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Dynamics of the Varying Force Constraints in the Friction Contact of Worm (screw-type) Gears

Motivations of work

- In many applications in mechanical engineering , worm (screw) gear drives are used to transmit more high power flows between rotating shafts due to the geometrical, friction and backlash features of gear mesh , has become an essential topic in multibody contact dynamics

- Therefore, the ability to incorporate the effect of the discontinuity contact internal forces in the dynamical rigid model is reasonable to introduce contact case modification properties and variable efficiency factors for high requirements of accuracy in computational design effort

- Therefore, to perform a detailed dynamic analysis and study of the Worm (Screw) Gear Drives by discontinuity approach of the varying force constraints and efficiency factors as well as left and right-hand side contact of the meshing teeth can be suitable for real-life mechanical systems.

Features of the Worm gear mesh contact

- The **worm** is a special type of gear that looks like a screw
- Sliding friction forces of contact between conjugated teeth can be represented analytically by friction angle which depends from contact materials (different types of bronze wheel and steel worm)
- The presence of backlash (clearance) has been utilized the effect of multi-tooth contact as well as contact between two active surface
- Thread-line slope of contact which depends from gear geometry and identifies a constant lead angle
- Worm gear mesh is rigid and defines rigid body contact modelling

Dynamical Model of the Worm (Screw)Drive system in translational motion (fig.1)

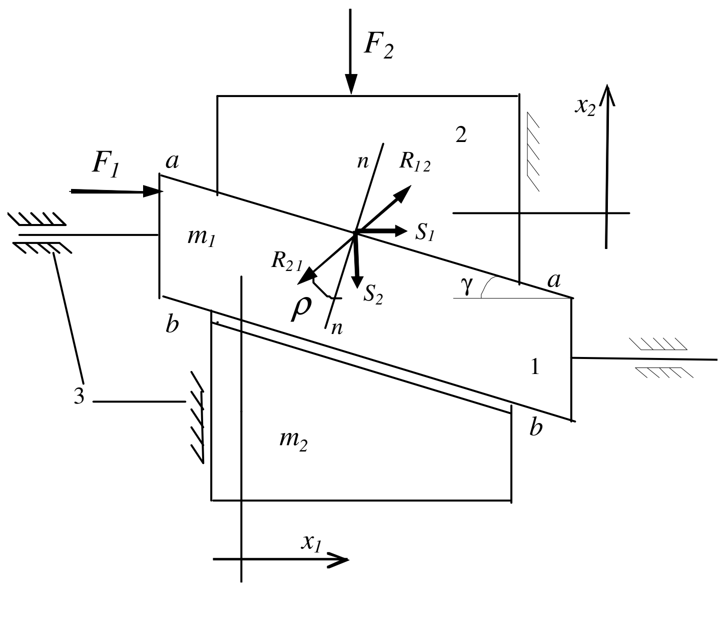


Fig.1 Two-Sided Wedge Mechanism

Model description and assumptions

- The **Two-Sided Wedge Model** (TSWM) is dynamically Structural-variable Multibody system with state dependent contacts and discontinuous handling events within simulation
- Conjugate relative motion of inertial wedge bodies marked by m1,m2 via one of active contact line (a-a) or (b-b) in rectangular joint co-ordinate x1,x2 is restricted by ideal guides 3
- An assembly motion upon one of the arbitrary main co-cordinate x1 or x2 is divided on a driving mass m1 and driven mass m2 respectively which moves together under applied external forces F1 and F2
- The slope line of each wedge is identifies lead angle of the worm (SCREW)

Modelling of constraint equations of kinematics

Kinematic constraints at TSWM

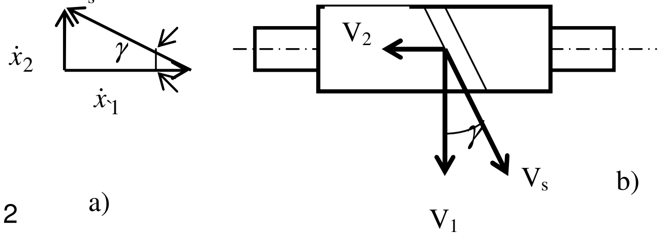
On acceleration level $$\ddot{x}_2(t) = \ddot{x}_1(t)tg\gamma$$

On velocity level $$\dot{x}_2(t) = \dot{x}_1(t)tg\gamma$$

On position level $$x_2(t) = x_1(t)tg\gamma$$

The Schematic Distribution of linear **v**elocities at the TSWM (a) and the worm gear mesh contact (b) on Fig.2

Sliding velocity



$$V_s = \frac{\dot{x}_1(t)}{\cos\gamma} = \frac{\dot{x}_2(t)}{\sin\gamma}$$

Fig 2

Plans of internal reduced forces S1 and S2

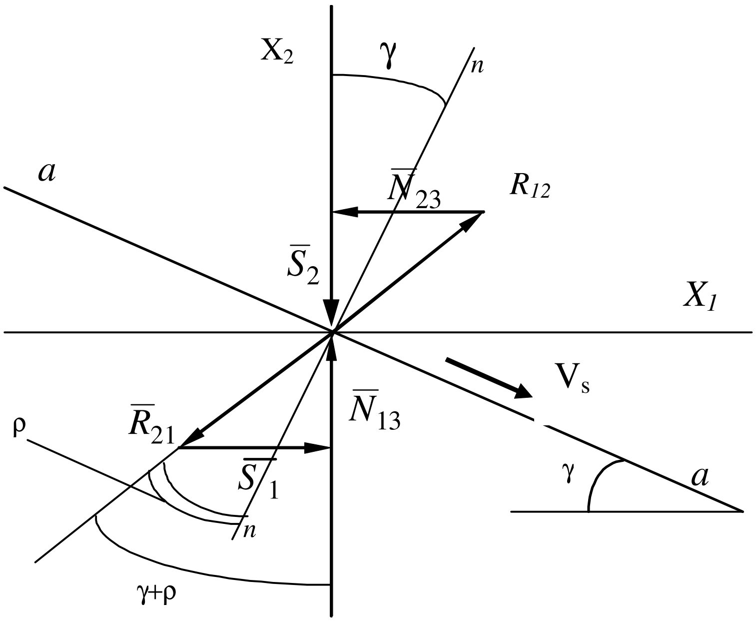


Fig.3 Force distribution plane on (a-a) active contact line modification

The internal reduced reactions (forces) S1 and S2 (fig.3-4) are the resultant dynamical loads takes from kinetostatic analysis for TSWM which identifies varying relationships and distribution of tangential forces Ft1 and Ft2 at the worm gear set

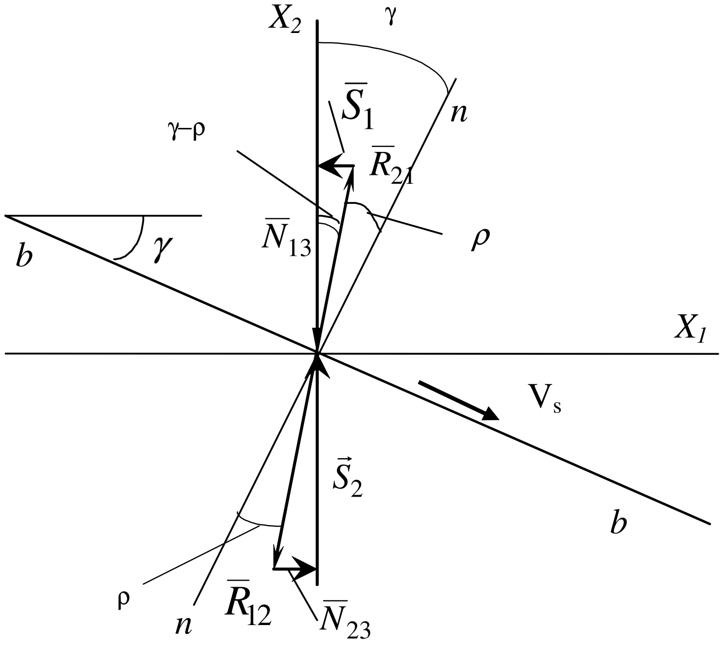


Fig.4 Force distribution plane on (b-b) active contact line modification

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Equation of Force constraints with Discontinuity Kernel (by analysis of the force planes on fig.3-4)

$$S_1 = \psi_j S_2, (j = 1,2)$$

where indices j´=1,2 denotes the corresponding type of a Force Transfer Function (FTF) between connected parts for each of qualitatively different regimes of motion

The regimes are classified by a following way

- Tractive regime for j=1 with direct energy and force flow (S1>0, S2<0)
- Inverse-tractive regime for j=2 with inverted energy and force flow (S1<0, S2>0)

The set of FTFs are call as **Discontinuity Kernel** resulted in

$$\psi_j, (j=1,2)$$

$$\psi_1 = -tg(\gamma + \rho), \psi_2 = -tg(\gamma - \rho)$$

Approximated formulae for the sliding friction angle as velocity function

One way to influence the sliding friction of the screw-type contact would be correct from approximation of experimental data in depends on sliding velocity Vs takes from the literature

Group number of contact materials of worm gear mesh	Analytical formulation of the friction angle depends on sliding velocity $\rho(V_s) = \left(aV_s^{\,b} + c\right)^{-1}$
I*	a=0,256;b=0,591;c=0,151
II*	a=0,174;b=0,642;c=0,140
III*	a=0,161;b=0,535;c=0,088

*Group I.Worm: hardened steel HRC>48. Wheel: Bronze (Sn 6-7%, Ni 1-2%)
**Group II. Worm: hardened steel 48>HRC>32. Wheel: Bronze (Sn 6-10%
***Group III.Worm: hardened steel HRC>48. Wheel: Bronze (Fe 1%)).

Varying Efficiency for the Dynamical Regime

- Tractive regime of motion is resulted in the direct force flow and efficiency factor

$$\eta = \left| \frac{S_1 \dot{x}_1}{S_2 \dot{x}_2} \right| = \frac{tg\gamma}{tg(\gamma + \rho)}$$

- Inverse-Tractive regime is resulted in the inverted force flow with two efficiency factors for $\gamma > \rho$ and $\rho > \gamma$

$$\mu = \left| \frac{S_2 \dot{x}_2}{S_1 \dot{x}_1} \right| = \frac{tg(\gamma - \rho)}{tg\gamma} \qquad \mu^{SL} = \left| \frac{S_2 \dot{x}_2}{S_1 \dot{x}_1} \right| = \frac{tg(\rho - \gamma)}{tg\gamma}$$

Mathematical Formulation

The mathematical formulation represented within independent state coordinate $x_1(t)$ and results in the ODEs form with Discontinuity Kernel (DK) at the right-hand side together with corresponding initial conditions

$$\ddot{x}_1(t) = \frac{F_1 - F_2 \psi_j}{m_1 - m_2 \psi_j tg\gamma}, (\psi_j = \{\psi_j, j = 1,2\}),$$

$$x_1(0) = x_{10}, \dot{x}_1(0) = \dot{x}_{10}$$

where

$$\psi_1 = -tg(\gamma + \rho(\dot{x}_1)), \psi_2 = -tg(\gamma - \rho(\dot{x}_1)),$$

$$\rho(\dot{x}_1) = 1/\left(a(\dot{x}_1 / \cos\gamma)^b + c\right) \qquad V_s = \frac{\dot{x}_1(t)}{\cos\gamma}$$

Defined monitor block and the switching function

Switching Function (SF) has been constructed on acceleration level by using following equation of so-called own acceleration

$$\varphi(t) = \ddot{x}_1^{\,own}(t) - \ddot{x}_2^{\,own}(t)\,\frac{1}{tg\gamma} \quad ,$$

¨ where $\ddot{x}_1^{own} = \dfrac{F_1(\dot{x}_1)}{m_1}$ own" accelerations of body 1

$\ddot{x}_2^{own} = \dfrac{F_2}{m_2}$ own" accelerations of body 2

Monitor Block and switching logic (additional element in the model) represented as

$$\psi_j = \begin{cases} \psi_1 \; if \;\; \varphi(t) > 0 & for\ the\ tractive\ regime \\ \psi_2 \; if \;\; \varphi(t) < 0 & inverse - tractive\ regime. \end{cases}$$

Implementation Model to the Motor Operated Valve (Simulation example)

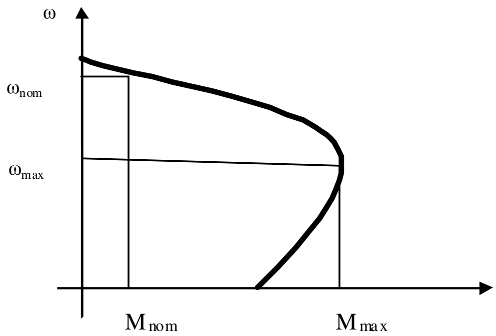
The simulation example is taken for the **Motor Operated Valve** (MOV) with worm gear reducer and AC Motor.

Input real life MOV parameters.
The ***AC motor*** technical characteristics: P=7,5 kW, n0=3000rpm; nn=2900 rpm, Mmax/Mn=2,2 ,smax=17%; Rotor inertia Ir= 0.0069 kgm2.

Worm gear reducer parameters:
reduction ratio u=40:1; diameter of worm d1=50 mm, Contact gear materials - group III , lead angle 4,76 deg

Model of External driving force F1 according to the AC motor characteristics (fig.5)

$$F_1 = \frac{M(\omega)}{R_1} = \frac{2F_{\max}}{\frac{V_0 - \dot{x}_1}{V_0 - V_{\max}} + \frac{V_0 - V_{\max}}{V_0 - \dot{x}_1}}$$



Where $V_0 = \omega_0 R_1$ - synchronous linear velocity $F_{\max} = M_{\max} / R_1$ -maximal motor force

$V_{\max} = \omega_{\max} R_1$ maximal linear velocity

R_1 worm pitch radius.

$\dot{x}_1$ - current linear velocity of driving body

Simulation on the force level

Within zero initial conditions and integration time interval from 0 to o,14 s
The following internal forces of Wedge Model are tested during simulation, calculated as

$$S_1(t) = F_1(\dot{x}_1) - m_1 \ddot{x}_1$$

$$S_2(t) = S_1(t) / \psi_j, \; j = \overline{1,2}$$

Dynamic peak values are compared with static tangential forces Ft1 and Ft2 at worm gear mesh by using dynamical factors K_{d1}, K_{d2}

$$K_{d1} = \left| S_1^m \right| / F_{t1} \qquad K_{d2} = \left| S_2^m \right| / F_{t2}$$

Table of reduced model parameters with m2, F2 variation according to MOV

Description	model designation	Value
Driving mass 1	m1	11.8 kg
Driven mass 2 (var)	m2	[100; 200,300]kg
Resistant force (var)	F2	[-5000;-10000;-15000] N
Wotm Lead angle	Gam	4,76 deg
Motor peak force	pk	2178 N
Motor synch linear speed	vc1	7.85 m/s
Motor max linear speed	vcm	6.5 m/s
Friction angle coeff	a,b,c	0.161, 0.535, 0.088

Fragment of the script m. file within Matlab ODE23s solver

```
y0=[0 0];% initial conditions in vector form
t0=0;tfinal=0.14;% time interval
[t,y]=ode23s('cpm1',ime,y0,options);
```

The code of monitor block and the switching function

```
function dy=cpm1(t,y)
globalm1 m2 pk vc1 vk1 gam a b c F2
V=(vc1-y(2))/(vc1-vk1);
F1=2*pk/(V+1./V);
fi=F1/m1-F2/m2/tan(gam);
ro=1/(a*(y(2)/cos(gam))^b+c);
ro=ro*atan(1)/45;
if sign(fi)==1
 lam=-(tan(gam+ro));
else lam=-(tan(gam-ro)); end
```

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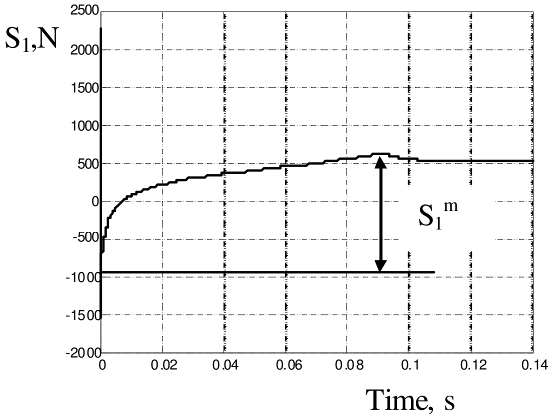
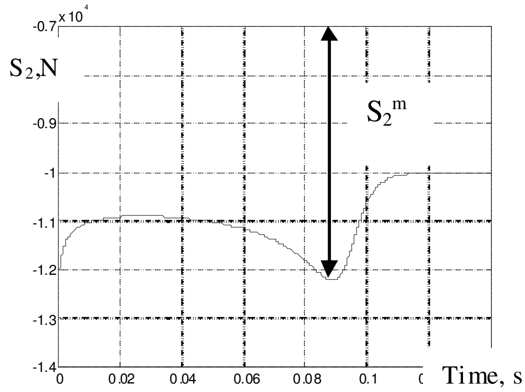
Plots of the internal forces S1(t), S2(t) with fixed peak points

for **F2=-10000 N**





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Summarized simulation results for the dynamical factors

Test case	m2 F2	S1, [N]	Kd1	S2 [N]	Kd2
1	100 kg; -5000 N;	800	0,81	-6300	0,76
2	100 kg; -10000 N;	1400	1,4	-11300	1,37
3	100 kg; -15000 N;	1800	1,84	-16500	2,0
4	200 kg; -5000 N;	900	0,91	-7600	0,92
5	200 kg; -10000 N;	1500	1,50	-12100	1,36

Conclusions

- **The discontinuity approach to handle Worm (screw) drive Mechanical System with Dynamicaly Varying Structure is presented.**
- **The conditions for exchanging regimes of motion and efficiency are observed by defined monitor block which is additional element in the math model.**
- **The switching function calculates the difference between own accelerations of systems parts.**
- **These model is simple implemented to the force simulation of Motor Operated valve with worm reducer.**
- **The results shows that the model methodology and numerical simulation in MATLAB ODE solver gives more high accuracy in compare with static force analysis**

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