

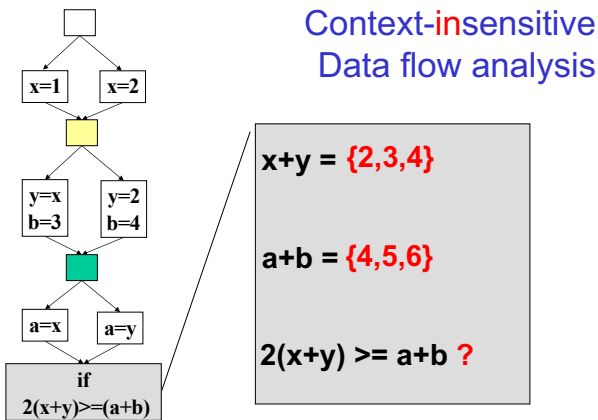
Outline

- Introduction to SSA
- Construction, Destruction
- How to capture analysis results
- Optimizations
 - Classic analyses and optimizations on SSA representations
 - Heap analyses and optimizations

Context-sensitive analysis revisited

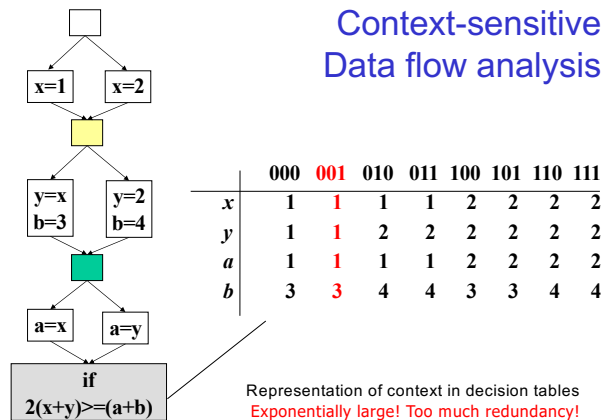
- Distinguish different call context of a method
- In general: Distinguish different execution path ending at and getting joined at a program point
 - Exponentially (in program size) many path in a sequential program
 - Exponentially many analysis values
- (Let away the problem of context sensitive analysis) How to capture the context-sensitive results efficiently?

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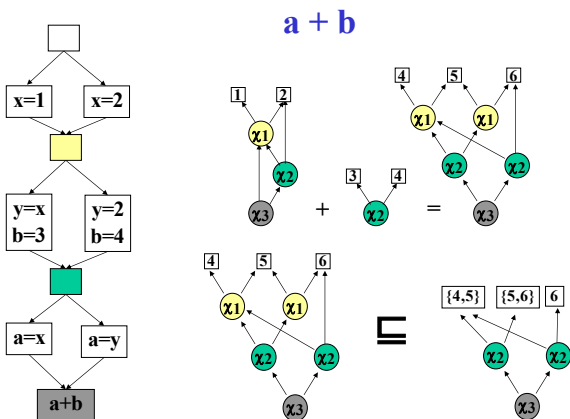
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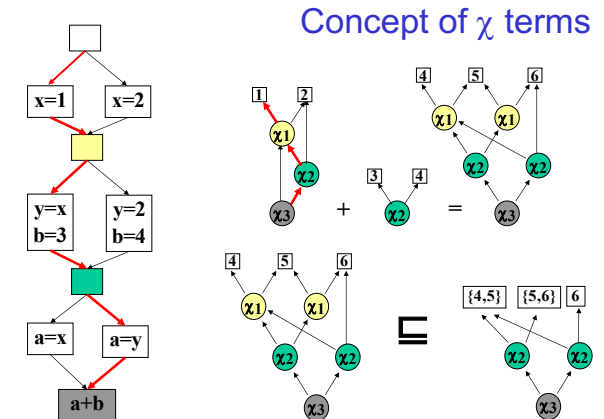
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Advantages of χ -Terms

- Compact representation of context sensitive information
- Delayed widening (recall abstract interpretation) of terms until no more memory: no unnecessary loss of information

$$\chi_3(\chi_2(\chi_1(4,5), \chi_1(5,6)), \chi_2(\chi_1(4,5), 6)) \sqsubseteq \chi_3(\chi_2(\{4,5\}, \{5,6\}), \chi_2(\{4,5\}, 6)) \sqsubseteq \chi_3(\{4,5,6\}, \{4,5,6\}) = \{4,5,6\} \sqsubseteq [4,6] \sqsubseteq \top$$

- χ -Terms are implementation of decision diagrams.
- Adaptation of OBDD implementation techniques.
- Symbolic computation with χ -terms (simplification of terms).
- Transition of the idea of SSA ϕ -function to data-flow values
- Particularly interesting for **address values** in order to distinguish **memory partitions** as long as possible

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Data-Flow Analyses (DFA) on SSA

- Each SSA node type n has a concrete semantics $[n]$,
 - On execution of n , map inputs i to outputs o and $o = [n](i)$
 - inputs i and outputs o are records of typed values (P1)
 - $[n] :: \text{type}(i_1) \times \dots \times \text{type}(i_k) \rightarrow \text{type}(o_1) \times \dots \times \text{type}(o_l)$
- Each static data-flow analysis abstracts from concrete semantics and values
 - Abstract semantics T_n is called transfer function of node type n
 - On analysis of n , map abstract analysis inputs $a(i)$ to abstract analysis outputs $a(o)$ and $a(o) = T_n(a(i))$
 - $T_n :: \text{type}(a(i_1)) \times \dots \times \text{type}(a(i_k)) \rightarrow \text{type}(a(o_1)) \times \dots \times \text{type}(a(o_l))$
- For each abstract analysis semantics value representation $U = \text{type}(a(\bullet))$ – analysis universe – there is a partial order relation \sqsubseteq and a meet operation \sqcup (supremum),
 - $\sqsubseteq : U \times U$
 - $\sqcup : U \times U \rightarrow U$ with $\sqcup = T_\phi$
 - (U, \sqsubseteq) defines a complete partial order

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DFA on SSA (cont.)

- Provided that transfer functions are monotone: $x \leq y \Rightarrow T_n(x) \leq T_n(y)$ with $x, y \in U_1 \times \dots \times U_k$ and \leq defined element-wise with \sqsubseteq of the respective analysis values U_i
- The following iteration terminates in a unique fixed point
 - Initialize the input of each node the SSA graph with the smallest (bottom) element of the Lattice/CPO corresponding its abstract semantics type
 - Initialize the *start* nodes of the SSA graph with proper abstract values of the corresponding its abstract types
 - Attach each node with its corresponding transfer function
 - Compute abstract output values of the nodes
 - Any fair random selection of nodes terminates
 - Fewer updates use SCC and interval analysis to determine a traversal strategy, backward problems analyzed analogously)

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Generalization to χ -terms

- Given such a context-insensitive analysis (lattices for abstract values, set of transfer functions, initialization of *start* node) we can **systematically construct** a **context-sensitive** analysis
- χ -term algebras X over abstract semantic values $a \in U$ introduced
 - $a \in U \Rightarrow a \in X_U$
 - $t_1, t_2 \in X \Rightarrow \chi(t_1, t_2) \in X_U$
 - Induces sensitive CPO for abstract values (X_U, \sqsubseteq) and for $a_1, a_2 \in U$ and $t_1, t_2, t_3, t_4 \in X_U$:
 - $a_1 \sqsubseteq a_2 \Rightarrow a_1 \sqsubseteq a_2$
 - $\chi(a_1, a_2) \sqsubseteq a_1 \sqcup a_2$
 - $t_1 \sqsubseteq t_3, t_2 \sqsubseteq t_4 \Rightarrow \chi(t_1, t_2) \sqsubseteq \chi(t_3, t_4)$
- New transfer functions on X_U are induced

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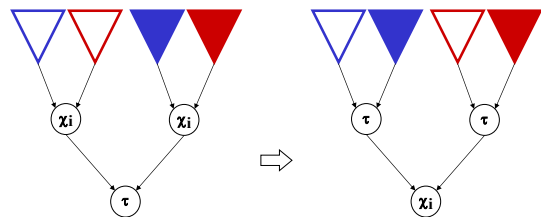
New transfer functions

- ϕ -node's transfer functions:
 - Insensitive: $T_\phi = \sqcup = a(i_1) \sqcup \dots \sqcup a(i_k)$
 - Sensitive: $S_\phi = \chi_b(a(i_1), \dots, a(i_k))$ with b block number of the ϕ -node
- Ordinary operation's (τ node's) transfer functions:
 - Insensitive (w.l.o.g. binary operation): $T_\tau : U_a \times U_b \rightarrow U_c$
 - Sensitive: $S_\tau : X_a \times X_b \rightarrow X_c$ and for $a_1, a_2 \in U_a, U_b$ and $t_1, t_2 \in X_a, X_b$:
 - $S_\tau(a_1, a_2) = T_\tau(a_1, a_2)$
 - $S_\tau(\chi_x(t_1, t_2), \chi_y(t_3, t_4)) = \chi_k(S_\tau(\text{cof}(\chi_x(t_1, t_2), \chi_k, 1), \text{cof}(\chi_y(t_3, t_4), \chi_k, 1))), S_\tau(\text{cof}(\chi_x(t_1, t_2), \chi_k, 2), \text{cof}(\chi_y(t_3, t_4), \chi_k, 2)))$ with k larger of x, y and cof of the co-factorization

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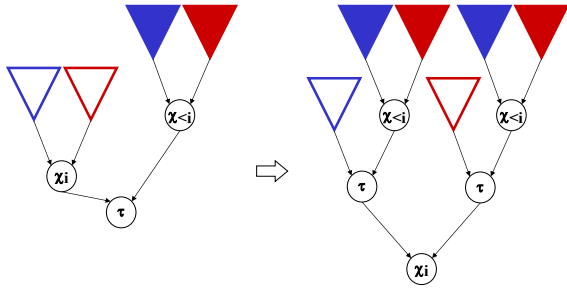
Sensitive Transfer Schema (case a)



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Sensitive Transfer Schema (case b)



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Co-factorization

- $\text{cof}(\chi_x(t_1, t_2), \chi_k, i)$ selects the i -th branch of a χ -term if $\chi_x = \chi_k$ and returns the whole χ -term, otherwise
- $\text{cof}(\chi_x(t_1, t_2), \chi_k, 1) = t_1$ iff $k = x$ (case a)
- $\text{cof}(\chi_x(t_1, t_2), \chi_k, 2) = t_2$ iff $k = x$ (case a)
- $\text{cof}(\chi_x(t_1, t_2), \chi_k, i) = \chi_x(t_1, t_2)$ iff $k > x$ (case b)

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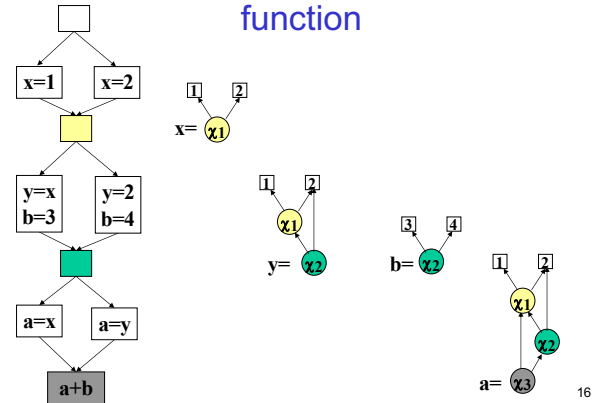
Example revisited: insensitive

- SSA node \oplus
- Semantic
 - $[\oplus] :: \text{Int} \times \text{Int} \rightarrow \text{Int}$
 - $[\oplus](a, b) = a + b$
- Abstract Int values $\{ \perp, 1, 2, \dots, \text{maxint}, \top \}$
- Context-insensitive transfer function:
 - $T_\oplus(\perp, x) = T_\oplus(x, \perp) = \perp$
 - $T_\oplus(\top, x) = T_\oplus(x, \top) = \top$
 - $T_\oplus(a, b) = [\oplus](a, b) = a + b$ for $a, b \in \text{Int}$
- Context-insensitive meet function
 - $T_\oplus(\perp, x) = T_\oplus(x, \perp) = x$
 - $T_\oplus(\top, x) = T_\oplus(x, \top) = \top$
 - $T_\oplus(x, x) = x$
 - $T_\oplus(x, y) = \top$

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Context-sensitive ϕ -node transfer function



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Context-sensitive $a \oplus b$

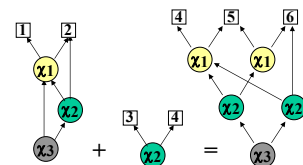
$$\begin{aligned}
 & S_\oplus(\chi_3(\chi_1(1,2), \chi_2(\chi_1(1,2), 2)), \chi_2(3,4)) \\
 &= \chi_3(S_\oplus(\text{cof}(\chi_3(\chi_1(1,2), \chi_2(\chi_1(1,2), 2)), \chi_3, 1), \text{cof}(\chi_2(3,4), \chi_3, 1)), \\
 & \quad S_\oplus(\text{cof}(\chi_3(\chi_1(1,2), \chi_2(\chi_1(1,2), 2)), \chi_3, 2), \text{cof}(\chi_2(3,4), \chi_3, 2))) \\
 &= \chi_3(S_\oplus(\chi_1(1,2), \chi_2(3,4)), \\
 & \quad S_\oplus(\chi_2(\chi_1(1,2), 2), \chi_2(3,4))) \\
 &= \chi_3(S_\oplus(S_\oplus(\text{cof}(\chi_1(1,2), \chi_2, 1), \text{cof}(\chi_2(3,4), \chi_2, 1))), \\
 & \quad S_\oplus(\text{cof}(\chi_1(1,2), \chi_2, 2), \text{cof}(\chi_2(3,4), \chi_2, 2))), \\
 & \quad \chi_2(S_\oplus(\text{cof}(\chi_2(\chi_1(1,2), 2), \chi_2, 1), \text{cof}(\chi_2(3,4), \chi_2, 1)), \\
 & \quad S_\oplus(\text{cof}(\chi_2(\chi_1(1,2), 2), \chi_2, 2), \text{cof}(\chi_2(3,4), \chi_2, 2)))) \\
 &= \chi_3(\chi_2(S_\oplus(\chi_1(1,2), 3), \\
 & \quad S_\oplus(\chi_1(1,2), 4)), \\
 & \quad \chi_2(S_\oplus(\chi_1(1,2), 3), \\
 & \quad S_\oplus(2,4)))
 \end{aligned}$$

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Context-sensitive $a \oplus b$ (cont.)

$$\begin{aligned}
 & S_\oplus(\chi_3(\chi_1(1,2), \chi_2(\chi_1(1,2), 2)), \chi_2(3,4)) = \dots \\
 &= \chi_3(\chi_2(S_\oplus(\chi_1(1,2), 3), S_\oplus(\chi_1(1,2), 4)), \chi_2(S_\oplus(\chi_1(1,2), 3), \\
 & \quad S_\oplus(2,4))) \\
 &= \chi_3(\chi_2(S_\oplus(\chi_1(1,2), 3), S_\oplus(\chi_1(1,2), 4)), \chi_2(S_\oplus(\chi_1(1,2), 3), \\
 & \quad 6)) \\
 &= \dots \\
 &= \chi_3(\chi_2(\chi_1(4,5), \chi_1(5,6)), \chi_2(\chi_1(4,5), 6))
 \end{aligned}$$



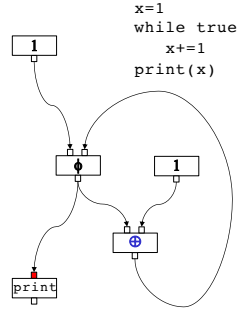
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Example

Given the SSA fragment on the left

- Perform **context-insensitive** data-flow analysis (using the definitions on the previous slides). What is the value at the entry of node x ?
- Perform **context-sensitive** data-flow analysis (using the definitions on the previous slides). What is the value at the entry of node x ?
- Why is the former less precise than the latter?
- Construct a scenario where you could take advantage of that precision in an optimization!

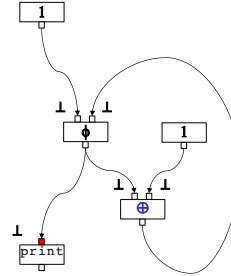


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Example (cont'd)

- Context-insensitive, initial situation:

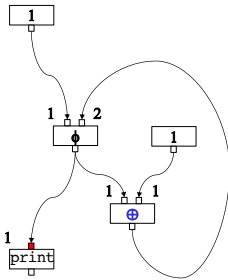


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Example (cont'd)

- Context-insensitive, after first iteration:

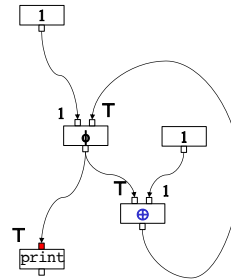


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Example (cont'd)

- Context-insensitive, after second iteration (stable):

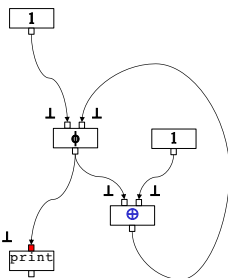


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Example (cont'd)

- Context-sensitive, initial situation:

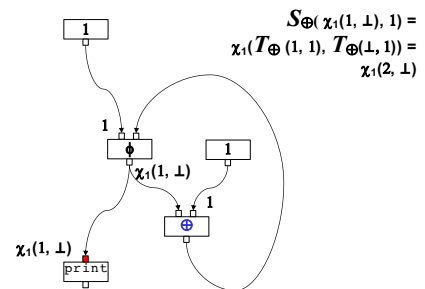


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Example (cont'd)

- Context-sensitive, after first iteration:



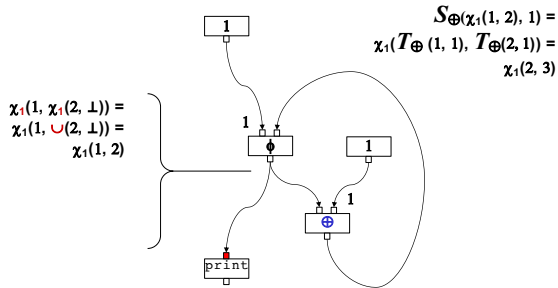
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$$\begin{aligned}
 S_{\oplus}(x_i(1, 1), 1) &= \\
 x_i(T_{\oplus}(1, 1), T_{\oplus}(1, 1)) &= \\
 x_i(2, 1) &
 \end{aligned}$$

Example (cont'd)

- Context-sensitive, after second iteration:

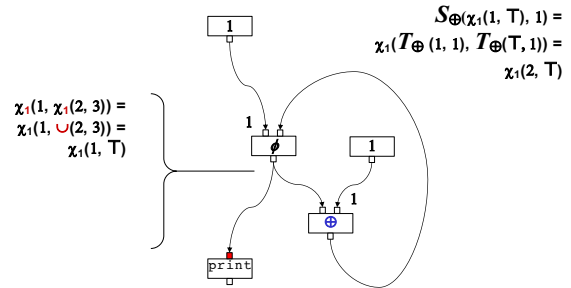


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Example (cont'd)

- Context-sensitive, after third iteration (stable):



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 - Classic analyses and optimizations on SSA representations
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Analyses and Optimizations

- Analyses in Compiler Construction allow to safely perform optimizations
- Cost model: runtime of a program
 - Statically only conservative approximations
 - Loop iterations
 - Conditional code
 - Even for linear code not known in advance:
 - Instruction scheduling
 - Cache access is data dependent
 - Instruction pipelining: execution time is not the sum of individual operations costs
- Alternative cost models:
 - memory size, power consumptions
 - Same non-decidability problem as for execution time
- Caution:** cost of a program \neq sum of costs of its elements

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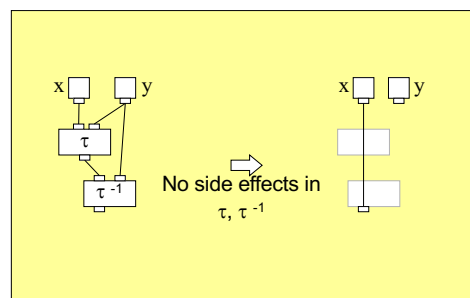
Optimization: Implementation

- Legal transformations in SSA-Graphs:
 - Simplifying transformations reduce the costs of a program
 - Preparative transformations allow the application of simplifying transformations
- Using
 - Algebraic Identities (e.g., Associative / Distributive law for certain operations)
 - Moving of operations
 - Reduction of dependencies
- Optimization is a sequence of **goal directed, legal simplifying and legal preparative** transformations
- Legibility proven
 - Locally by checking preconditions
 - Due to static data-flow analyses

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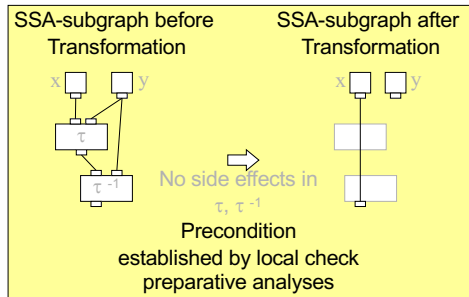
Algebraic Identity: Elimination of Operations and its Inverse



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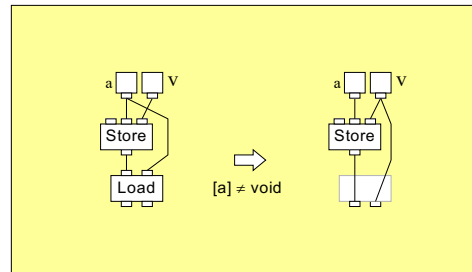
Graph Rewrite Schema



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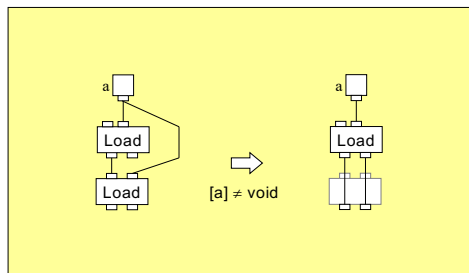
Elimination of Memory Operation and its Inverse



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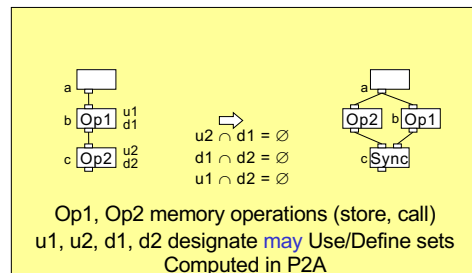
Elimination of Duplicated Memory Operations



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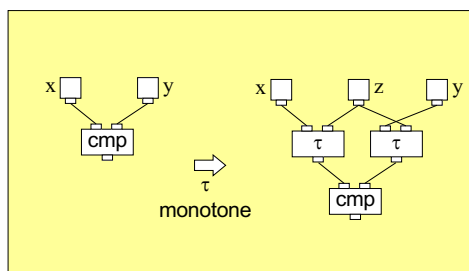
Elimination of non-essential dependencies



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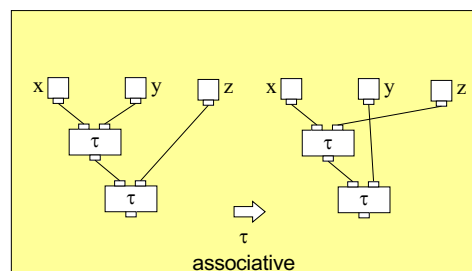
Algebraic Identity: Invariant Compares



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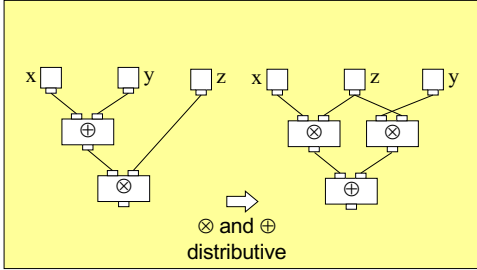
Associative Law



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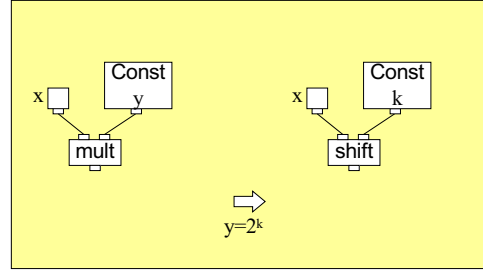
Distributive Law



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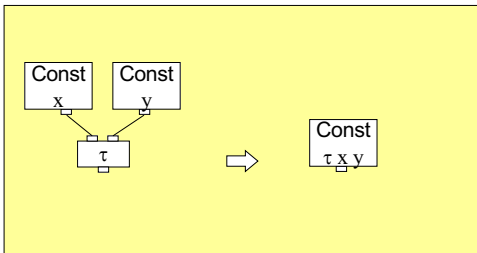
Operator Simplification



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Constant Folding

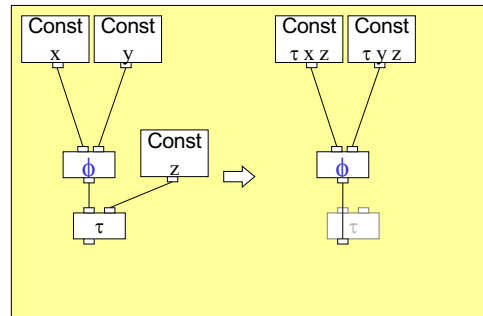


Evaluation using source algebra or target algebra (if allowed by source language)

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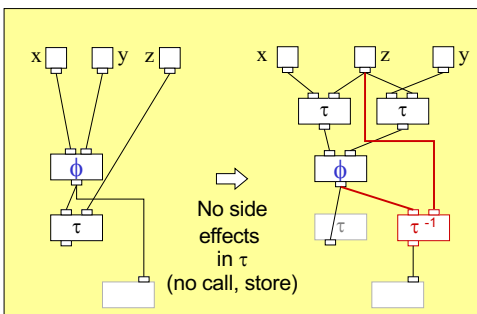
Constant folding over φ-functions



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General: Moving arithmetic operations over φ-functions



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Optimizations

- Strength reduction:
 - Bauer & Samelson 1959
 - Replace expensive by cheap operations
 - Loops contain multiplications with iteration variable,
 - These operations could be replaced by add operations (Induction analysis)
 - One of the oldest optimizations: already in Fortran I-compiler (1954/55) and Algol 58/60- compiler
- Partial redundancy elimination (PRE):
 - Morel & Renvoise 1978
 - Eliminate partially redundant computations
 - SSA eliminates all static but not dynamic redundancies
 - Problem on SSA: which is the best block to perform the computation
 - Move loop invariant computations out of loops, into conditionals
 - subsumes a number of simpler optimization

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Example: Strength reduction

```

for (i=0; i<n; i++){
  for (j=0; j<n; j++){
    b[i,j]=a[i,j];
  }
}
//Original loop body:
ao = i*n*d + j*d
aij = >a[0,0]<+ao
bo = i*n*d + j*d
bij = >b[0,0]<+bo
<bij> = <aij>

```

```

adda=>a[0,0]<
addb=>b[0,0]<
d=4
addend=adda+n*n*d;
jump (addend==adda) END
LOOP:
<addb>=<adda>
adda=adda+d
addb=addb+d
jump LOOP
END:
exit

```

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Induction Analysis Idea

- Find **Induction variable** i for a loop using DFA
 - i is **induction variable** if in loop only assignments of form $i := i+c$ with loop constant c or, recursively, $i := c'*i'+c''$ with i' induction variable and loop constants c', c''
 - c is a **loop constant** iff c does not change value in loop, i.e.
 - c is static constant,
 - c defined in enclosing loop
- Example (cont'd), consider the inner loop:
 - for (j=0; j<n; j++){...}
 - Direct induction variable: j , as $j=j+1$ ($c=1$)
 - Indirect induction variable: $ao=i*n*d+j*d$ ($c'=d, c''=i*n*d$)
 - Note that $i*n*d$ and d are loop constants for the inner loop

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Induction Transformation Idea

- Transformation goal: values of induction variables should grow linearly with iteration; add operations replace mul t operations
- Transformation:
 - Let i_0 initialization of i and induction variables, $i := i+c$ and $i' := c'*i'+c''$
 - New variable ia initialized $ia := c'*i_0+c''$
 - At loop end insert $ia = ia + c'*c$
 - Replace consistently operands i' by ia
 - Remove all assignments to i, i' and i, i' themselves if i is not used elsewhere (DFA)
- Example:
 - Before: loop $ao = i*n*d+j*d \dots j++$ end loop
 - After: $aoa = i*n*d$ loop $\dots aoa = aoa + d$ end loop

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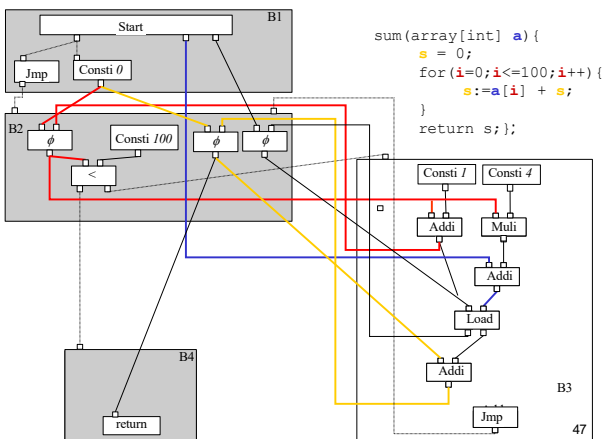
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Induction Analysis: Implementation

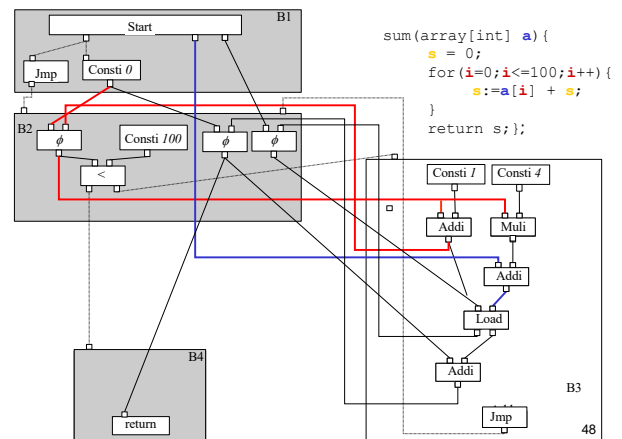
- Assume initially optimistically: all variables are induction variables
- Finding induction variable i for a loop follows definition
- Iteratively until fix point: i is not induction variable if **not**:
 - $i := i+c$ with loop constants c (direct induction variable)
 - $i := c'*i'+c''$ with i' induction variable and loop constants c', c'' (indirect induction variable)
- On SSA, simplifications of that analysis are possible
 - Any direct loop variable corresponds to a cyclic subgraph over $i := \phi(i_1 \dots i_n)$
 - Find Strongly Connected Component (SCC) and check those subgraphs for the direct induction variable condition first
 - Then find the indirect induction variables

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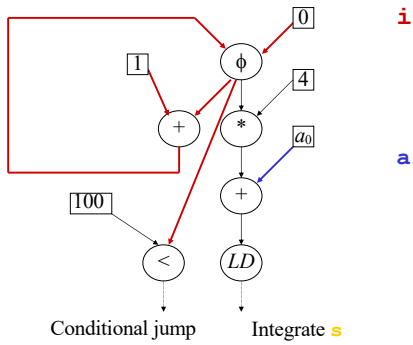


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Induction Variables (Schematic)

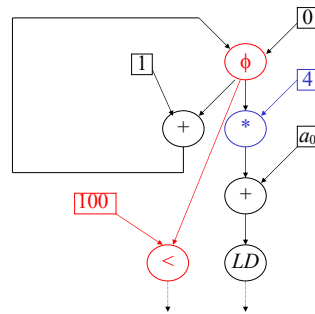


Conditional jump Integrate s

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Move \times over ϕ -function

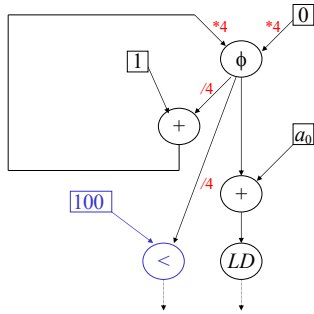


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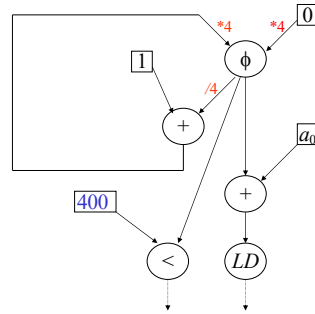
Invariant Comparison ($\times 4$) then constant folding and removal of operation and its invariant ($/4 \times 4$)



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Distributive Law then constant folding

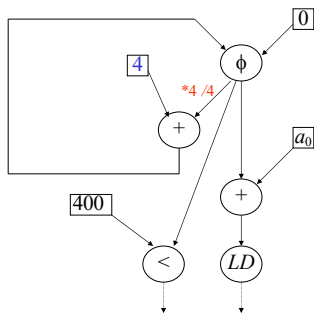


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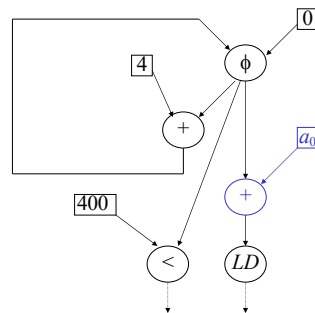
Removal of operation and its inverse



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Move $+$ over ϕ -function

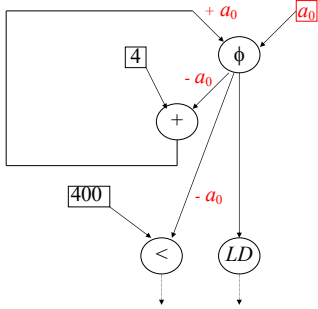


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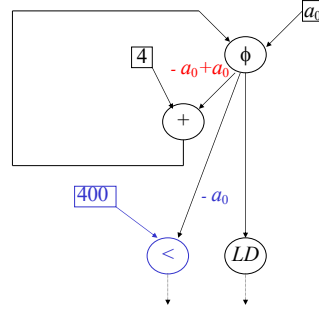
Associative Law



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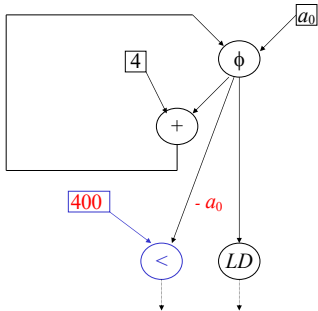
Removal of operation and its inverse



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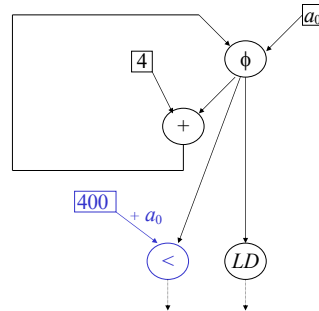
Invariant Comparison (+a₀) then removal of operation and its inverse



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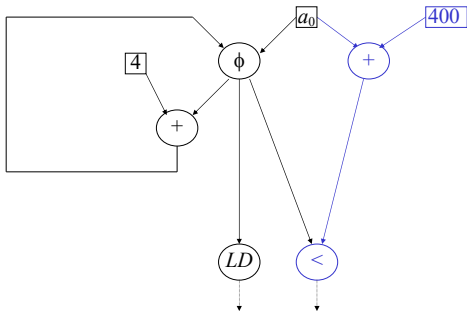
Rearranging the drawing



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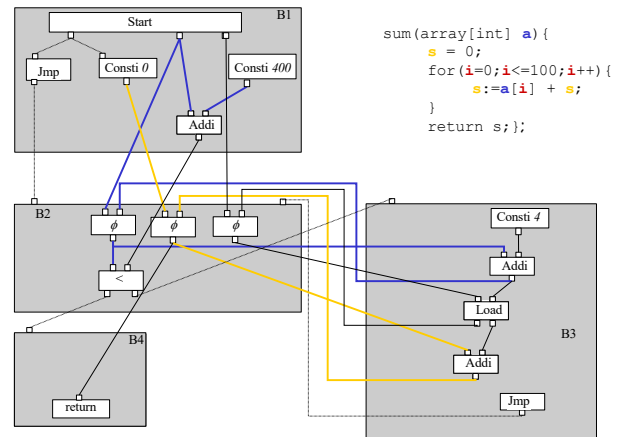
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Result



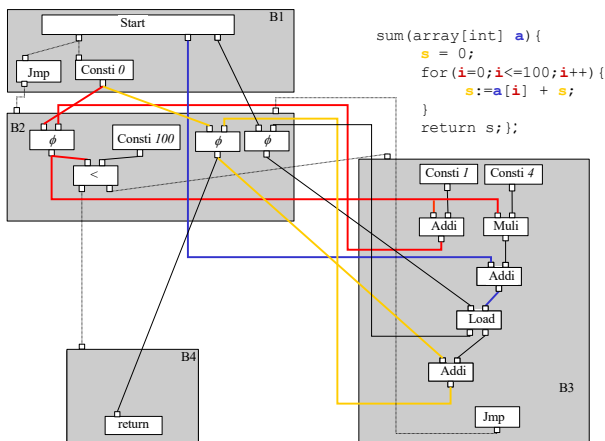
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Observations

- Order of optimizations matters in theory:
 - Application of one optimization might destroy precondition of another
 - Optimization can ruin the effects of the previous once
- Optimal order unclear (in scientific papers usual statements like: "Assume my optimization is the last ...")
- Simultaneous optimization too complex
- Usually, first optimization gives 15% sum of remaining 5%, independent of the chosen optimizations
- Might differ in certain application domains, e.g., in numerical applications operator simplification gives factor >2, cache optimization factor 2-5

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Optimizations on Memory

- Elimination of memory accesses.
- Elimination of object creations.
- Elimination nonessential dependencies.
- Those are **normalizing** transformations for further **analyses**
- Nothing new under the sun:
 - Define abstract values, addresses, memory
 - Define context-insensitive transfer functions for memory relevant SSA nodes (Load, Store, Call) (discussed already)
 - Generalization to context-sensitive analyses (discussed already)
 - Optimizations as graph transformations (discussed already)

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Further Optimizations

- Constant evaluation (simple transformation rule)
- Constant propagation (iterative application of that rule)
- Copy propagation (on SSA construction)
- Dead code elimination (on SSA construction)
- Common subexpression elimination (on SSA construction)
- Specialization of basic blocks, procedures, i.e. cloning
- Procedure inlining
- Control flow simplifications
- Loop transformations (Splitting/merging/unrolling)
- Bound check eliminations
- Cache optimizations (array access, object layout)
- Parallelization
- ...

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Outline

- Introduction to SSA
- Construction, Destruction
- How to capture analysis results
- Optimizations
 - Classic analyses and optimizations on SSA representations
 - Heap analyses and optimizations

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Memory Values

- Differentiation by **Name Schema**
- Distinguish e.g.:
 - local arrays with different name
 - disjoint index sets in an array (odd/even etc.)
 - different types of heap objects
 - objects with same type but statically different creation program point
 - objects with same creation program point but with statically different path to that creation program point (execution context, context-sensitive)

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Abstract values, addresses, memory

- References and values
 - allocation site lattice φ^0 abstracts from objects 0
 - arbitrary lattice X abstracts from values like `Integer` or `Boolean`
 - abstract heap memory M :
 - $0 \times F \rightarrow \varphi^0$ (F set of fields with reference object semantics)
 - $0 \times V \rightarrow X$ (V set of fields with value semantics)
- Arrays
 - Treated as objects
 - Abstract heap memory M :
 - $0 \times F[] \times I \rightarrow 0$ ($F[]$ set of fields with type array of reference object)
 - $0 \times V[] \times I \rightarrow X$ ($V[]$ set of fields with type array of value)
 - I an arbitrary integer value lattice (e.g., constant or interval lattice)
- Abstract address $Addr \subseteq \varphi^0 \times F \times I$ (F set of field names)
 - object-field-(index) triples where index might be ignored

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

Updates of Memory

- Given an abstract object-field of a store operation
- In general, this abstract object points to more than one real memory cell
- A store operation overwrites only one of these cells, all others contain the same value
- Hence, a store to an abstract object-field **adds** a new possible (abstract) value - **weak update**
- Only if guaranteed that abstract object-field-(index) matches one and only one concrete address, a new (abstract) value **overwrites** the old value - **strong update**
- Auxiliary function:
 - $update(mem, o, f, v) = v$ (if strong update possible)
 - $= mem(o, f) \cup v$ (otherwise, weak update)

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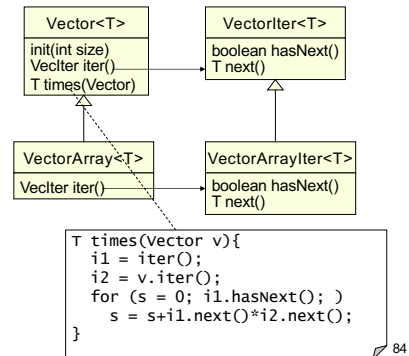
Transfer functions (insensitive, no arrays)

- $T_{store,f}(mem, addr, v)$:
 $(update(mem, o_1, f, v) \dots update(mem, o_k, f, v))$
 $addr = \{o_1 \dots o_k\}$

- $T_{load,f}(mem, addr)$:
 $(mem, mem(o_1) \cup \dots \cup mem(o_k))$
 $addr = \{o_1 \dots o_k\}$

- $T_{alloc,type,id}(mem)$:
 $(mem(o_{id}, f_i) \mapsto \perp, \dots, mem(o_{id}, f_k) \mapsto \perp), \{o_{id}\}$
 $\{f_1 \dots f_k\}$ fields of $Type$


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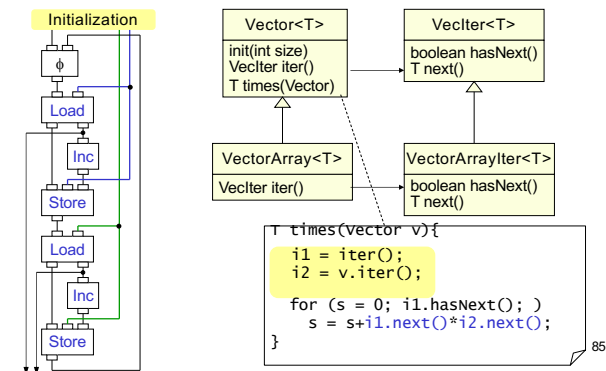
Example: Main Loop Inner Product Algorithm



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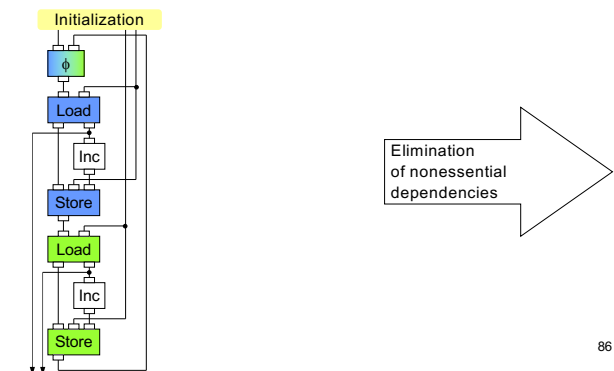
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Example: SSA



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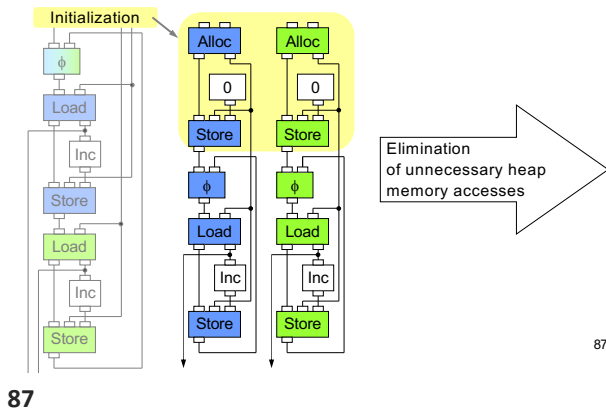
Actually, two Iterators



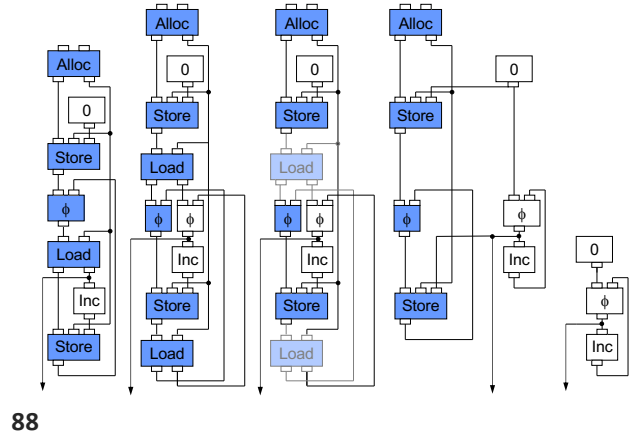
86

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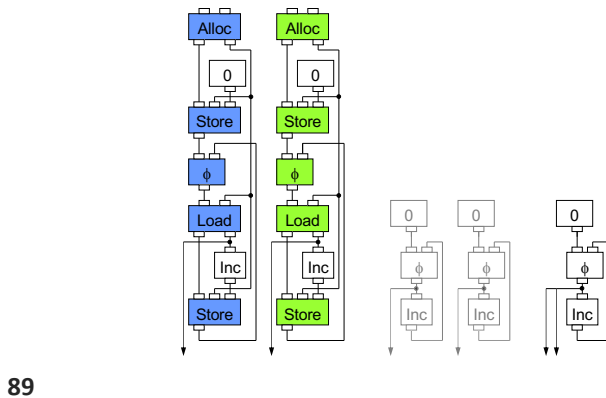
Initialization: disjoint memory guaranteed



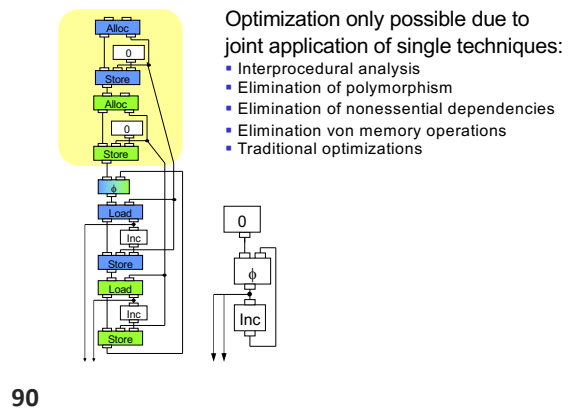
Memory objects replaced by values



Value numbering proves equivalence



Example revisited



- Optimization only possible due to joint application of single techniques:
- Interprocedural analysis
 - Elimination of polymorphism
 - Elimination of nonessential dependencies
 - Elimination von memory operations
 - Traditional optimizations

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