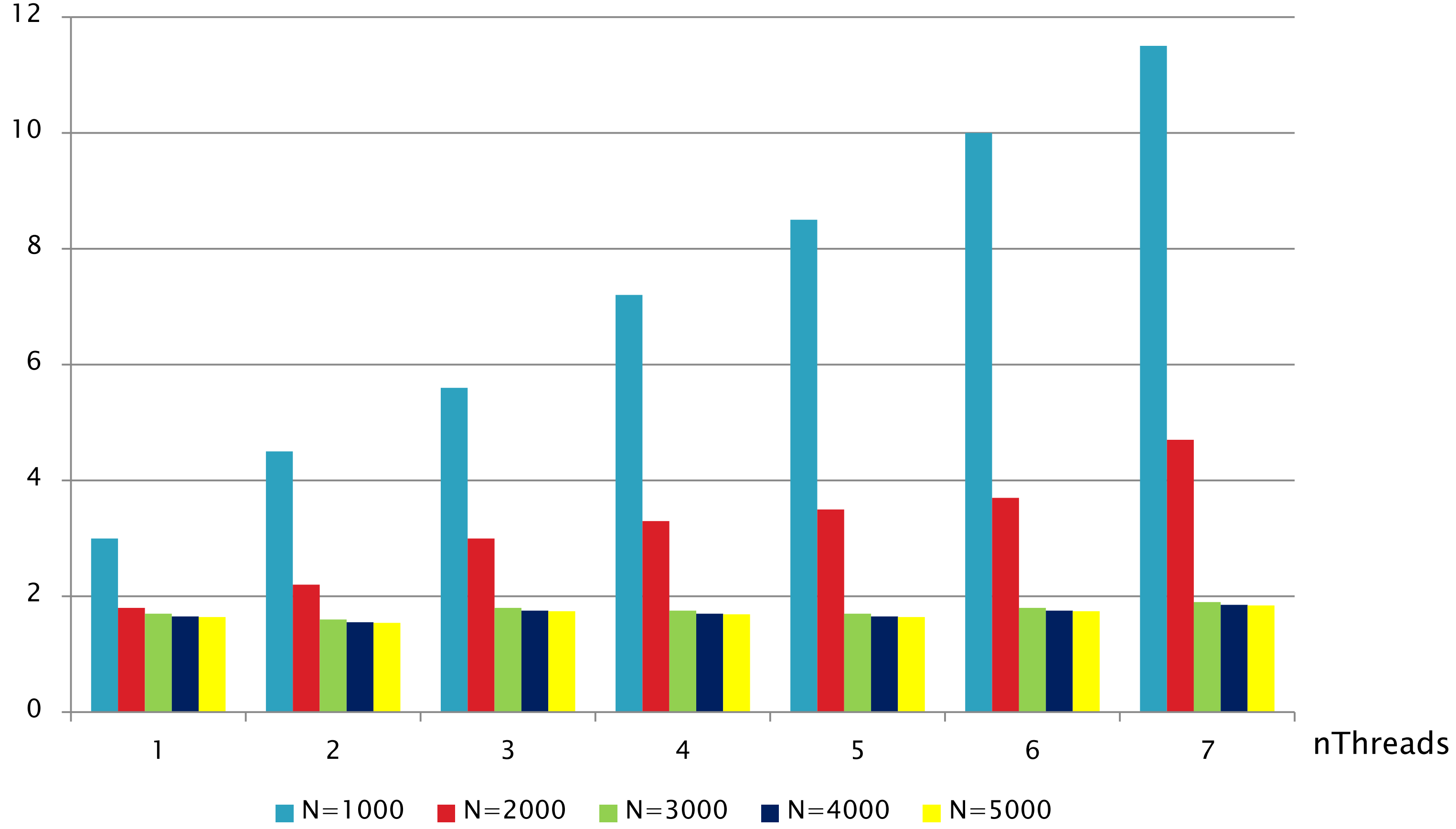
```
C=1/A(j,j)
A(j+1:n)=A(j+1:n)*c
```

Cyclic Column distribution

- Pivots array
- Locks array
- Pivot holder
  - Eliminate (i) on column(i+1)
  - Search (i+1)
  - Store pivot (i+1) position
  - Prepare colmn (i+1)
  - Free lock (i+1)
  - Eliminate (i) on rest of scope



Cyclic Column distribution speed up



What went wrong?!

- The original algorithm requires pivot columns to be prepared in order while the whole matrix is accessed for each pivot column.
- For large input sizes; the cache is evicted many times for each iteration and there is no reuse of data in the cache.
- False sharing on pivots and locks array.

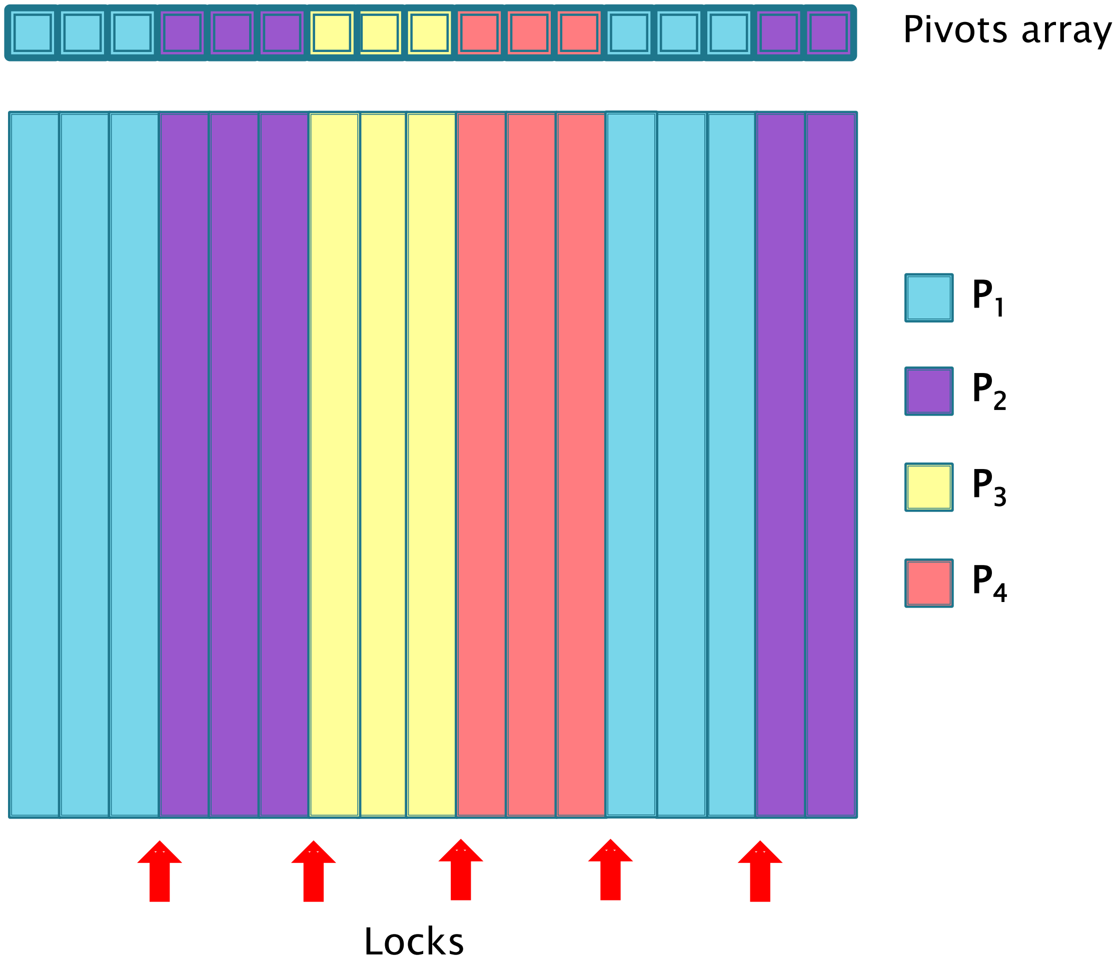
Improving!

- Double elimination on pivot holders.
  - Knowledge of two pivots allow data reuse.

- Each column is an accumulation of eliminations using previous columns!
  - Make more pivots available each step and eliminate each column using several pivots while it is in the cache.

Cyclic Block distribution

- Block of pivots
- Increase work/iter.
- Increase locality
- Less locks
- Load balancing?!



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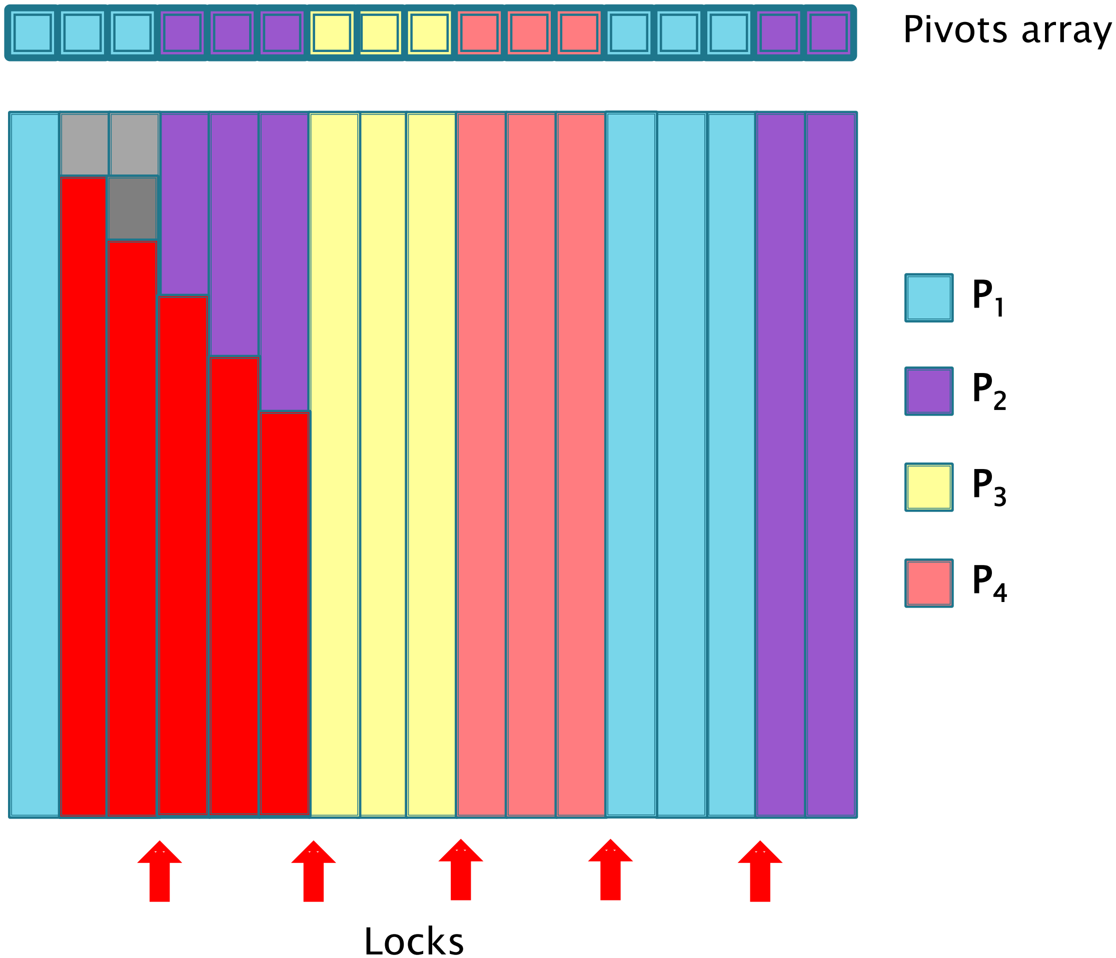
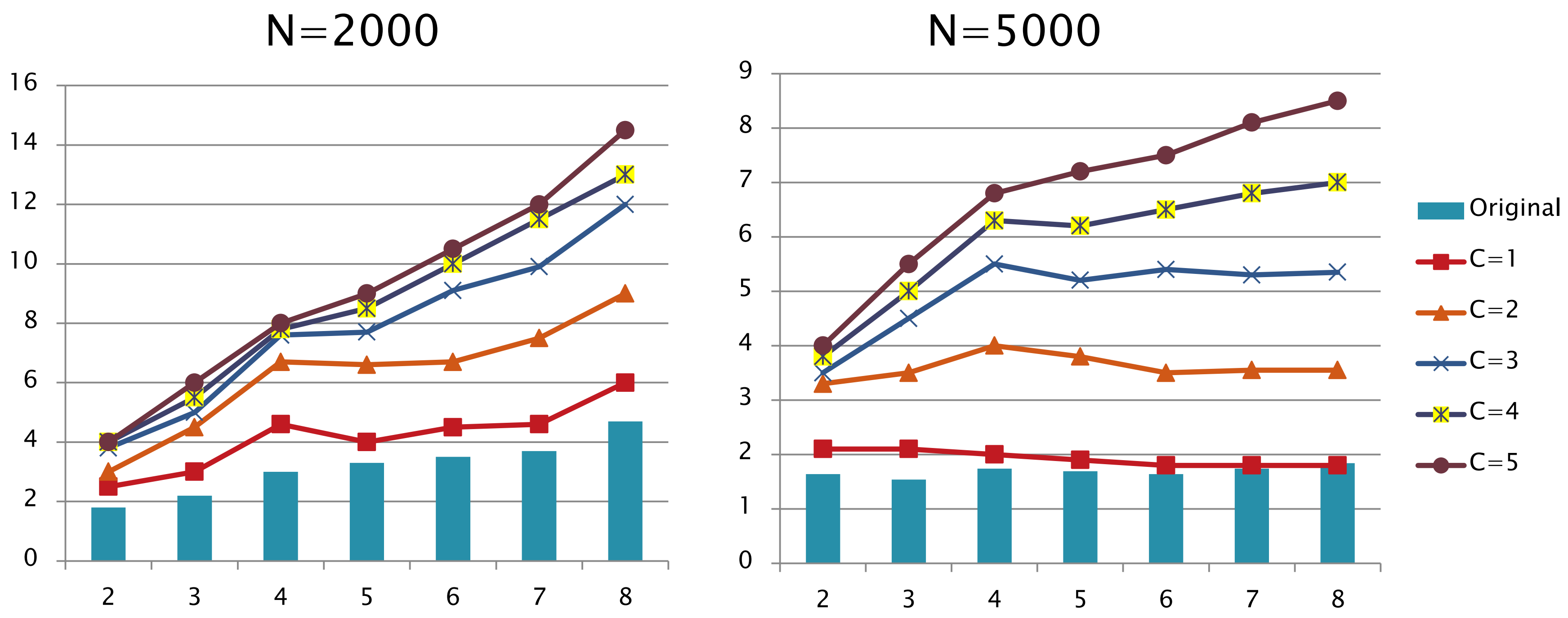
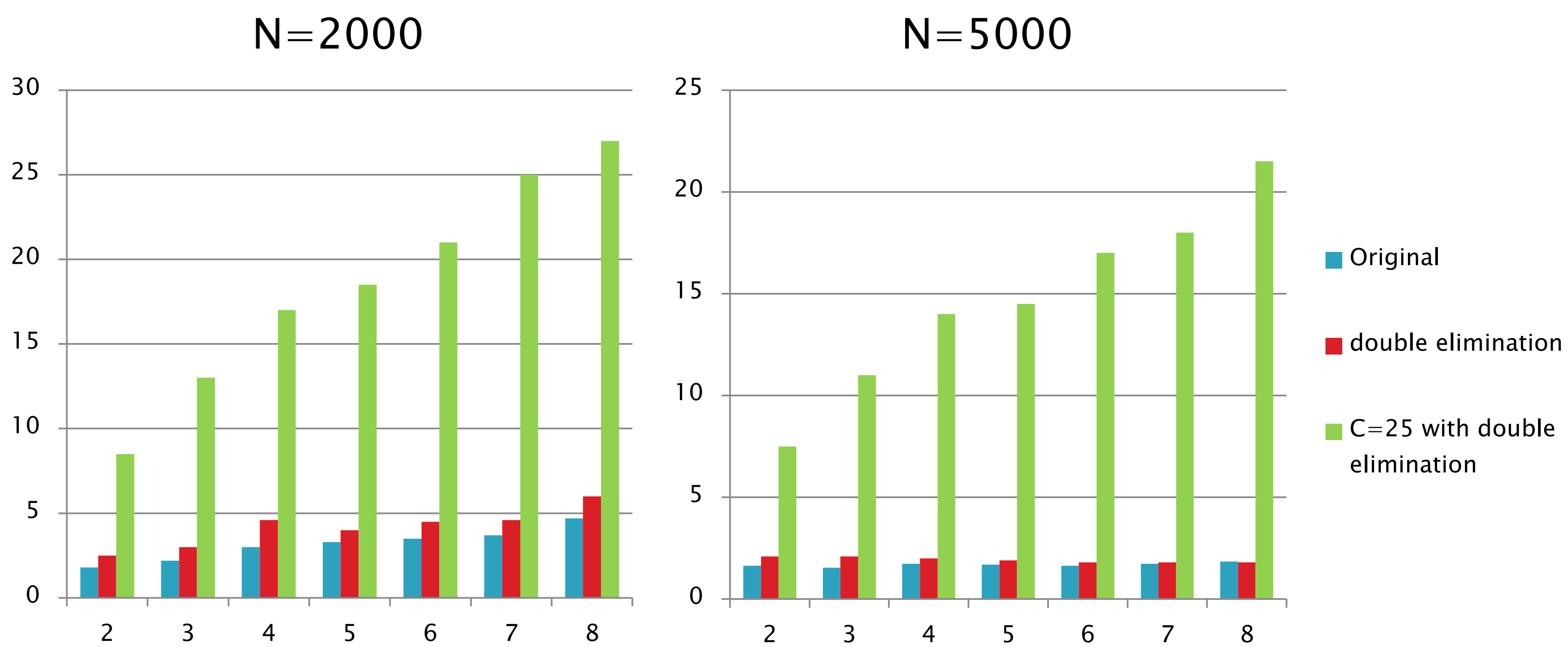


Speed up for block distribution



Speed up as C=25



Conclusions and future work

- Scalable performance on multicores is highly dependent on application implementation, data layout and access patterns.

- Cache and memory access optimization techniques is vital for performance despite the loss of readability.

- Future work:
  - Adaptive blocking scheme that changes the block size as a function of the matrix size, cache settings, and number of cores.

Questions