

Description logics

Description Logics

- A family of KR formalisms, based on FOPL decidable, supported by automatic reasoning systems
 - Used for modelling of application domains
 - Classification of concepts and individuals
concepts (unary predicates), subconcept (subsumption), roles (binary predicates), individuals (constants), constructors for building concepts, equality ...
- [Baader et al. 2002]

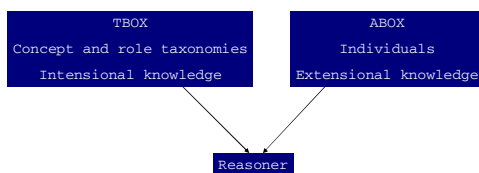
Applications

- n software management
- n configuration management
- n natural language processing
- n clinical information systems
- n information retrieval
- n ...
- n Ontologies and the Web

Outline

- n DL languages
 - syntax and semantics
- n DL reasoning services
 - algorithms, complexity
- n DL systems
- n DLs for the web

Tbox and Abox



Syntax - \mathcal{AL}

- R atomic role, A atomic concept
- $C, D \rightarrow A \mid$ (atomic concept)
- $\top \mid$ (universal concept, top)
- $\perp \mid$ (bottom concept)
- $\neg A \mid$ (atomic negation)
- $C \cap D \mid$ (conjunction)
- $\forall R.C \mid$ (value restriction)
- $\exists R.T \mid$ (limited existential quantification)

$\mathcal{AL}[X]$

C $\neg C$ (concept negation)

\mathcal{U} $C \sqcup D$ (disjunction)

\mathcal{E} $\exists R.C$ (existential quantification)

\mathcal{N} $\geq n R, \leq n R$ (number restriction)

\mathcal{Q} $\geq n R.C, \leq n R.C$ (qualified number restriction)

Example

Team

Team $\sqcap \geq 10$ hasMember

Team $\sqcap \geq 11$ hasMember
 $\sqcap \forall$ hasMember.Soccer-player

$\mathcal{AL}[X]$

\mathcal{R} $R \sqcap S$ (role conjunction)

\mathcal{I} R^- (inverse roles)

\mathcal{H} (role hierarchies)

\mathcal{F} $u_1 = u_2, u_1 \neq u_2$ (feature (dis)agreements)

$S[X]$

S \mathcal{ALC} + transitive roles

$SHIQ$ \mathcal{ALC} + transitive roles

+ role hierarchies

+ inverse roles

+ number restrictions

Tbox

\sqcap Terminological axioms:

$\sqsubset C = D$ ($R = S$)

$\sqsubset C \sqsubseteq D$ ($R \sqsubseteq S$)

\sqsubset (disjoint $C D$)

\sqcap An equality whose left-hand side is an atomic concept is a definition.

\sqcap A finite set of definitions T is a Tbox (or terminology) if no symbolic name is defined more than once.

Example Tbox

Soccer-player $\sqsubseteq T$

Team $\sqsubseteq \geq 2$ hasMember

Large-Team = Team $\sqcap \geq 10$ hasMember

S-Team = Team $\sqcap \geq 11$ hasMember
 $\sqcap \forall$ hasMember.Soccer-player

DL as sublanguage of FOPL

Team(this)
^
($\exists x_1, \dots, x_{11}$:
hasMember(this, x1) ^ ... ^ hasMember(this, x11)
^ $x_1 \neq x_2$ ^ ... ^ $x_{10} \neq x_{11}$)
^
($\forall x$: hasMember(this, x) \rightarrow Soccer-player(x))

Abox

- n Assertions about individuals:
 - C(a)
 - R(a,b)

Example

Ida-member(Sture)

Individuals in the description language

- n $O \{i_1, \dots, i_k\}$ (one-of)
- n R:a (fills)

Example

(S-Team \sqcap hasMember:Sture)(IDA-FF)

Knowledge base

A knowledge base is a tuple $\langle T, A \rangle$
where T is a Tbox and A is an Abox.

Example KB

Soccer-player \subseteq T
 Team $\subseteq \geq 2$ hasMember
 Large-Team = Team $\cap \geq 10$ hasMember
 S-Team = Team $\cap \geq 11$ hasMember
 $\cap \forall$ hasMember.Soccer-player

Ida-member(Sture)

(S-Team \cap hasMember:Sture)(IDA-FF)

\mathcal{AL} (Semantics)

An interpretation I consists of a non-empty set Δ^I (the domain of the interpretation) and an interpretation function \cdot^I which assigns to every atomic concept A a set $A^I \subseteq \Delta^I$ and to every atomic role R a binary relation $R^I \subseteq \Delta^I \times \Delta^I$.

The interpretation function is extended to concept definitions using inductive definitions.

\mathcal{AL} (Semantics)

$C, D \rightarrow A$ (atomic concept)

\top (universal concept) $\top^I = \Delta^I$

\perp (bottom concept) $\perp^I = \emptyset$

$\neg A$ (atomic negation) $(\neg A)^I = \Delta^I \setminus A^I$

$C \cap D$ (conjunction) $(C \cap D)^I = C^I \cap D^I$

$\forall R.C$ (value restriction) $(\forall R.C)^I = \{a \in \Delta^I \mid \forall b. (a,b) \in R^I \rightarrow b \in C^I\}$

$\exists R.T$ (limited existential quantification) $(\exists R.T)^I = \{a \in \Delta^I \mid \exists b. (a,b) \in R^I \wedge b \in T^I\}$

\mathcal{ALC} (Semantics)

$(\neg C)^I = \Delta^I \setminus C^I$

$(C \cup D)^I = C^I \cup D^I$

$(\geq n R)^I = \{a \in \Delta^I \mid \#\{b \in \Delta^I \mid (a,b) \in R^I\} \geq n\}$

$(\leq n R)^I = \{a \in \Delta^I \mid \#\{b \in \Delta^I \mid (a,b) \in R^I\} \leq n\}$

$(\exists R.C)^I = \{a \in \Delta^I \mid \exists b \in \Delta^I : (a,b) \in R^I \wedge b \in C^I\}$

Semantics

Individual i

$i^I \in \Delta^I$

Unique Name Assumption:

if $i_1 \neq i_2$ then $i_1^I \neq i_2^I$

Semantics

An interpretation \cdot^I is a model for a terminology T iff

$C^I = D^I$ for all $C = D$ in T

$C^I \subseteq D^I$ for all a $C \subseteq D$ in T

$C^I \cap D^I = \emptyset$ for all (disjoint $C D$) in T

Semantics

An interpretation \mathcal{I} is a model for a knowledge base $\langle T, A \rangle$ iff

\mathcal{I} is a model for T

$a^{\mathcal{I}} \in C^{\mathcal{I}}$ for all $C(a)$ in A
 $\langle a^{\mathcal{I}}, b^{\mathcal{I}} \rangle \in R^{\mathcal{I}}$ for all $R(a,b)$ in A

Semantics - acyclic Tbox

$Bird = Animal \cap \forall Skin.Feather$

$\Delta^{\mathcal{I}} = \{tweety, goofy, fea1, fur1\}$

$Animal^{\mathcal{I}} = \{tweety, goofy\}$

$Feather^{\mathcal{I}} = \{fea1\}$

$Skin^{\mathcal{I}} = \{\langle tweety, fea1 \rangle, \langle goofy, fur1 \rangle\}$

$Bird^{\mathcal{I}} = \{tweety\}$

Semantics - cyclic Tbox

$QuietPerson = Person \cap \forall Friend.QuietPerson$
 $(A = F(A))$

$\Delta^{\mathcal{I}} = \{john, sue, andrea, bill\}$

$Person^{\mathcal{I}} = \{john, sue, andrea, bill\}$

$Friend^{\mathcal{I}} = \{\langle john, sue \rangle, \langle andrea, bill \rangle, \langle bill, bill \rangle\}$

$QuietPerson^{\mathcal{I}} = \{john, sue\}$

$QuietPerson^{\mathcal{I}} = \{john, sue, andrea, bill\}$

Semantics - cyclic Tbox

Descriptive semantics: $A = F(A)$ is a constraint stating that A has to be some solution for the equation.

- Not appropriate for defining concepts
- Necessary and sufficient conditions for concepts

$Human = Mammal \cap \exists Parent$

$\cap \forall Parent.Human$

Semantics - cyclic Tbox

Least fixpoint semantics: $A = F(A)$ specifies that A is to be interpreted as the smallest solution (if it exists) for the equation.

- Appropriate for inductively defining concepts

$DAG = EmptyDAG \cup (Node \cap \forall Arc.DAG)$

$Human = Mammal \cap \exists Parent \cap \forall Parent.Human$

$Human = \perp$

Semantics - cyclic Tbox

Greatest fixpoint semantics: $A = F(A)$ specifies that A is to be interpreted as the greatest solution (if it exists) for the equation.

- Appropriate for defining concepts whose individuals have circularly repeating structure

$FoB = Blond \cap \exists Child.FoB$

$Human = Mammal \cap \exists Parent \cap \forall Parent.Human$

$Horse = Mammal \cap \exists Parent \cap \forall Parent.Horse$

$Human = Horse$

Open world vs closed world semantics

Databases: closed world reasoning
 database instance represents one interpretation
 absence of information interpreted as negative information
 “complete information”
 query evaluation is finite model checking
DL: open world reasoning
 Abox represents many interpretations (its models)
 absence of information is lack of information
 “incomplete information”
 query evaluation is logical reasoning

Open world vs closed world semantics

hasChild(Jocasta, Oedipus)
 hasChild(Jocasta, Polyneikes)
 hasChild(Oedipus, Polyneikes)
 hasChild(Polyneikes, Thersandros)
 patricide(Oedipus)
 \neg patricide(Thersandros)

Does it follow from the Abox that
 \exists hasChild.(patricide \cap \exists hasChild. \neg patricide)(Jocasta) ?

Reasoning services

- n Satisfiability of concept
- n Subsumption between concepts
- n Equivalence between concepts
- n Disjointness of concepts

- n Classification

- n Instance checking
- n Realization
- n Retrieval
- n Knowledge base consistency

Reasoning services

- n Satisfiability of concept
 - n C is satisfiable w.r.t. \mathcal{T} if there is a model I of \mathcal{T} such that C^I is not empty.
- n Subsumption between concepts
 - n C is subsumed by D w.r.t. \mathcal{T} if $C^I \subseteq D^I$ for every model I of \mathcal{T} .
- n Equivalence between concepts
 - n C is equivalent to D w.r.t. \mathcal{T} if $C^I = D^I$ for every model I of \mathcal{T} .
- n Disjointness of concepts
 - n C and D are disjoint w.r.t. \mathcal{T} if $C^I \cap D^I = \emptyset$ for every model I of \mathcal{T} .

Reasoning services

- n Reduction to subsumption
 - n C is unsatisfiable iff C is subsumed by \perp
 - n C and D are equivalent iff C is subsumed by D and D is subsumed by C
 - n C and D are disjoint iff $C \cap D$ is subsumed by \perp

- n The statements also hold w.r.t. a Tbox.

Reasoning services

- n Reduction to unsatisfiability
 - n C is subsumed by D iff $C \cap \neg D$ is unsatisfiable
 - n C and D are equivalent iff both $(C \cap \neg D)$ and $(D \cap \neg C)$ are unsatisfiable
 - n C and D are disjoint iff $C \cap D$ is unsatisfiable

- n The statements also hold w.r.t. a Tbox.

Tableau algorithms

- n To prove that C subsumes D:
 - If C subsumes D, then it is impossible for an individual to belong to D but not to C.
 - Idea: Create an individual that belongs to D and not to C and see if it causes a contradiction.
 - If **always** a contradiction (clash) then subsumption is proven. Otherwise, we have found a model that contradicts the subsumption.

Tableau algorithms

- n Based on constraint systems.
 - $S = \{x: \neg C \cap D\}$
 - Add constraints according to a set of propagation rules
 - Until clash or no constraint is applicable

Tableau algorithms – de Morgan rules

- $\neg \neg C \quad C$
- $\neg (A \cap B) \quad \neg A \cup \neg B$
- $\neg (A \cup B) \quad \neg A \cap \neg B$
- $\neg (\forall R.C) \quad \exists R.(\neg C)$
- $\neg (\exists R.C) \quad \forall R.(\neg C)$

Tableau algorithms – constraint propagation rules

- n $S \cap \{x:C_1, x:C_2\} \cup S$
if $x: C_1 \cap C_2$ in S
and either $x:C_1$ or $x:C_2$ is not in S
- n $S \cup \{x:D\} \cup S$
if $x: C_1 \cup C_2$ in S and neither $x:C_1$ or $x:C_2$
is in S, and $D = C_1$ or $D = C_2$

Tableau algorithms – constraint propagation rules

- n $S \cap \{y:C\} \cup S$
if $x: \forall R.C$ in S and xRy in S and $y:C$ is not
in S
- n $S \cap \{xRy, y:C\} \cup S$
if $x: \exists R.C$ in S and y is a new variable and
there is no z such that both xRz and $z:C$
are in S

Example

- n ST: Tournament
 $\cap \exists \text{ hasParticipant.Swedish}$
- n SBT: Tournament
 $\cap \exists \text{ hasParticipant.}(\text{Swedish} \cap \text{Belgian})$

Example 1

n SBT => ST?
n S = { x:
 $\neg(\text{Tournament} \cap \exists \text{ hasParticipant.Swedish})$
 $\cap (\text{Tournament}$
 $\cap \exists \text{ hasParticipant.}(\text{Swedish} \cap \text{Belgian}))$
}

Example 1

n S = { x:
 $(\neg\text{Tournament}$
 $\cup \forall \text{ hasParticipant.}\neg \text{Swedish})$
 $\cap (\text{Tournament}$
 $\cap \exists \text{ hasParticipant.}(\text{Swedish} \cap \text{Belgian}))$
}

Example 1

\cap -rule:
n S = {
x: $(\neg\text{Tournament}$
 $\cup \forall \text{ hasParticipant.}\neg \text{Swedish})$
 $\cap (\text{Tournament}$
 $\cap \exists \text{ hasParticipant.}(\text{Swedish} \cap \text{Belgian}))$,
x: $\neg\text{Tournament}$
 $\cup \forall \text{ hasParticipant.}\neg \text{Swedish}$,
x: Tournament,
x: $\exists \text{ hasParticipant.}(\text{Swedish} \cap \text{Belgian})$
}

Example 1

\exists -rule:
n S = {
x: $(\neg\text{Tournament} \cup \forall \text{ hasParticipant.}\neg \text{Swedish})$
 $\cap (\text{Tournament}$
 $\cap \exists \text{ hasParticipant.}(\text{Swedish} \cap \text{Belgian}))$,
x: $\neg\text{Tournament}$
 $\cup \forall \text{ hasParticipant.}\neg \text{Swedish}$,
x: Tournament,
x: $\exists \text{ hasParticipant.}(\text{Swedish} \cap \text{Belgian})$,
x hasParticipant y, y: $(\text{Swedish} \cap \text{Belgian})$
}

Example 1

\cap -rule:
n S = {x: $(\neg\text{Tournament} \cup \forall \text{ hasParticipant.}\neg \text{Swedish})$
 $\cap (\text{Tournament}$
 $\cap \exists \text{ hasParticipant.}(\text{Swedish} \cap \text{Belgian}))$,
x: $\neg\text{Tournament} \cup \forall \text{ hasParticipant.}\neg \text{Swedish}$,
x: Tournament,
x: $\exists \text{ hasParticipant.}(\text{Swedish} \cap \text{Belgian})$,
x hasParticipant y, y: $(\text{Swedish} \cap \text{Belgian})$,
y: Swedish, y: Belgian }

Example 1

U-rule, choice 1
n S = { x: $(\neg\text{Tournament} \cup \forall \text{ hasParticipant.}\neg \text{Swedish})$
 $\cap (\text{Tournament}$
 $\cap \exists \text{ hasParticipant.}(\text{Swedish} \cap \text{Belgian}))$,
x: $\neg\text{Tournament} \cup \forall \text{ hasParticipant.}\neg \text{Swedish}$,
x: Tournament,
x: $\exists \text{ hasParticipant.}(\text{Swedish} \cap \text{Belgian})$,
x hasParticipant y, y: $(\text{Swedish} \cap \text{Belgian})$,
y: Swedish, y: Belgian,
x: $\neg\text{Tournament}$
}
clash

Example 1

U-rule, choice 2

```

n S = {x: (¬Tournament U ∇ hasParticipant.¬ Swedish)
  ∩ (Tournament
  ∩ ∃ hasParticipant.(Swedish ∩ Belgian)),
x: ¬Tournament U ∇ hasParticipant.¬ Swedish,
x: Tournament,
x: ∃ hasParticipant.(Swedish ∩ Belgian),
x hasParticipant y, y: (Swedish ∩ Belgian),
y: Swedish, y: Belgian,
x: ∇ hasParticipant.¬ Swedish
}

```

Example 1

choice 2 – continued

∇-rule

```

n S = {
x: (¬Tournament U ∇ hasParticipant.¬ Swedish)
  ∩ (Tournament ∩ ∃ hasParticipant.(Swedish ∩ Belgian)),
x: ¬Tournament U ∇ hasParticipant.¬ Swedish,
x: Tournament,
x: ∃ hasParticipant.(Swedish ∩ Belgian),
x hasParticipant y, y: (Swedish ∩ Belgian),
y: Swedish, y: Belgian,
x: ∇ hasParticipant.¬ Swedish,
y: ¬ Swedish
}

clash

```

Example 2

n ST => SBT?

```

n S = { x:
  ¬ (Tournament
  ∩ ∃ hasParticipant.(Swedish ∩ Belgian))
  ∩ (Tournament ∩ ∃ hasParticipant.Swedish)
}

```

Example 2

```

n S = { x:
  (¬Tournament
  U ∇ hasParticipant.(¬ Swedish U ¬ Belgian))
  ∩ (Tournament ∩ ∃ hasParticipant.Swedish)
}

```

Example 2

∩-rule

```

n S = {
x: (¬Tournament
  U ∇ hasParticipant.(¬ Swedish U ¬ Belgian))
  ∩ (Tournament ∩ ∃ hasParticipant.Swedish),
x: (¬Tournament
  U ∇ hasParticipant.(¬ Swedish U ¬ Belgian)),
x: Tournament,
x: ∃ hasParticipant.Swedish
}

```

Example 2

∃-rule

```

n S = {
x: (¬Tournament
  U ∇ hasParticipant.(¬ Swedish U ¬ Belgian))
  ∩ (Tournament ∩ ∃ hasParticipant.Swedish),
x: (¬Tournament
  U ∇ hasParticipant.(¬ Swedish U ¬ Belgian)),
x: Tournament,
x: ∃ hasParticipant.Swedish,
x hasParticipant y, y: Swedish
}

```

Example 2

U-rule, choice 1
 $\mathcal{S} = \{$
 $x: (\neg \text{Tournament}$
 $\quad U \forall \text{ hasParticipant.}(\neg \text{Swedish } U \neg \text{Belgian}))$
 $\quad \cap (\text{Tournament } \cap \exists \text{ hasParticipant.Swedish}),$
 $x: (\neg \text{Tournament}$
 $\quad U \forall \text{ hasParticipant.}(\neg \text{Swedish } U \neg \text{Belgian})),$
 $x: \text{Tournament},$
 $x: \exists \text{ hasParticipant.Swedish},$
 $x \text{ hasParticipant } y, y: \text{Swedish},$
 $x: \neg \text{Tournament}$
 $\quad \}$
 clash

Example 2

U-rule, choice 2
 $\mathcal{S} = \{$
 $x: (\neg \text{Tournament}$
 $\quad U \forall \text{ hasParticipant.}(\neg \text{Swedish } U \neg \text{Belgian}))$
 $\quad \cap (\text{Tournament } \cap \exists \text{ hasParticipant.Swedish}),$
 $x: (\neg \text{Tournament}$
 $\quad U \forall \text{ hasParticipant.}(\neg \text{Swedish } U \neg \text{Belgian})),$
 $x: \text{Tournament},$
 $x: \exists \text{ hasParticipant.Swedish},$
 $x \text{ hasParticipant } y, y: \text{Swedish},$
 $x: \forall \text{ hasParticipant.}(\neg \text{Swedish } U \neg \text{Belgian})$
 $\quad \}$

Example 2

choice 2 continued
 \forall -rule
 $\mathcal{S} = \{$
 $x: (\neg \text{Tournament}$
 $\quad U \forall \text{ hasParticipant.}(\neg \text{Swedish } U \neg \text{Belgian}))$
 $\quad \cap (\text{Tournament } \cap \exists \text{ hasParticipant.Swedish}),$
 $x: (\neg \text{Tournament}$
 $\quad U \forall \text{ hasParticipant.}(\neg \text{Swedish } U \neg \text{Belgian})),$
 $x: \text{Tournament},$
 $x: \exists \text{ hasParticipant.Swedish},$
 $x \text{ hasParticipant } y, y: \text{Swedish},$
 $x: \forall \text{ hasParticipant.}(\neg \text{Swedish } U \neg \text{Belgian}),$
 $y: (\neg \text{Swedish } U \neg \text{Belgian})$
 $\quad \}$

Example 2

choice 2 continued
 U-rule, choice 2.1
 $\mathcal{S} = \{$
 $x: (\neg \text{Tournament}$
 $\quad U \forall \text{ hasParticipant.}(\neg \text{Swedish } U \neg \text{Belgian}))$
 $\quad \cap (\text{Tournament } \cap \exists \text{ hasParticipant.Swedish}),$
 $x: (\neg \text{Tournament}$
 $\quad U \forall \text{ hasParticipant.}(\neg \text{Swedish } U \neg \text{Belgian})),$
 $x: \text{Tournament},$
 $x: \exists \text{ hasParticipant.Swedish},$
 $x \text{ hasParticipant } y, y: \text{Swedish},$
 $x: \forall \text{ hasParticipant.}(\neg \text{Swedish } U \neg \text{Belgian}),$
 $y: (\neg \text{Swedish } U \neg \text{Belgian}),$
 $y: \neg \text{Swedish}$
 $\quad \}$ clash

Example 2

choice 2 continued
 U-rule, choice 2.2
 $\mathcal{S} = \{$
 $x: (\neg \text{Tournament}$
 $\quad U \forall \text{ hasParticipant.}(\neg \text{Swedish } U \neg \text{Belgian}))$
 $\quad \cap (\text{Tournament } \cap \exists \text{ hasParticipant.Swedish}),$
 $x: (\neg \text{Tournament}$
 $\quad U \forall \text{ hasParticipant.}(\neg \text{Swedish } U \neg \text{Belgian})),$
 $x: \text{Tournament},$
 $x: \exists \text{ hasParticipant.Swedish},$
 $x \text{ hasParticipant } y, y: \text{Swedish},$
 $x: \forall \text{ hasParticipant.}(\neg \text{Swedish } U \neg \text{Belgian}),$
 $y: (\neg \text{Swedish } U \neg \text{Belgian}),$
 $y: \neg \text{Belgian}$
 $\quad \}$ ok, model

Complexity - languages

Overview available via the DL home page at
<http://dl.kr.org>

Example tractable language:

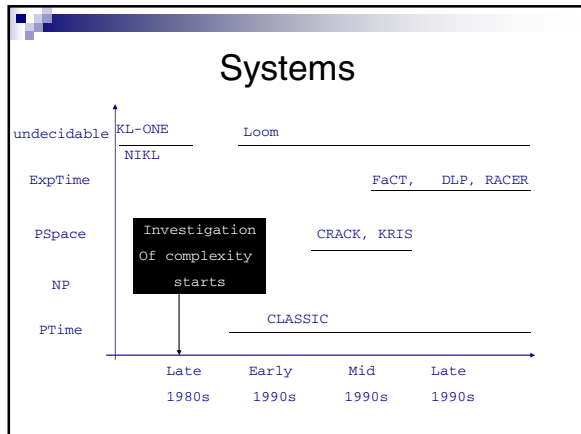
$A, T, \perp, \neg A, C \cap D, \forall R.C, \geq n R, \leq n R$

Reasons for intractability:

choices, e.g. $C \cup D$
 exponential size models,
 e.g. interplay universal and existential quantification

Reasons for undecidability:

e.g. role-value maps $R=S$



Systems

- n Overview available via the DL home page at <http://dl.kr.org>
- n Current systems include: CEL, Cerebra Enginer, FaCT++, fuzzyDL, Hermit, KAON2, MSPASS, Pellet, QuOnto, RacerPro, SHER

Extensions

- n Time
- n Defaults
- n Part-of
- n Knowledge and belief
- n Uncertainty (fuzzy, probabilistic)

DAML+OIL Class Constructors

Constructor	DL Syntax	Example
intersectionOf	$C_1 \sqcap \dots \sqcap C_n$	Human \sqcap Male
unionOf	$C_1 \sqcup \dots \sqcup C_n$	Doctor \sqcup Lawyer
complementOf	$\neg C$	\neg Male
oneOf	$\{x_1 \dots x_n\}$	{john, mary}
toClass	$\forall P.C$	\forall hasChild.Doctor
hasClass	$\exists P.C$	\exists hasChild.Lawyer
hasValue	$\exists P.\{x\}$	\exists citizenOf.{USA}
minCardinalityQ	$\geq n.P.C$	≥ 2 hasChild.Lawyer
maxCardinalityQ	$\leq n.P.C$	≤ 1 hasChild.Male
cardinalityQ	$= n.P.C$	$= 1$ hasParent.Female

- ☛ XMLS **datatypes** as well as classes
- ☛ Arbitrarily complex **nesting** of constructors
 - E.g., $\text{Person} \sqcap \forall \text{hasChild} . (\text{Doctor} \sqcup \exists \text{hasChild} . \text{Doctor})$

EDBT 2002, DAML+OIL - p.1332

DAML+OIL Axioms

Axiom	DL Syntax	Example
subClassOf	$C_1 \sqsubseteq C_2$	Human \sqsubseteq Animal \sqcap Biped
sameClassAs	$C_1 \equiv C_2$	Man \equiv Human \sqcap Male
subPropertyOf	$P_1 \sqsubseteq P_2$	hasDaughter \sqsubseteq hasChild
samePropertyAs	$P_1 \equiv P_2$	cost \equiv price
sameIndividualAs	$\{x_1\} \equiv \{x_2\}$	{President_Bush} \equiv {G_W_Bush}
disjointWith	$C_1 \sqsubseteq \neg C_2$	Male $\sqsubseteq \neg$ Female
differentIndividualFrom	$\{x_1\} \sqsubseteq \neg \{x_2\}$	{john} $\sqsubseteq \neg$ {peter}
inverseOf	$P_1 \equiv P_2^-$	hasChild \equiv hasParent $^-$
transitiveProperty	$P^+ \sqsubseteq P$	ancestor $^+$ \sqsubseteq ancestor
uniqueProperty	$\top \sqsubseteq \leq 1 P$	$\top \sqsubseteq \leq 1$ hasMother
unambiguousProperty	$\top \sqsubseteq \leq 1 P^-$	$\top \sqsubseteq \leq 1$ isMotherOf $^-$

- ☛ Axioms (mostly) **reducible to subClass/PropertyOf**

EDBT 2002, DAML+OIL - p.1432

OWL

- n OWL-Lite, OWL-DL, OWL-Full: increasing expressivity
- n A legal OWL-Lite ontology is a legal OWL-DL ontology is a legal OWL-Full ontology
- n OWL-DL: expressive description logic, decidable
- n XML-based
- n RDF-based (OWL-Full is extension of RDF, OWL-Lite and OWL-DL are extensions of a restriction of RDF)

OWL-Lite

- n **Class**, subClassOf, equivalentClass
- n intersectionOf (only named classes and restrictions)
- n **Property**, subPropertyOf, equivalentProperty
- n domain, range (global restrictions)
- n inverseOf, TransitiveProperty (*), SymmetricProperty, FunctionalProperty, InverseFunctionalProperty
- n allValuesFrom, someValuesFrom (local restrictions)
- n minCardinality, maxCardinality (only 0/1)
- n **Individual**, sameAs, differentFrom, AllDifferent

(*) restricted

OWL-DL

- n **Type separation** (class cannot also be individual or property, property cannot be also class or individual), Separation between DatatypeProperties and ObjectProperties
- n **Class –complex classes**, subClassOf, equivalentClass, *disjointWith*
- n *intersectionOf*, *unionOf*, *complementOf*
- n **Property**, subPropertyOf, equivalentProperty
- n domain, range (global restrictions)
- n inverseOf, TransitiveProperty (*), SymmetricProperty, FunctionalProperty, InverseFunctionalProperty
- n allValuesFrom, someValuesFrom (local restrictions), *oneOf*, *hasValue*
- n minCardinality, maxCardinality
- n **Individual**, sameAs, differentFrom, AllDifferent

(*) restricted

References

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