Current Status of Homotopy-based Initialization in OpenModelica

10^{th} OpenModelica Annual Workshop

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Contents

Homotopy Method

Why homotopy?

- Some nonlinear algebraic systems are hard to solve with standard methods
- Especially during initialization:
  - Large systems with lots of unknowns (algebraic variables, discrete variables, states, ...)
  - Good start values are needed for convergence

Homotopy Method

Why homotopy?

- Some nonlinear algebraic systems are hard to solve with standard methods
- Especially during initialization:
  - Large systems with lots of unknowns (algebraic variables, discrete variables, states, ...)
  - Good start values are needed for convergence

Idea of Homotopy

- Starting from solving a simplified problem the problem is transformed to the original one successively

→ Continuation method

→ Convergence is possible even without accurate initial guess values

Homotopy Method

General Approach

Nonlinear Problem

Solve non-linear equation system

$$\underline{F}(\underline{x}^*) = \underline{0}$$

with given start vector $\underline{x}_0 \in \mathbb{R}^n$.

Simple Example

$$f(x) = 2x - 4 + sin(2\pi x),$$

$$x_0 = 0.5, \quad x^* = 2.$$

Possible Homotopy Functions

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Nonlinear Problem

Solve non-linear equation system

$$\underline{F}(\underline{x}^*) = \underline{0}$$

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Possible Homotopy Functions

- Fixpoint-Homotopy:

$$\underline{H}(\underline{x}, \lambda) = \lambda \underline{F}(\underline{x}) + (1 - \lambda)(\underline{x} - \underline{x}_0) = \underline{0}$$

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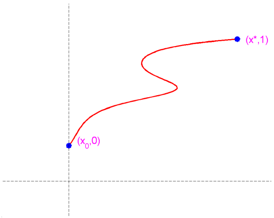
Homotopy Path (Fixpoint)



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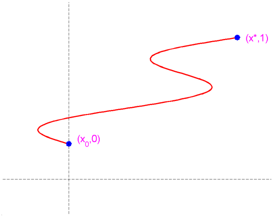

Homotopy Method

General Approach

- Modelica offers a homotopy operator for the homotopy-based initialization[1]

  → The homotopy function is defined by the user

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- Homotopy operator for initialization:

$$homotopy(actual = actual, simplified = simplified) = \lambda actual + (1-\lambda) simplified$$

[1] Modelica Language Specification 3.4, p. 23

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Homotopy Iteration

Start with $(\underline{x}_0, \underbrace{\lambda_0}_{=0})$ and $\underline{H}(\underline{x}_0, \lambda_0) = \underline{0}$.

Determine $(\underline{x}_{i+1}, \lambda_{i+1})$ with $\underline{H}(\underline{x}_{i+1}, \lambda_{i+1}) = \underline{0}$.

Stop, when $\lambda_m = 1$ yields.

$$\Rightarrow \underline{H}(\underline{x}_m, \underbrace{\lambda_m}_{=1}) = \underline{F}(\underline{x}_m) = \underline{0} \Rightarrow \underline{x}^* = \underline{x}_m.$$

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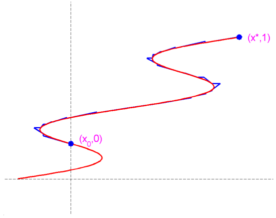

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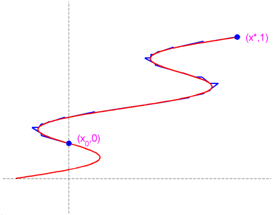

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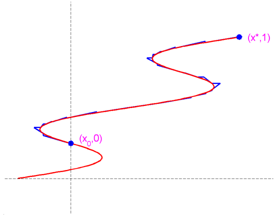

Homotopy Approaches in OM

(OMCompiler v1.13.0-dev.305+g3f877ad)

- Several different homotopy approaches are available in OpenModelica:
  - Homotopy solver as nonlinear system solver
    - Introduces the homotopy function by itself (fixpoint, newton)
    - Default fallback method if newton algorithm fails
  - Homotopy approaches for solving the initial systems
    - Usage of homotopy operator
    - Local/global homotopy with equidistant λ-steps
    - Local/global homotopy with the use of the homotopy solver (adaptive step size)

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- Each nonlinear system with homotopy operator is solved separately with its own homotopy iteration

Modelica Model

```
model homotopy4 "testsuite/simulation/modelica/initialization/homotopy4.mos"
  Real x,y,z,x1,y1,z1,x2,y2,z2,x3,y3,z3,a,b,c,d,e,f;
  parameter Real p1 = 10;
equation
  a = sin(p1* time + 3.1415/2);

  x + 5 = z + a;
  -y + 10 = z + p1;
  x - y + 9 = z;

  b = sin(a*x);

  // (Beginning of homotopy iteration loop)
  c = homotopy(b^2, b);
  // (End of homotopy iteration loop)

  x1 + 5 = z1 + c;
  -y1 + 10 = z1 + p1;
  x1 - y1 + 9 = z1;

  d = x1 + y1 + z1;

  // Beginning of homotopy iteration loop
  homotopy(x2^2, x2) + 5 = z2 + 5;
  homotopy(y2^2, -d) + 10 = z2 + p1;
  x2 + y2 + 9 = z2;
  // End of homotopy iteration loop

  e = sin(d*x2);

  x3 + 5 = z3 + e;
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  f = x + y + z + a + b + c + d + e + x1 + y1 + z1 + x2 + y2 + z2
         + x3 + y3 + z3;
end homotopy4;
```

Homotopy Approaches in OM

Local homotopy

- Each nonlinear system with homotopy operator is solved separately with its own homotopy iteration

- Faster alternative, because homotopy is only performed for components which are directly affected

- Previous homotopy iterations do not affect later systems (problematic if homotopy operator is used on a variable that is an input to the system)

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Modelica Spec. 3.4, p. 26

It is recommended to perform (conceptually) one homotopy iteration over the whole model, and not several homotopy iterations over the respective non-linear algebraic equation systems. The reason is that the following structure can be present:

$$\underline{w} = \underline{f_1}(\underline{x}) \quad \text{// has homotopy operator}$$

$$\underline{0} = \underline{f_2}(\underline{\dot{x}}, \underline{x}, \underline{z}, \underline{w})$$

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- The above structure can be handled with homotopy

- The homotopy iteration loop can become quite large

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Homotopy Approaches in OM

Homotopy with equidistant step size

- Homotopy with equidistant step size is available for both the local and global version

OM Compiler Flags

```
--homotopyApproach=equidistantLocal
--homotopyApproach=equidistantGlobal
```

- Homotopy with equidistant step size is available for both the local and global version
- The user defines a number of homotopy steps to be performed
  → λ is increased equidistantly
  (e.g. 3 steps ⇒ $\lambda = 0, 0.5, 1$)

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OM Runtime Flags

```
-ils=<value>  (default: 4)
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Homotopy Approaches in OM

Homotopy with equidistant step size

- Homotopy with equidistant step size is available for both the local and global version
- The user defines a number of homotopy steps to be performed
  $\rightarrow$ λ is increased equidistantly
  (e.g. 3 steps $\Rightarrow \lambda = 0, 0.5, 1$)
- No sensitive homotopy parameters that influence the convergence
- λ can only be increased and does not react to changes in the properties of the homotopy path
  $\rightarrow$ Homotopy path is not followed correctly and can be lost easily

OM Compiler Flags

```
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OM Runtime Flags

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Backend preparation for the equidistant global approach:

- Creation and optimization of a separate λ_0-DAE
  - Duplicate DAE and internally set `--initOptModules+=replaceHomotopyWithSimplified`

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- Creation and optimization of a separate λ_0-DAE
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Default

The **equidistant global** approach is currently the default homotopy method for initialization. It is activated as **fallback method** if the normal initialization fails.

OM Runtime Flags

`-homotopyOnFirstTry`

to directly use the homotopy method to solve the initialization problem.

Homotopy Approaches in OM

Homotopy with adaptive step size

- Homotopy with adaptive step size is available for both the local and global version

OM Compiler Flags

```
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Homotopy Approaches in OM

Homotopy with adaptive step size

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- Use of homotopy solver with predictor-corrector steps
  - → λ is increased/decreased according to the properties of the path

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OM Runtime Flags

... *will be introduced on the following slides* ...

- Homotopy with adaptive step size is available for both the local and global version

- Use of homotopy solver with predictor-corrector steps
  - → λ is increased/decreased according to the properties of the path

- Homotopy parameter λ is calculated by the homotopy algorithm to follow the homotopy path

- Many parameters influence the behaviour of the algorithm (e.g. initial step size, tolerances, ...)

OM Compiler Flags

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OM Runtime Flags

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Backend preparation for the adaptive approaches:

- Inlining of the homotopy operator
  - Internally set `--initOptModules+=inlineHomotopy`
- λ is introduced as a variable in each component with homotopy operator
  - Internally set `--initOptModules+=generateHomotopyComponents`

  $\rightarrow$ system size: $n \times (n+1)$
- Symbolic calculation of the Jacobian of the $n \times (n+1)$ system

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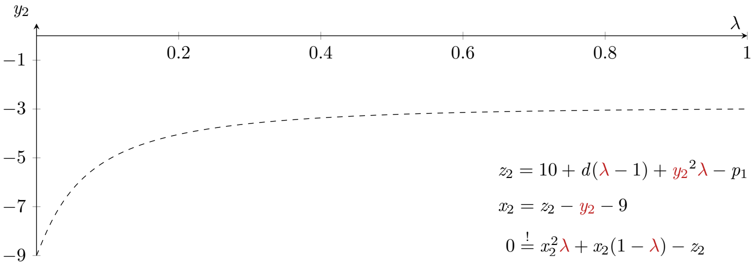
Additionally for the **global** approach:

- Creation and optimization of a separate λ_0-DAE
  - Duplicate DAE and internally set `--initOptModules+=replaceHomotopyWithSimplified`
- Determination of the smallest homotopy iteration loop in the original initialization system
  - Internally set `--initOptModules+=generateHomotopyComponents`
  - Beginning: First BLT block in which a homotopy operator is present
  - End: Last BLT block that describes a non-linear algebraic equation system
  - Creation of one nonlinear system

OM Runtime Flags

Homotopy Path Calculation (omc homotopy4.mos --homotopyApproach=adaptiveLocal)



$$z_2 = 10 + d(\lambda - 1) + {y_2}^2\lambda - p_1$$

$$x_2 = z_2 - y_2 - 9$$

$$0 \overset{!}{=} x_2^2\lambda + x_2(1-\lambda) - z_2$$

Procedure

OM Runtime Flags

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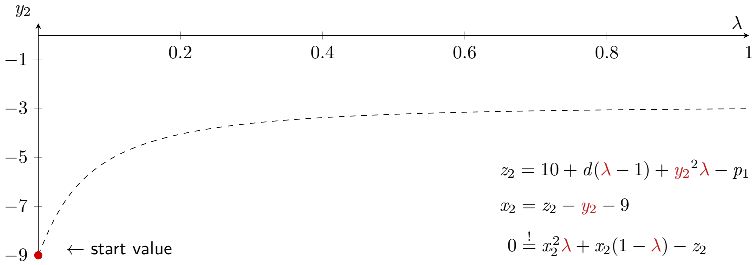

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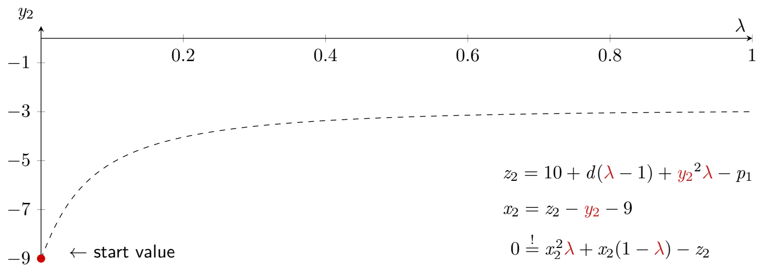

Procedure: Predictor Step

Derive:

$$\underline{H}(\underline{x}(s), \lambda(s)) = \underline{0}$$

Homotopy Approaches in OM

Homotopy with adaptive step size: The Homotopy Algorithm

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OM Runtime Flags

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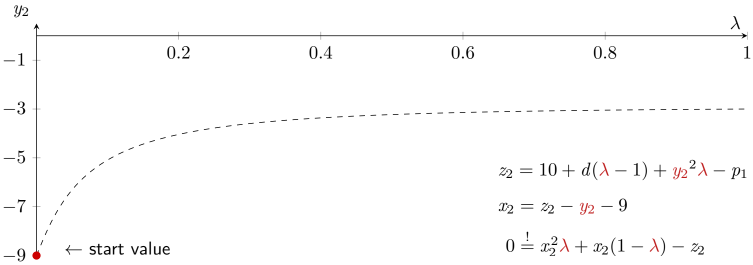

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OM Runtime Flags

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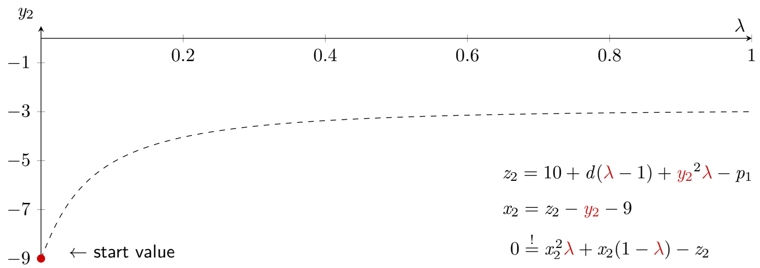

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Solve linear system:

$$J_{\underline{H}}(\underline{x}_i, \lambda_i) \cdot \underline{v}_i = \underline{0}$$

$J_{\underline{H}} \in \mathbb{R}^{(n,n+1)}$, Jacobian

$$0 \overset{!}{\leq} (\underline{v}^i)^T \cdot \underline{v}^{i-1}$$

OM Runtime Flags

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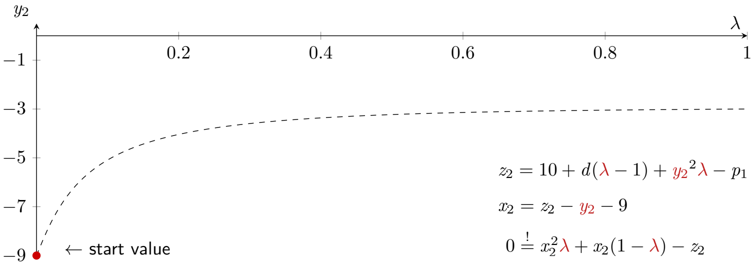

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Perform predictor step:

$$\begin{pmatrix} \underline{x}^\#_{i+1} \\ \lambda^\#_{i+1} \end{pmatrix} = \begin{pmatrix} \underline{x}_i \\ \lambda_i \end{pmatrix} + \tau_i \cdot \underline{v}_i,$$

τ_i step size, $\underline{v}_i$ normalized direction.

OM Runtime Flags

`-homTauStart=<value>`

to define the initial step size τ_0.

Default: 0.2

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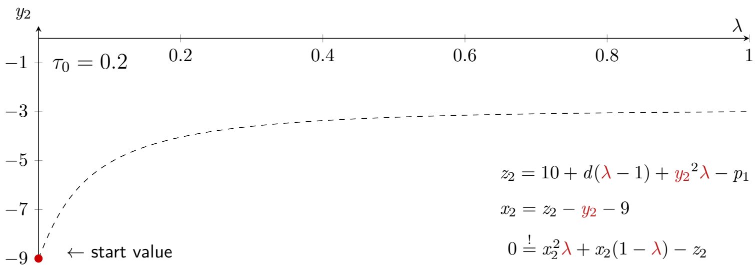

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$$0 \overset{!}{\leq} (\underline{v}^i)^T \cdot \underline{v}^{i-1}$$

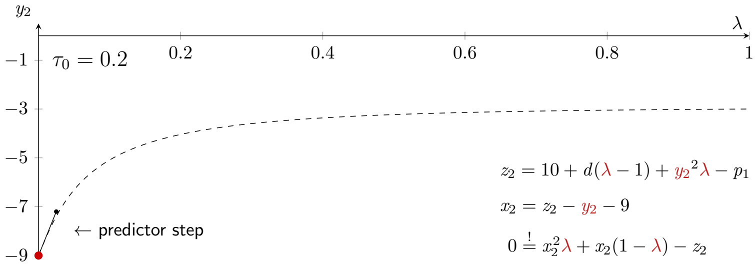
Perform predictor step:

$$\begin{pmatrix} \underline{x}_{i+1}^{\#} \\ \lambda_{i+1}^{\#} \end{pmatrix} = \begin{pmatrix} \underline{x}_i \\ \lambda_i \end{pmatrix} + \tau_i \cdot \underline{v}_i,$$

τ_i step size, $\underline{v}_i$ normalized direction.

OM Runtime Flags

Homotopy Path Calculation (omc homotopy4.mos --homotopyApproach=adaptiveLocal)



Procedure: Predictor Step

Derive:

$$\underline{H}(\underline{x}(s), \lambda(s)) = \underline{0}$$
$$\Rightarrow \frac{\partial \underline{H}}{\partial \underline{x}} \cdot \underline{x}'(s) + \frac{\partial \underline{H}}{\partial \lambda} \cdot \lambda'(s) = \underline{0}$$

Solve linear system:

$$J_{\underline{H}}(\underline{x}_i, \lambda_i) \cdot \underline{v}_i = \underline{0}$$

$J_{\underline{H}} \in \mathbb{R}^{(n,n+1)}$, Jacobian

$$0 \overset{!}{\leq} (\underline{v}^i)^T \cdot \underline{v}^{i-1}$$

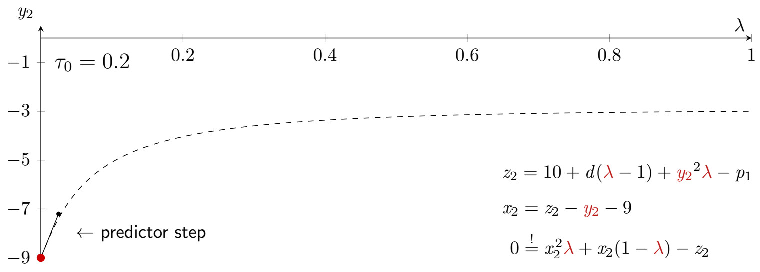
Perform predictor step:

$$\begin{pmatrix} \underline{x}_{i+1}^{\#} \\ \lambda_{i+1}^{\#} \end{pmatrix} = \begin{pmatrix} \underline{x}_i \\ \lambda_i \end{pmatrix} + \tau_i \cdot \underline{v}_i,$$

τ_i step size, $\underline{v}_i$ normalized direction.

OM Runtime Flags

Homotopy Path Calculation (omc homotopy4.mos --homotopyApproach=adaptiveLocal)



Procedure: Predictor Step

- Assert?
  - If $\underline{H}(\underline{x}^{\#}_{i+1}, \lambda^{\#}_{i+1})$ cannot be calculated, τ is decreased by a factor: $\tau := \frac{\tau}{homTauDecFacPredictor}$

→ No assert in the example

Perform predictor step:

$$\begin{pmatrix} \underline{x}^{\#}_{i+1} \\ \lambda^{\#}_{i+1} \end{pmatrix} = \begin{pmatrix} \underline{x}_i \\ \lambda_i \end{pmatrix} + \tau_i \cdot \underline{v}_i,$$

τ_i step size, $\underline{v}_i$ normalized direction.

Homotopy Approaches in OM

Homotopy with adaptive step size: The Homotopy Algorithm

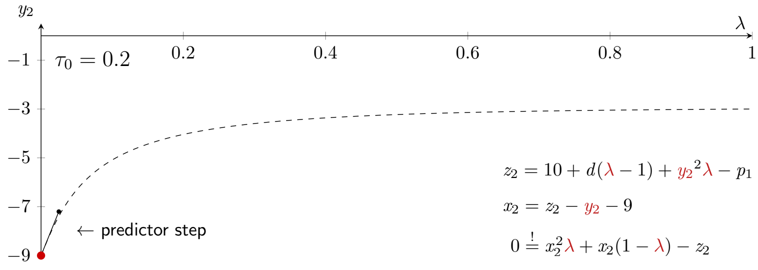
OM Runtime Flags

-homTauDecFacPredictor
=<value>

to define the factor by which τ is decreased if the predictor step fails.

Default: 2.0

Homotopy Path Calculation (omc homotopy4.mos --homotopyApproach=adaptiveLocal)



Procedure: Predictor Step

- Assert?
  - If $\underline{H}(\underline{x}^{\#}_{i+1}, \lambda^{\#}_{i+1})$ cannot be calculated, τ is decreased by a factor:
    $\tau := \frac{\tau}{homTauDecFacPredictor}$

→ No assert in the example

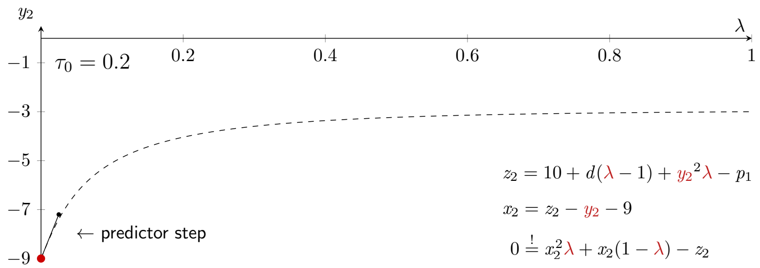
Perform predictor step:

$$\begin{pmatrix} \underline{x}^{\#}_{i+1} \\ \lambda^{\#}_{i+1} \end{pmatrix} = \begin{pmatrix} \underline{x}_i \\ \lambda_i \end{pmatrix} + \tau_i \cdot \underline{v}_i,$$

τ_i step size, $\underline{v}_i$ normalized direction.

OM Runtime Flags

Homotopy Path Calculation (omc homotopy4.mos --homotopyApproach=adaptiveLocal)



Procedure: Corrector Step

<u>Newton</u>:

Get back to the homotopy path by Newton iteration steps with start values $(\underline{x}^{\#}_{i+1}, \lambda^{\#}_{i+1})$ until

$$\underline{H}(\underline{x}_{i+1}, \lambda_{i+1}) \approx \underline{0}.$$

Homotopy Approaches in OM

Homotopy with adaptive step size: The Homotopy Algorithm

OM Runtime Flags

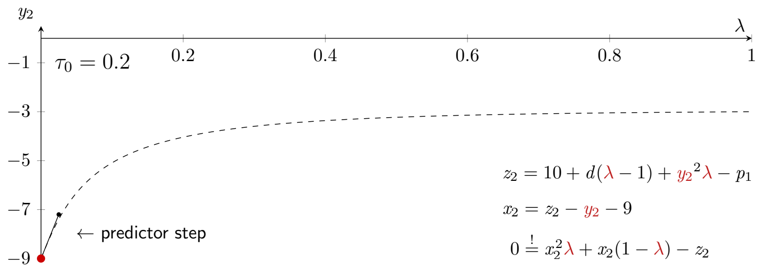
`-homHEps=<value>`

to define the tolerance for the residuals in the Newton corrector step.

Default: 1e-5

(In the last step ($\lambda = 1$) the global accuracy is used.)

Homotopy Path Calculation (omc homotopy4.mos --homotopyApproach=adaptiveLocal)



Procedure: Corrector Step

Newton:
Get back to the homotopy path by Newton iteration steps with start values $(\underline{x}^{\#}_{i+1}, \lambda^{\#}_{i+1})$ until

$$\underline{H}(\underline{x}_{i+1}, \lambda_{i+1}) \approx \underline{0}.$$

Backtracing Strategies:

- Fix one coordinate:

$$x^{j}_{i+1} - x^{\#,j}_{i+1} = 0$$

- Orthogonal to tangent vector:

$$(\underline{x}_{i+1} - \underline{x}^{\#}_{i+1})^T \cdot \underline{v}_i = 0$$

Homotopy Approaches in OM

Homotopy with adaptive step size: The Homotopy Algorithm

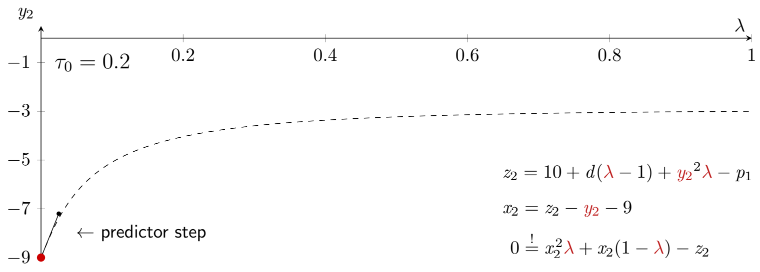
OM Runtime Flags

```
-homMaxNewtonSteps
=<value>
```

to define the maximum number of iterations in the Newton corrector step.

Default: 20

Homotopy Path Calculation (omc homotopy4.mos --homotopyApproach=adaptiveLocal)



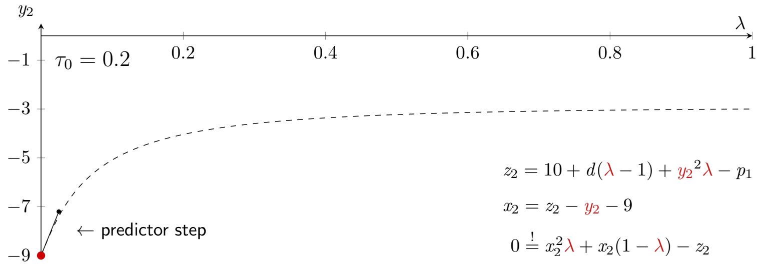
Procedure: Corrector Step

<u>Newton</u>:
Get back to the homotopy path by Newton iteration steps with start values $(\underline{x}^{\#}_{i+1}, \lambda^{\#}_{i+1})$ until

$$\underline{H}(\underline{x}_{i+1}, \lambda_{i+1}) \approx \underline{0}.$$

OM Runtime Flags

Homotopy Path Calculation (omc homotopy4.mos --homotopyApproach=adaptiveLocal)



Procedure: Corrector Step

Newton:

Get back to the homotopy path by Newton iteration steps with start values $(\underline{x}^{\#}_{i+1}, \lambda^{\#}_{i+1})$ until

$$\underline{H}(\underline{x}_{i+1}, \lambda_{i+1}) \approx \underline{0}.$$

Backtracing Strategies:

1. Fix one coordinate:

$$x^{j_i}_{i+1} - x^{\#j_i}_{i+1} = 0$$

2. Orthogonal to tangent vector:

$$\left(\underline{x}_{i+1} - \underline{x}^{\#}_{i+1}\right)^T \cdot \underline{v}_i = 0$$

Homotopy Approaches in OM

Homotopy with adaptive step size: The Homotopy Algorithm

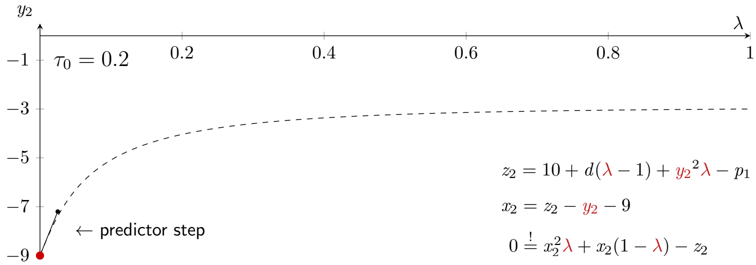
OM Runtime Flags

`-homBacktraceStrategy =<fix/orthogonal>`

to define the backtracing strategy in the Newton corrector step.

Default: fix

Homotopy Path Calculation (omc homotopy4.mos --homotopyApproach=adaptiveLocal)



$$z_2 = 10 + d(\lambda - 1) + {y_2}^2\lambda - p_1$$

$$x_2 = z_2 - y_2 - 9$$

$$0 \stackrel{!}{=} x_2^2\lambda + x_2(1-\lambda) - z_2$$

Procedure: Corrector Step

Newton:
Get back to the homotopy path by Newton iteration steps with start values $(\underline{x}^{\#}_{i+1}, \lambda^{\#}_{i+1})$ until

$$\underline{H}(\underline{x}_{i+1}, \lambda_{i+1}) \approx \underline{0}.$$

Backtracing Strategies:

1. Fix one coordinate:

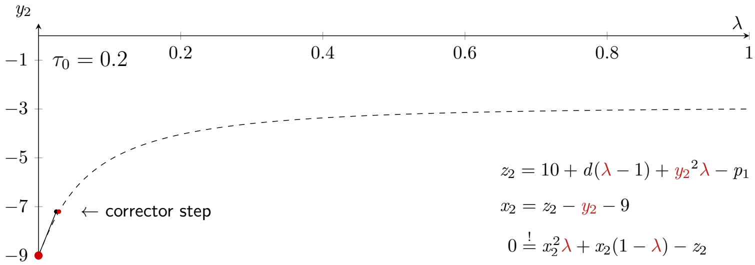
$$x^{j_i}_{i+1} - x^{\# j_i}_{i+1} = 0$$

2. Orthogonal to tangent vector:

$$(\underline{x}_{i+1} - \underline{x}^{\#}_{i+1})^T \cdot \underline{v}_i = 0$$

OM Runtime Flags

Homotopy Path Calculation (omc homotopy4.mos --homotopyApproach=adaptiveLocal)



Procedure: Corrector Step

Newton:

Get back to the homotopy path by Newton iteration steps with start values $(\underline{x}^{\#}_{i+1}, \lambda^{\#}_{i+1})$ until

$$\underline{H}(\underline{x}_{i+1}, \lambda_{i+1}) \approx \underline{0}.$$

Backtracing Strategies:

1. Fix one coordinate:

$$x^{j_i}_{i+1} - x^{\# j_i}_{i+1} = 0$$

2. Orthogonal to tangent vector:

$$\left(\underline{x}_{i+1} - \underline{x}^{\#}_{i+1}\right)^T \cdot \underline{v}_i = 0$$

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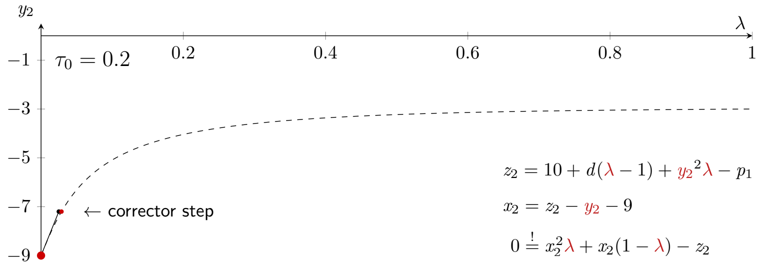
OM Runtime Flags

Homotopy Path Calculation (omc homotopy4.mos --homotopyApproach=adaptiveLocal)



$$z_2 = 10 + d(\lambda - 1) + {y_2}^2\lambda - p_1$$

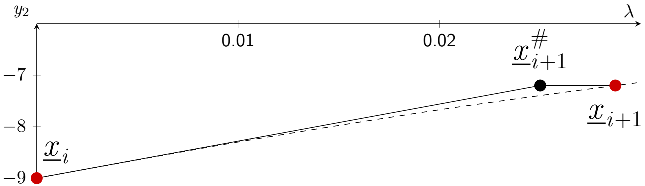
$$x_2 = z_2 - y_2 - 9$$

$$0 \stackrel{!}{=} x_2^2\lambda + x_2(1-\lambda) - z_2$$

Procedure: Check Step Acceptance and Adapt Step Size

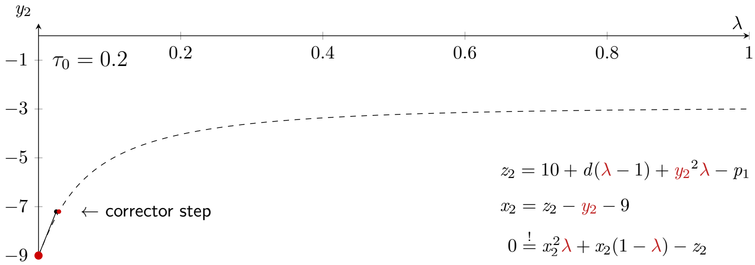
Calculate the bending:

$$b = \frac{\|\underline{x}_{i+1} - \underline{x}^{\#}_{i+1}\|}{\|\underline{x}^{\#}_{i+1} - \underline{x}_i\|}$$



OM Runtime Flags

Homotopy Path Calculation (omc homotopy4.mos --homotopyApproach=adaptiveLocal)



$$z_2 = 10 + d(\lambda - 1) + {y_2}^2\lambda - p_1$$

$$x_2 = z_2 - y_2 - 9$$

$$0 \overset{!}{=} x_2^2\lambda + x_2(1-\lambda) - z_2$$

Procedure: Check Step Acceptance and Adapt Step Size

Calculate the bending:

$$b = \frac{\|x_{i+1} - x_{i+1}^{\#}\|}{\|x_{i+1}^{\#} - x_i\|}$$

$b > homAdaptBend$

→ Reject step!

→ $\tau := \frac{\tau}{homTauDecFac}$

→ Perform predictor and corrector step again with new τ

$b < homAdaptBend$

→ Accept step!

→ $b < \frac{homAdaptBend}{homTauIncThreshold}$

→ $\tau := \tau \cdot homTauIncFac$

Homotopy Approaches in OM

Homotopy with adaptive step size: The Homotopy Algorithm

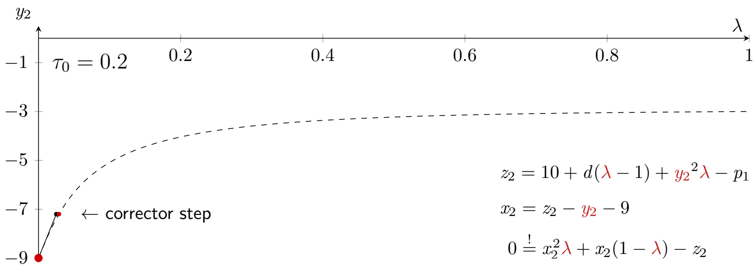
OM Runtime Flags

`-homAdaptBend=<value>`

to define the maximum trajectory bending to accept the homotopy step.

Default: 0.5

Homotopy Path Calculation (omc homotopy4.mos --homotopyApproach=adaptiveLocal)



Procedure: Check Step Acceptance and Adapt Step Size

Calculate the bending:

$$b = \frac{\|x_{i+1} - x^{\#}_{i+1}\|}{\|x^{\#}_{i+1} - x_i\|}$$

$b > homAdaptBend$

→ Reject step!

→ $\tau := \frac{\tau}{homTauDecFac}$

→ Perform predictor and corrector step again with new τ

$b < homAdaptBend$

→ Accept step!

→ $b < \frac{homAdaptBend}{homTauIncThreshold}$

→ $\tau := \tau \cdot homTauIncFac$

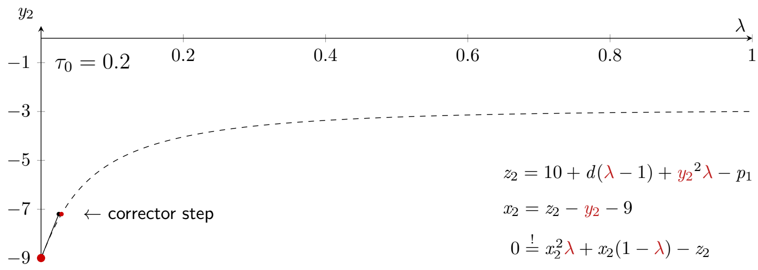
OM Runtime Flags

`-homTauDecFac=<value>`

to define the factor by which τ is decreased if the bending is too high.

Default: 10.0

Homotopy Path Calculation (omc homotopy4.mos --homotopyApproach=adaptiveLocal)



Procedure: Check Step Acceptance and Adapt Step Size

Calculate the bending:

$$b = \frac{\|x_{i+1} - x^{\#}_{i+1}\|}{\|x^{\#}_{i+1} - x_i\|}$$

$b > homAdaptBend$

- → Reject step!
- → $\tau := \frac{\tau}{homTauDecFac}$
- → Perform predictor and corrector step again with new τ

$b < homAdaptBend$

- → Accept step!
- → $b < \frac{homAdaptBend}{homTauIncThreshold}$
  - → $\tau := \tau \cdot homTauIncFac$

Homotopy Approaches in OM

Homotopy with adaptive step size: The Homotopy Algorithm

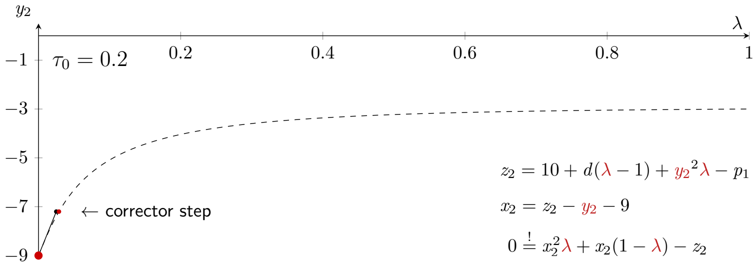
OM Runtime Flags

`-homTauMin=<value>`

to define the minimum step size τ.

Default: 1e-4

Homotopy Path Calculation (omc homotopy4.mos --homotopyApproach=adaptiveLocal)



Procedure: Check Step Acceptance and Adapt Step Size

Calculate the bending:

$$b = \frac{\|x_{i+1} - x^{\#}_{i+1}\|}{\|x^{\#}_{i+1} - x_i\|}$$

$b > homAdaptBend$

- → Reject step!
- → $\tau := \frac{\tau}{homTauDecFac}$
- → Perform predictor and corrector step again with new τ

$b < homAdaptBend$

- → Accept step!
- → $b < \frac{homAdaptBend}{homTauIncThreshold}$
  - → $\tau := \tau \cdot homTauIncFac$

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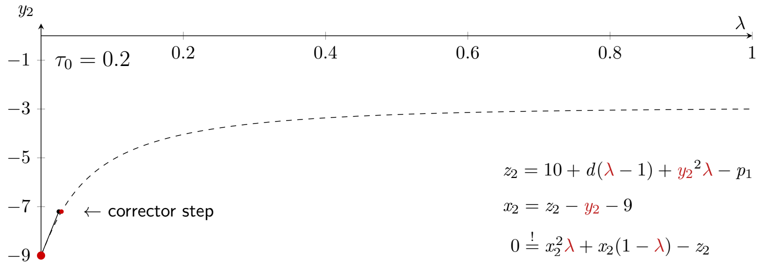
OM Runtime Flags

`-homTauIncThreshold =<value>`

to define a threshold for increasing τ.

Default: 10.0

Homotopy Path Calculation (omc homotopy4.mos --homotopyApproach=adaptiveLocal)



Procedure: Check Step Acceptance and Adapt Step Size

Calculate the bending:

$$b = \frac{\|x_{i+1} - x^{\#}_{i+1}\|}{\|x^{\#}_{i+1} - x_i\|}$$

$b > homAdaptBend$

→ Reject step!

→ $\tau := \frac{\tau}{homTauDecFac}$

→ Perform predictor and corrector step again with new τ

$b < homAdaptBend$

→ Accept step!

→ $b < \frac{homAdaptBend}{homTauIncThreshold}$

→ $\tau := \tau \cdot homTauIncFac$

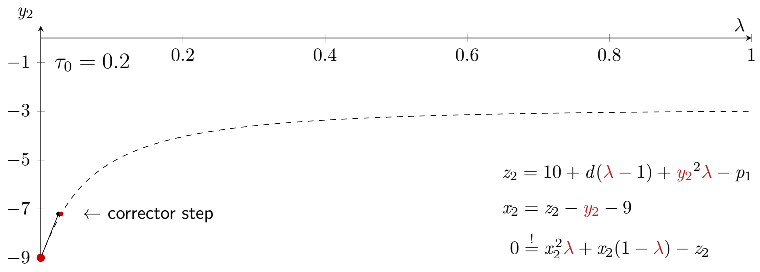
OM Runtime Flags

`-homTauIncFac=<value>`

to define the factor by which τ is increased if the bending is lower than the threshold.

Default: 2.0

Homotopy Path Calculation (omc homotopy4.mos --homotopyApproach=adaptiveLocal)



Procedure: Check Step Acceptance and Adapt Step Size

Calculate the bending:

$$b = \frac{\|x_{i+1} - x^{\#}_{i+1}\|}{\|x^{\#}_{i+1} - x_i\|}$$

$b > homAdaptBend$

→ Reject step!

→ $\tau := \dfrac{\tau}{homTauDecFac}$

→ Perform predictor and corrector step again with new τ

$b < homAdaptBend$

→ Accept step!

→ $b < \dfrac{homAdaptBend}{homTauIncThreshold}$

→ $\tau := \tau \cdot homTauIncFac$

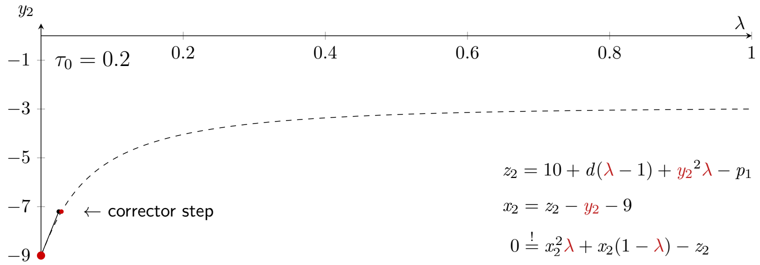
OM Runtime Flags

`-homTauMax=<value>`

to define the maximum step size τ.

Default: 10.0

Homotopy Path Calculation (omc homotopy4.mos --homotopyApproach=adaptiveLocal)



Procedure: Check Step Acceptance and Adapt Step Size

Calculate the bending:

$$b = \frac{\|x_{i+1} - x^{\#}_{i+1}\|}{\|x^{\#}_{i+1} - x_i\|}$$

$b > homAdaptBend$

$\rightarrow$ Reject step!

$\rightarrow \tau := \frac{\tau}{homTauDecFac}$

$\rightarrow$ Perform predictor and corrector step again with new τ

$b < homAdaptBend$

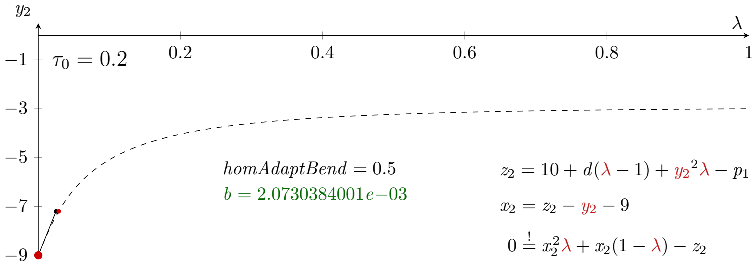
$\rightarrow$ Accept step!

$\rightarrow b < \frac{homAdaptBend}{homTauIncThreshold}$

$\rightarrow \tau := \tau \cdot homTauIncFac$

OM Runtime Flags

Homotopy Path Calculation (omc homotopy4.mos --homotopyApproach=adaptiveLocal)



$$z_2 = 10 + d(\lambda - 1) + {y_2}^2\lambda - p_1$$

$$x_2 = z_2 - y_2 - 9$$

$$0 \overset{!}{=} x_2^2\lambda + x_2(1-\lambda) - z_2$$

Procedure: Check Step Acceptance and Adapt Step Size

Calculate the bending:

$$b = \frac{\|x_{i+1} - x^{\#}_{i+1}\|}{\|x^{\#}_{i+1} - x_i\|}$$

$b > homAdaptBend$

$\rightarrow$ Reject step!

$\rightarrow \tau := \frac{\tau}{homTauDecFac}$

$\rightarrow$ Perform predictor and corrector step again with new τ

$b < homAdaptBend$

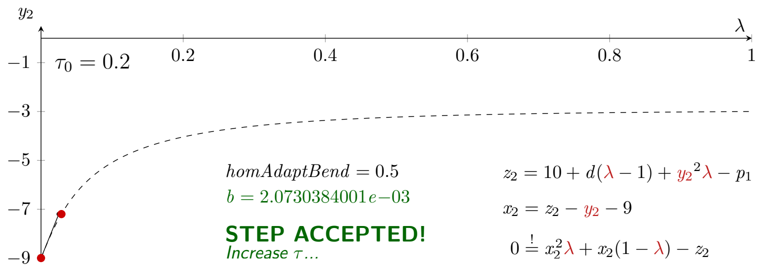
$\rightarrow$ Accept step!

$\rightarrow b < \frac{homAdaptBend}{homTauIncThreshold}$

$\rightarrow \tau := \tau \cdot homTauIncFac$

OM Runtime Flags

Homotopy Path Calculation (omc homotopy4.mos --homotopyApproach=adaptiveLocal)



Procedure: Check Step Acceptance and Adapt Step Size

Calculate the bending:

$$b = \frac{\|\underline{x}_{i+1} - \underline{x}^{\#}_{i+1}\|}{\|\underline{x}^{\#}_{i+1} - \underline{x}_i\|}$$

$b > homAdaptBend$

- $\rightarrow$ Reject step!
- $\rightarrow \tau := \dfrac{\tau}{homTauDecFac}$
- $\rightarrow$ Perform predictor and corrector step again with new τ

$b < homAdaptBend$

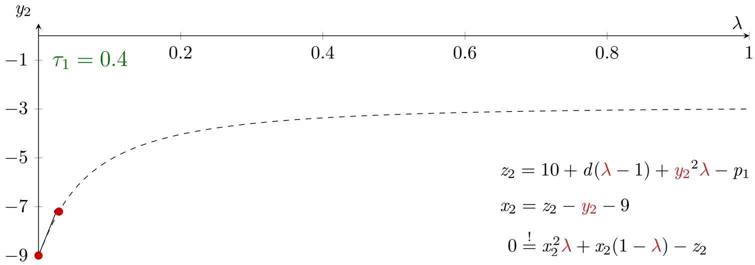
- $\rightarrow$ Accept step!
- $\rightarrow b < \dfrac{homAdaptBend}{homTauIncThreshold}$
  - $\rightarrow \tau := \tau \cdot homTauIncFac$

Homotopy Approaches in OM

Homotopy with adaptive step size: The Homotopy Algorithm

OM Runtime Flags

Homotopy Path Calculation (omc homotopy4.mos --homotopyApproach=adaptiveLocal)



$$z_2 = 10 + d(\lambda - 1) + {y_2}^2\lambda - p_1$$

$$x_2 = z_2 - y_2 - 9$$

$$0 \stackrel{!}{=} x_2^2\lambda + x_2(1-\lambda) - z_2$$

Procedure

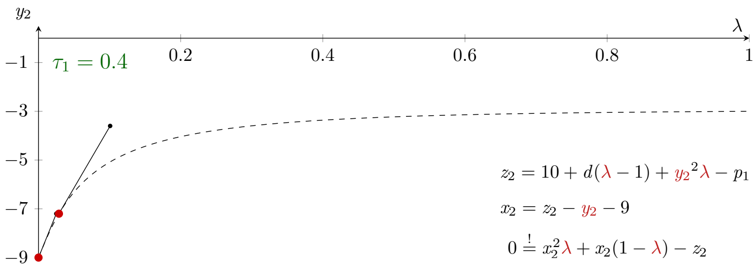
Perform further predictor and corrector steps...

Homotopy Approaches in OM

Homotopy with adaptive step size: The Homotopy Algorithm

OM Runtime Flags

Homotopy Path Calculation (omc homotopy4.mos --homotopyApproach=adaptiveLocal)

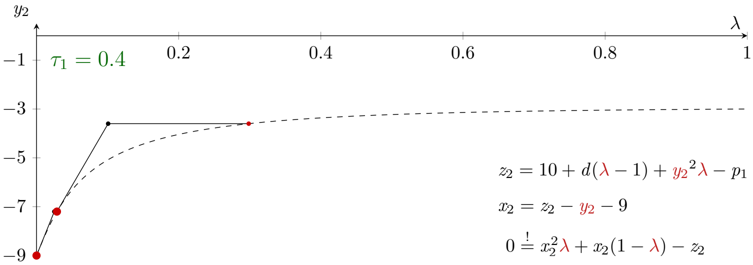


Procedure

Perform further predictor and corrector steps...

OM Runtime Flags

Homotopy Path Calculation (omc homotopy4.mos --homotopyApproach=adaptiveLocal)



$$z_2 = 10 + d(\lambda - 1) + {y_2}^2\lambda - p_1$$

$$x_2 = z_2 - y_2 - 9$$

$$0 \stackrel{!}{=} x_2^2\lambda + x_2(1-\lambda) - z_2$$

Procedure

Perform further predictor and corrector steps...

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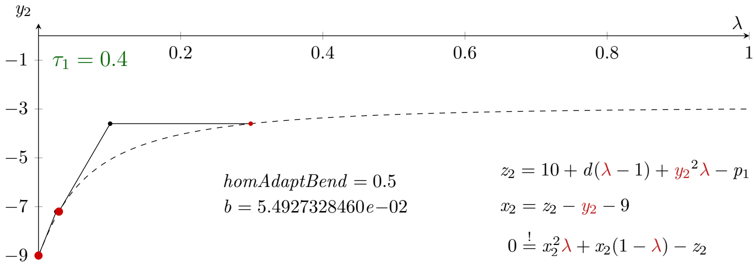
OM Runtime Flags

Homotopy Path Calculation (omc homotopy4.mos --homotopyApproach=adaptiveLocal)

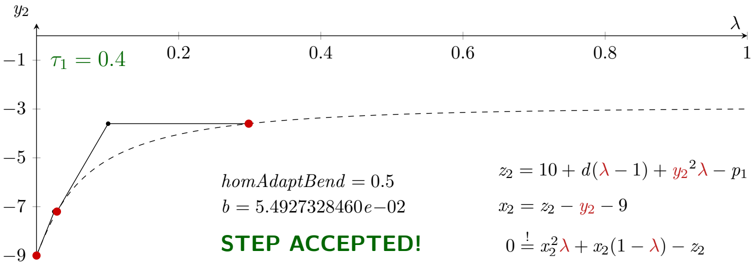


Procedure

Perform further predictor and corrector steps...

OM Runtime Flags

Homotopy Path Calculation (omc homotopy4.mos --homotopyApproach=adaptiveLocal)



$homAdaptBend = 0.5$

$b = 5.4927328460e{-}02$

STEP ACCEPTED!

$$z_2 = 10 + d(\lambda - 1) + {y_2}^2\lambda - p_1$$

$$x_2 = z_2 - y_2 - 9$$

$$0 \stackrel{!}{=} x_2^2\lambda + x_2(1 - \lambda) - z_2$$

Procedure

Perform further predictor and corrector steps...

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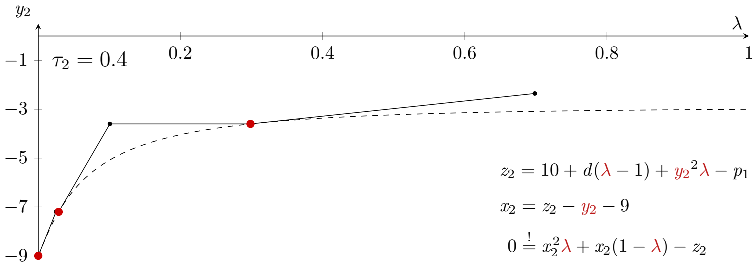
OM Runtime Flags

Homotopy Path Calculation (omc homotopy4.mos --homotopyApproach=adaptiveLocal)



Procedure

Perform further predictor and corrector steps...

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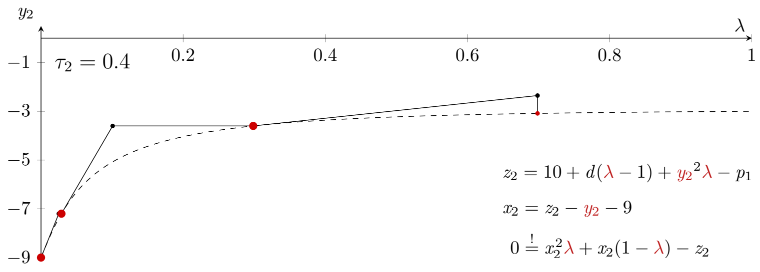
OM Runtime Flags

Homotopy Path Calculation (omc homotopy4.mos --homotopyApproach=adaptiveLocal)



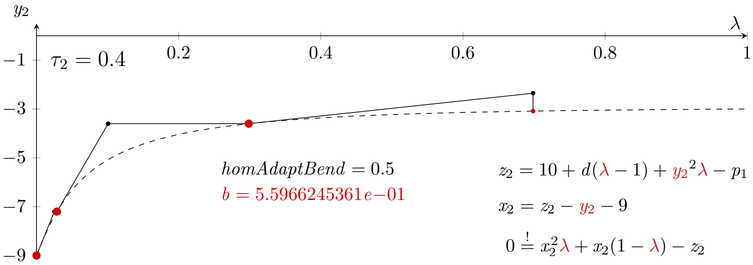
Procedure

Perform further predictor and corrector steps...

FH Bielefeld University of Applied Sciences

OM Runtime Flags

Homotopy Path Calculation (omc homotopy4.mos --homotopyApproach=adaptiveLocal)



$$z_2 = 10 + d(\lambda - 1) + {y_2}^2\lambda - p_1$$

$$x_2 = z_2 - y_2 - 9$$

$$0 \stackrel{!}{=} x_2^2\lambda + x_2(1-\lambda) - z_2$$

Procedure

Perform further predictor and corrector steps...

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OM Runtime Flags

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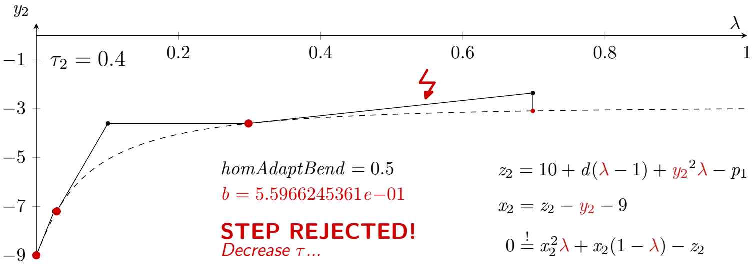

$homAdaptBend = 0.5$

$b = 5.5966245361e{-}01$

STEP REJECTED!

Decrease τ...

$$z_2 = 10 + d(\lambda - 1) + {y_2}^2\lambda - p_1$$

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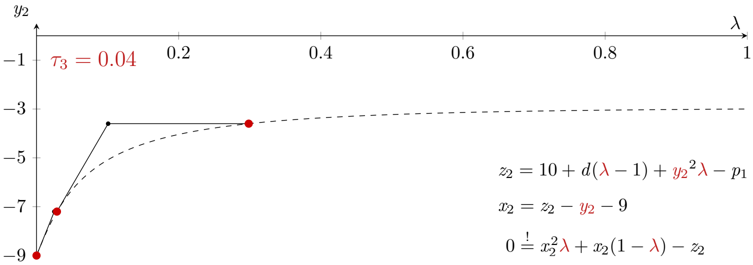

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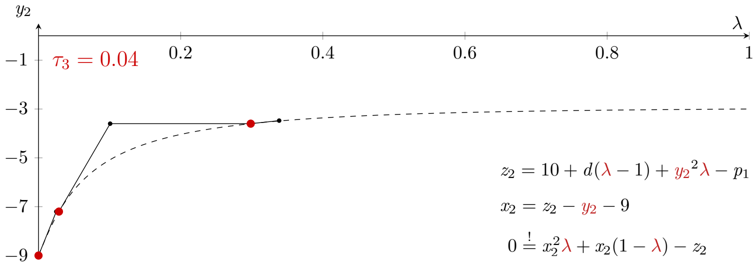

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Homotopy Approaches in OM

Homotopy with adaptive step size: The Homotopy Algorithm

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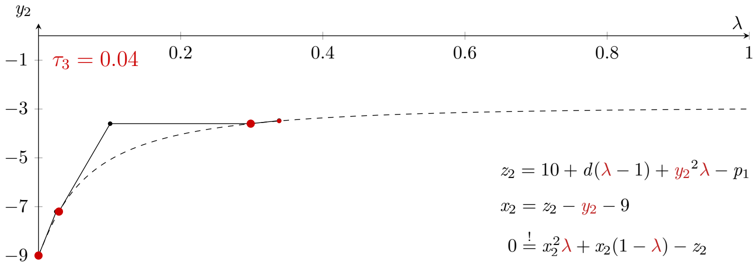

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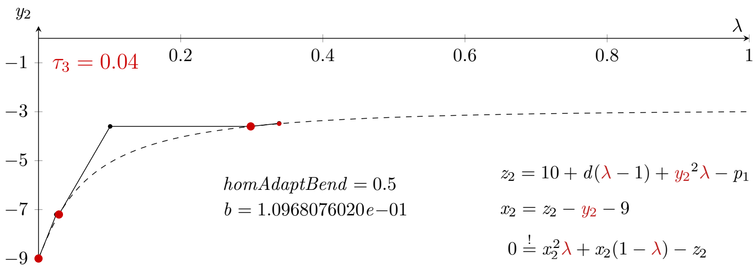

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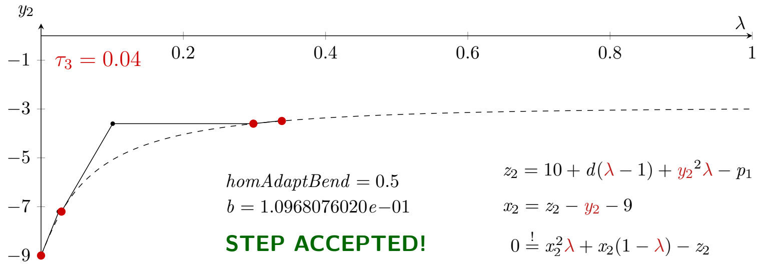

$homAdaptBend = 0.5$

$b = 1.0968076020e{-}01$

STEP ACCEPTED!

$$z_2 = 10 + d(\lambda - 1) + {y_2}^2\lambda - p_1$$

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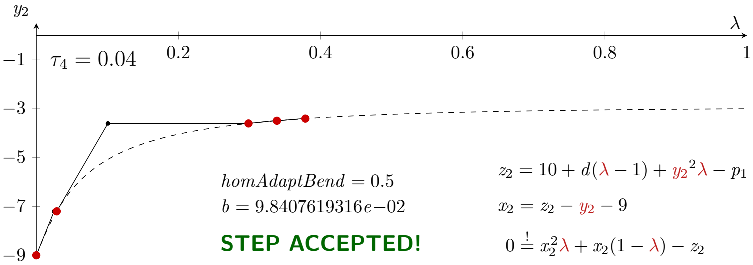

$homAdaptBend = 0.5$

$b = 9.8407619316e{-}02$

STEP ACCEPTED!

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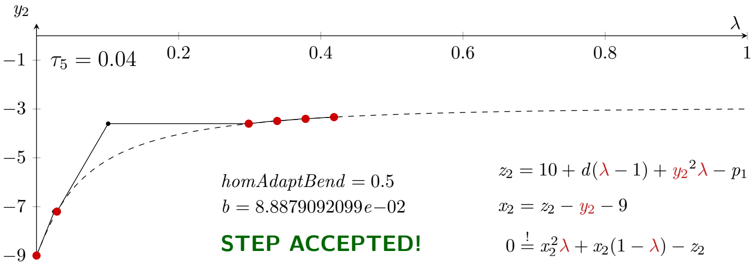

$homAdaptBend = 0.5$

$b = 8.8879092099e{-}02$

STEP ACCEPTED!

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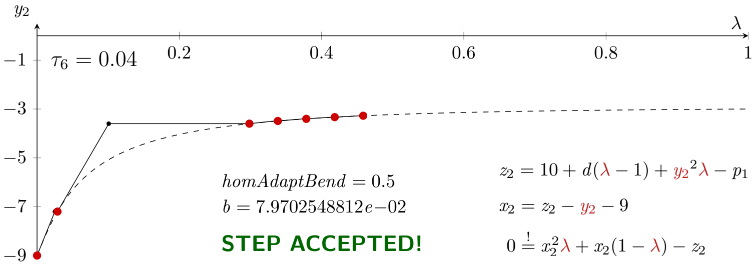

$homAdaptBend = 0.5$

$b = 7.9702548812e{-}02$

STEP ACCEPTED!

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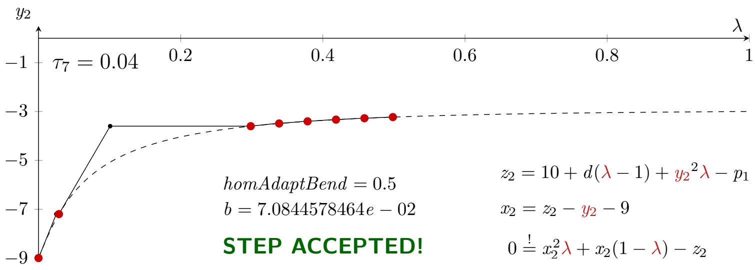

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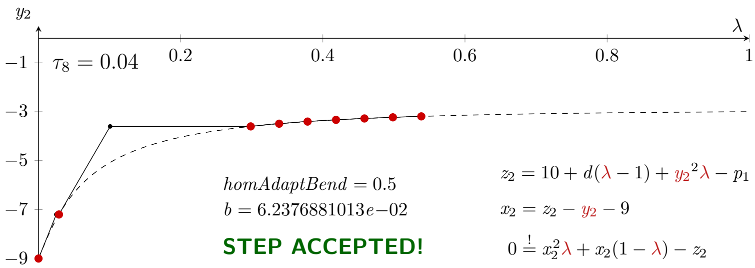

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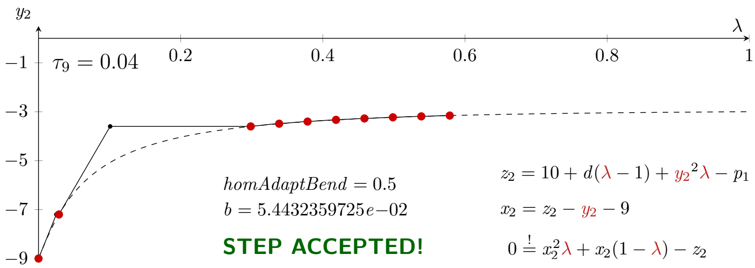

$homAdaptBend = 0.5$

$b = 5.4432359725e{-}02$

STEP ACCEPTED!

$$z_2 = 10 + d(\lambda - 1) + {y_2}^2\lambda - p_1$$

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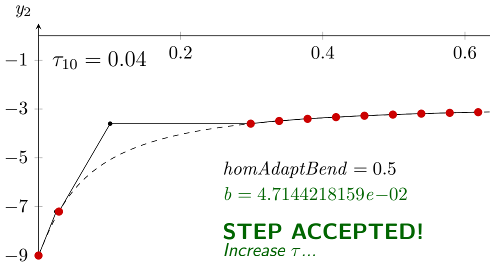

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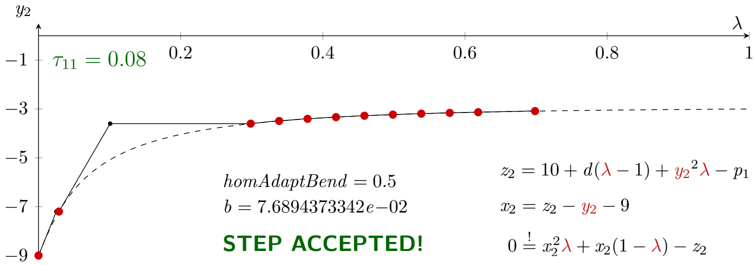

$homAdaptBend = 0.5$

$b = 7.6894373342e{-}02$

STEP ACCEPTED!

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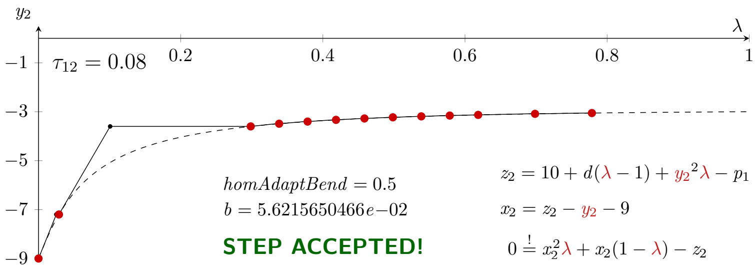

$homAdaptBend = 0.5$

$b = 5.6215650466e{-02}$

STEP ACCEPTED!

$$z_2 = 10 + d(\lambda - 1) + {y_2}^2\lambda - p_1$$

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Procedure

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OM Runtime Flags

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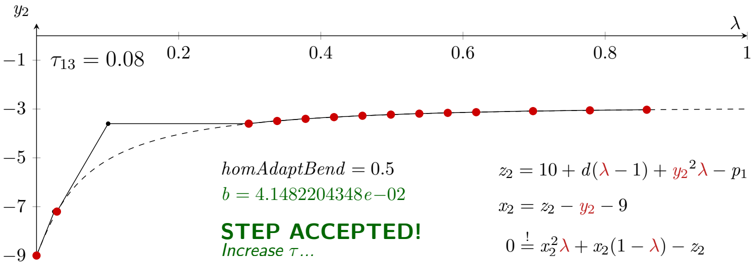

$homAdaptBend = 0.5$

$b = 4.1482204348e{-}02$

STEP ACCEPTED!

Increase τ...

$z_2 = 10 + d(\lambda - 1) + {y_2}^2\lambda - p_1$

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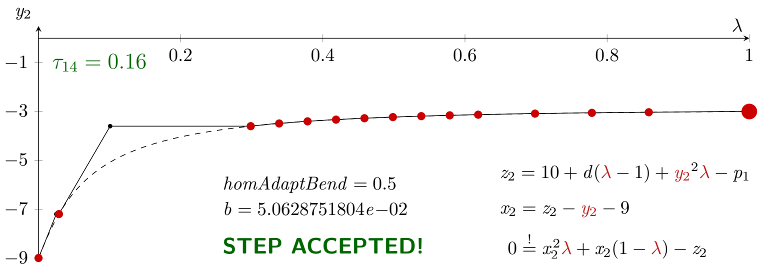

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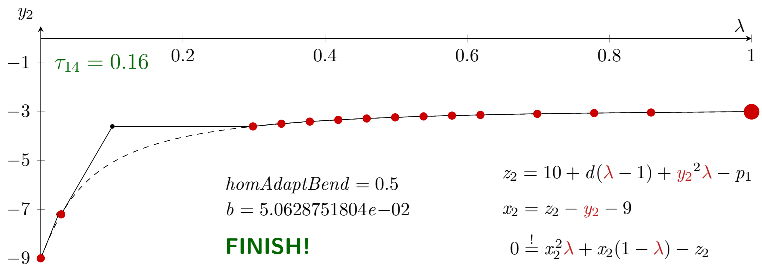


Procedure: Finish or Break

- Finish:
  - The system is solved successfully for $\lambda = 1$.
- Break:
  - $homMaxLambdaSteps$ is reached
  - $homMaxTries$ for one λ is reached
  - $\tau = homTauMin$ and current step fails
  - $\lambda < -1$

Homotopy Approaches in OM

Homotopy with adaptive step size: The Homotopy Algorithm

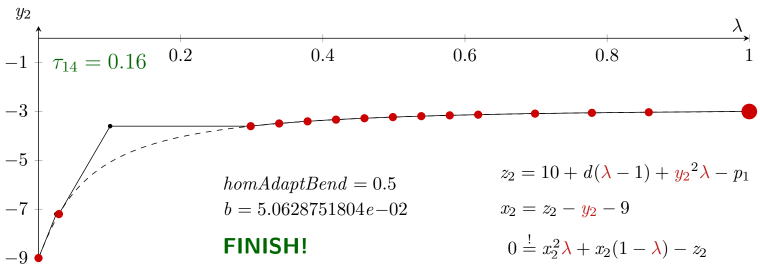
OM Runtime Flags

`-homMaxLambdaSteps =<value>`

to define the maximum number of homotopy λ-steps as a termination criterion.

Default: 100·sizeOfSystem

Homotopy Path Calculation (omc homotopy4.mos --homotopyApproach=adaptiveLocal)



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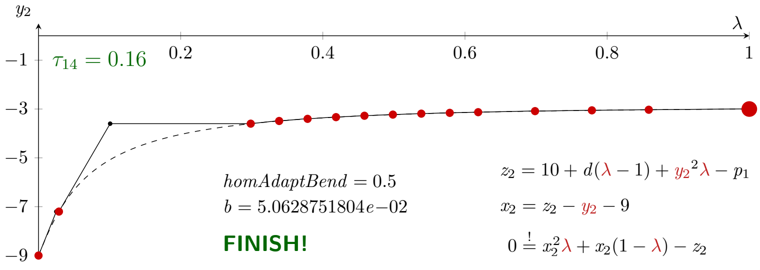
OM Runtime Flags

`-homMaxTries=<value>`

to define the maximum number of predictor/corrector step tries for one λ-step as a termination criterion.

Default: 10

Homotopy Path Calculation (omc homotopy4.mos --homotopyApproach=adaptiveLocal)



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Characteristic Features of the Homotopy Algorithm

Start Direction

- λ does not have to be monotonically nondecreasing
- The continuation direction is handled by the algorithm

$\rightarrow$ Problem: For $\lambda = 0$ the correct start direction is unknown

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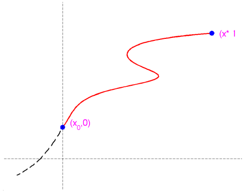
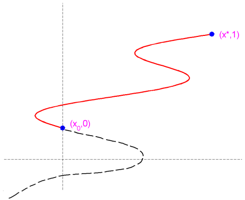
Use positive start direction:



Use negative start direction:



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- Homotopy algorithm first tries the positive direction
  - If no convergence, try negative direction

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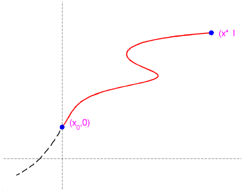

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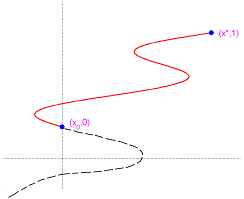

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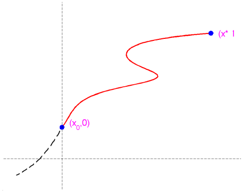
OM Runtime Flags

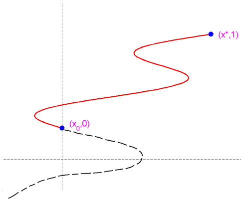
`-homNegStartDir`

to start the continuation method in the negative direction first.
If no convergence, the positive direction is tried.

Use positive start direction:



Use negative start direction:



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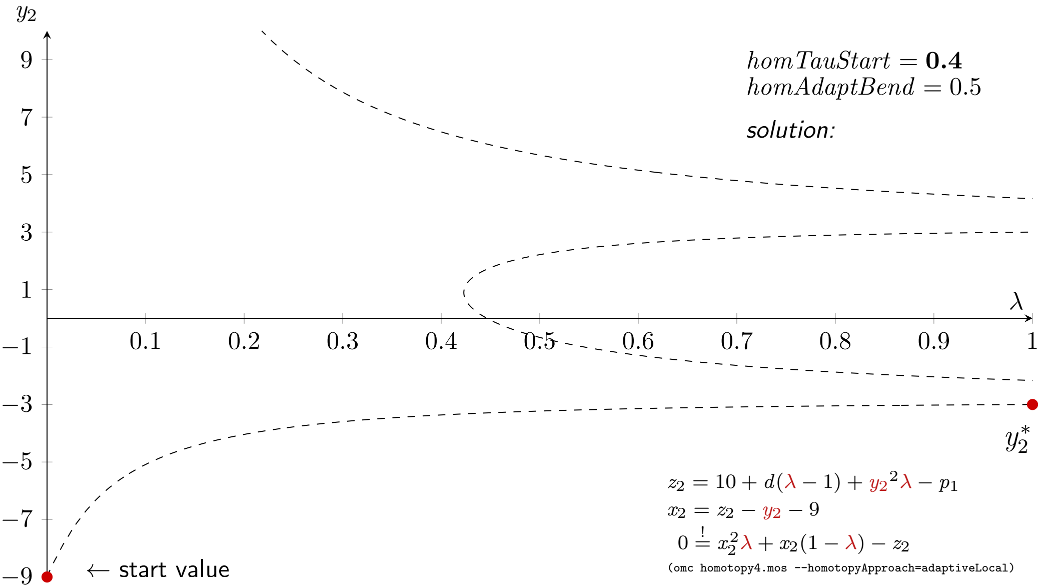

$homTauStart = \mathbf{0.4}$
$homAdaptBend = 0.5$

solution:

$$z_2 = 10 + d(\lambda - 1) + {y_2}^2\lambda - p_1$$
$$x_2 = z_2 - y_2 - 9$$
$$0 \stackrel{!}{=} x_2^2\lambda + x_2(1-\lambda) - z_2$$

(omc homotopy4.mos --homotopyApproach=adaptiveLocal)

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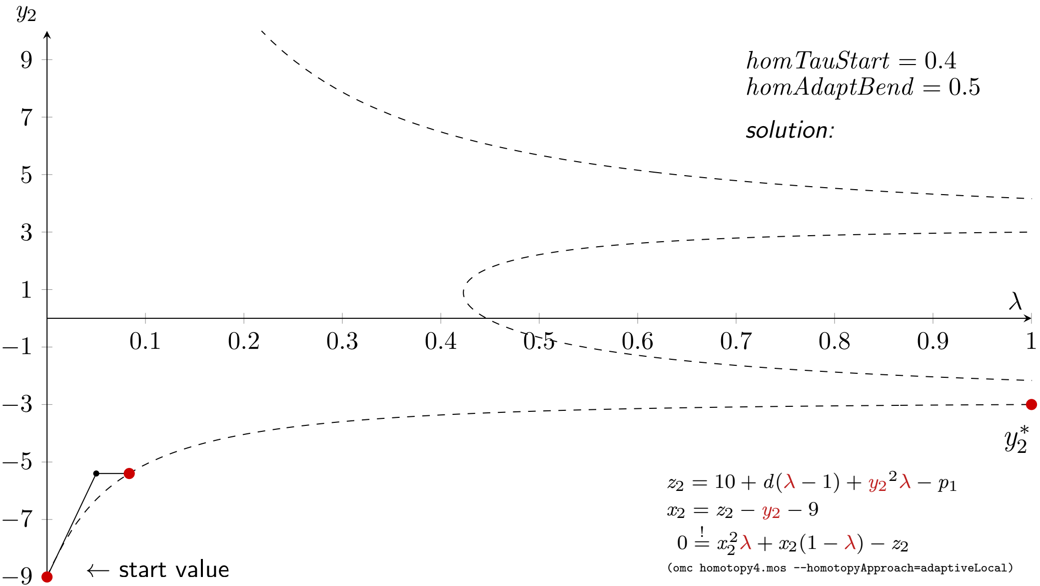

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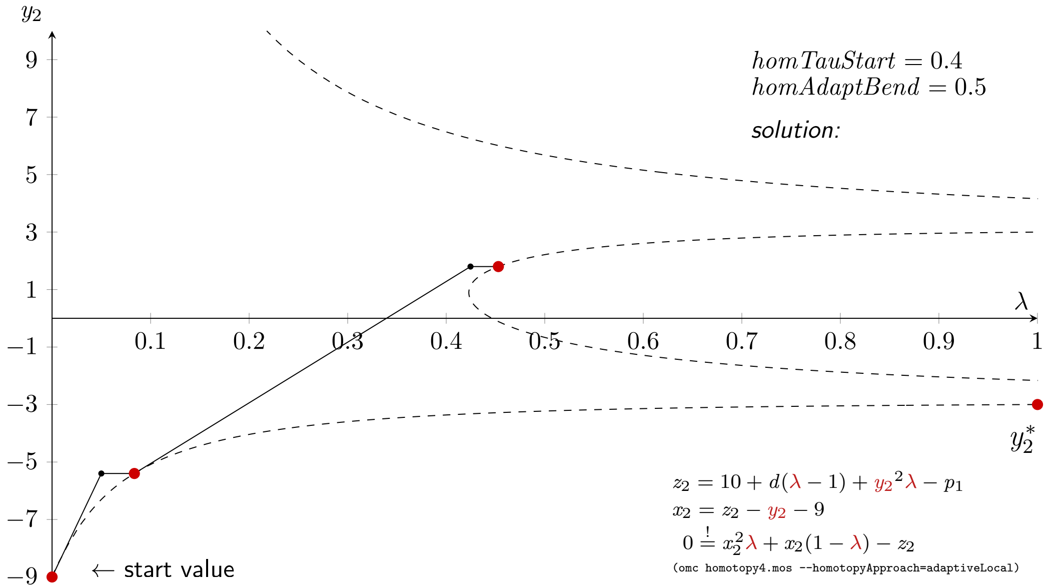

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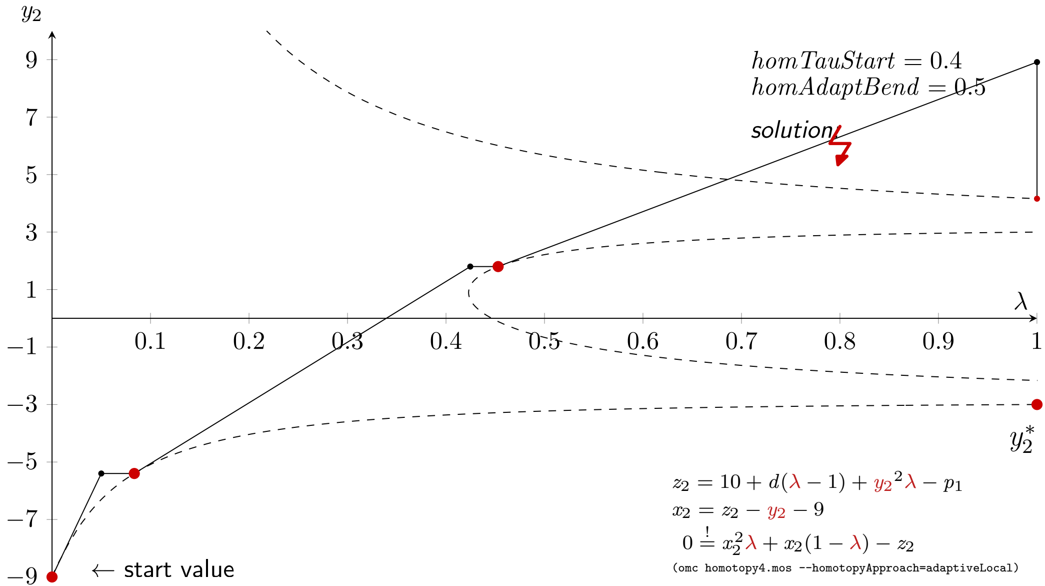

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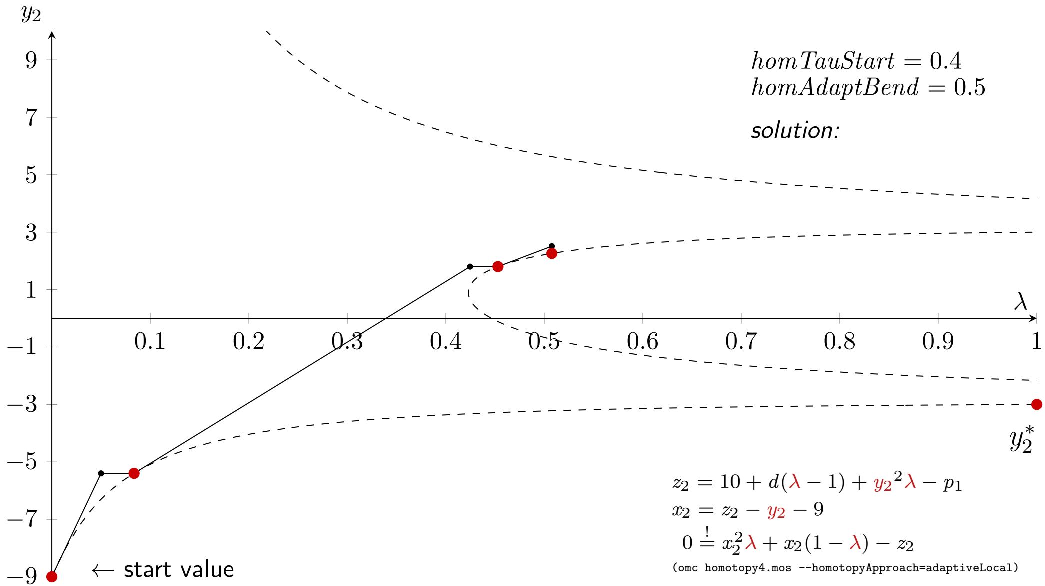

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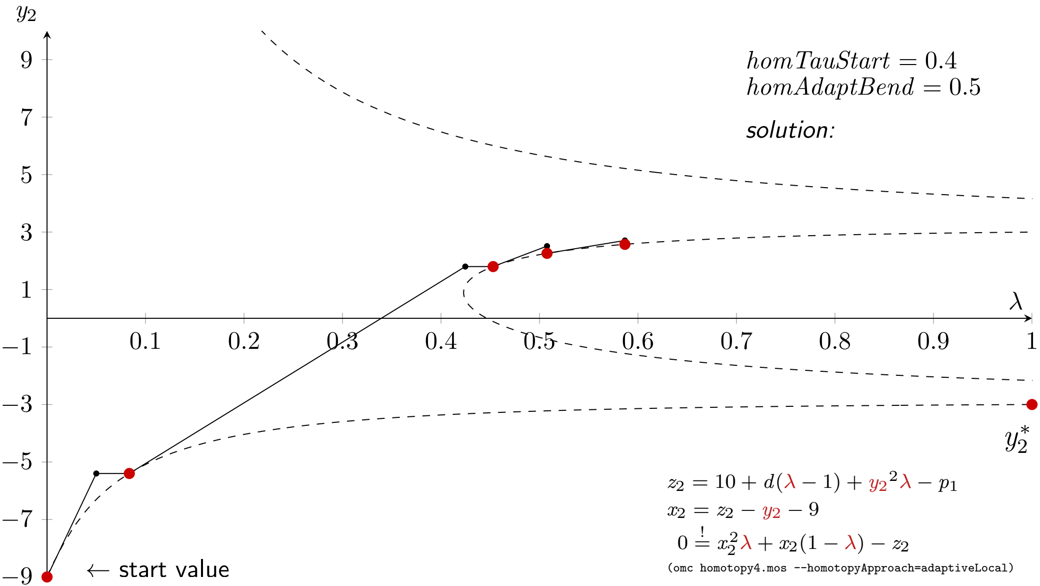

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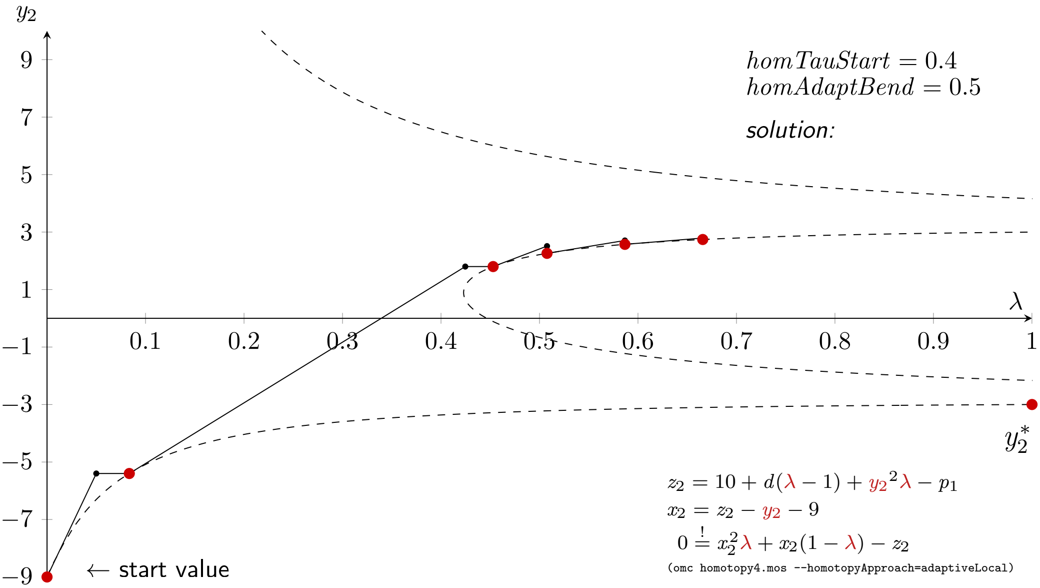

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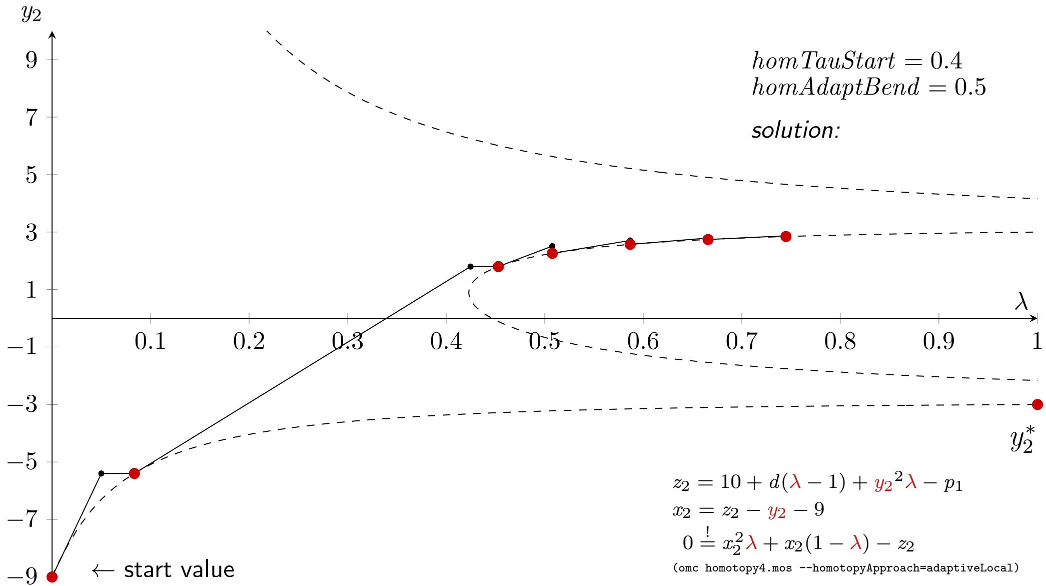

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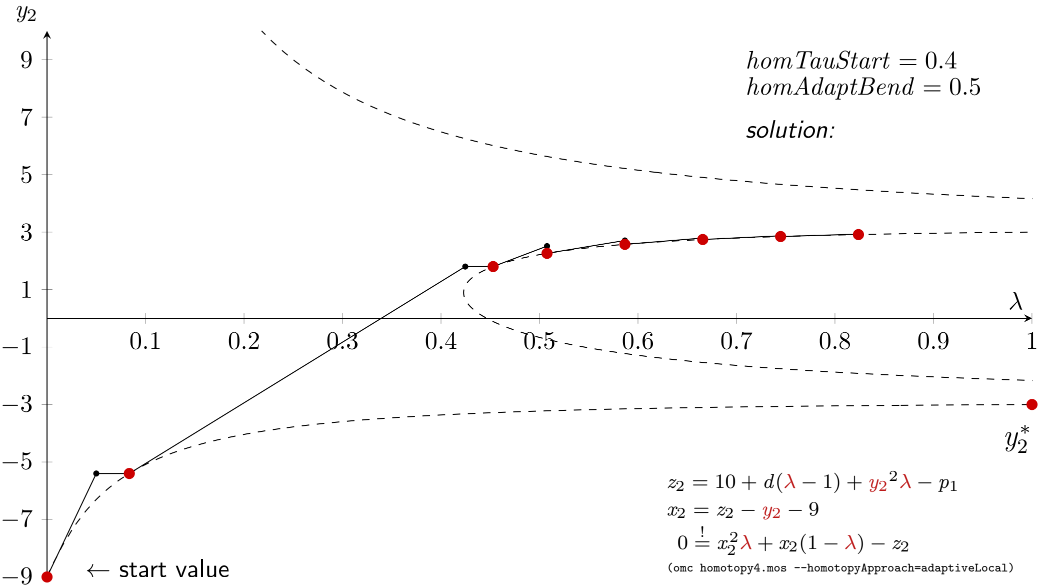

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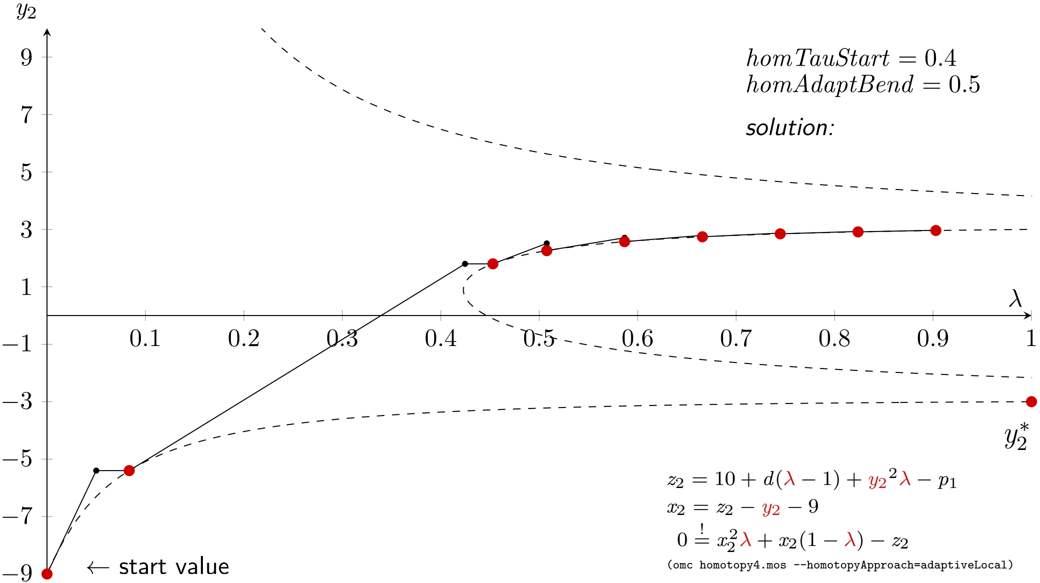


$$z_2 = 10 + d(\lambda - 1) + {y_2}^2\lambda - p_1$$

$$x_2 = z_2 - y_2 - 9$$

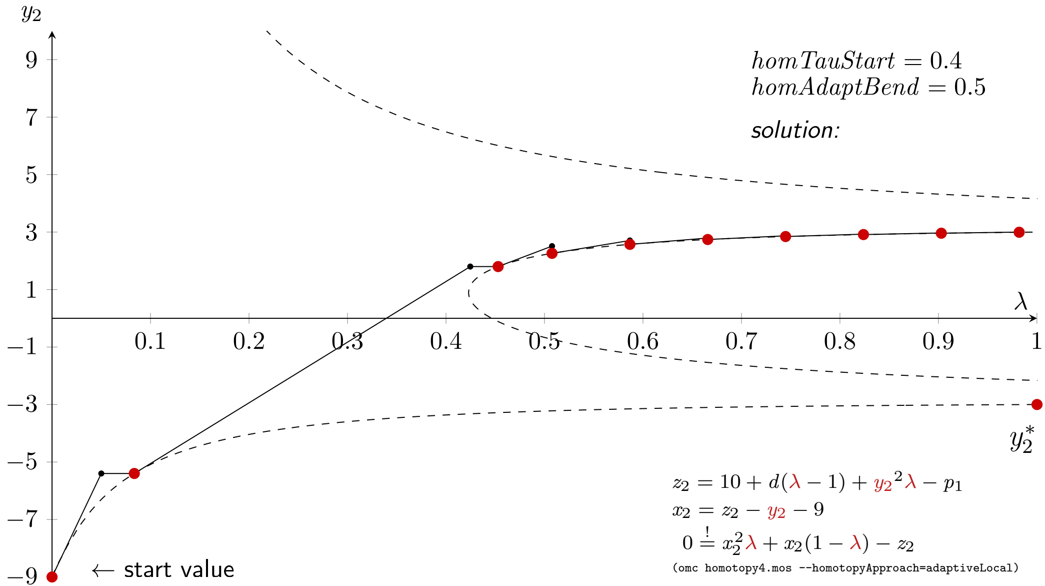
$$0 \stackrel{!}{=} x_2^2\lambda + x_2(1-\lambda) - z_2$$

`(omc homotopy4.mos --homotopyApproach=adaptiveLocal)`

Characteristic Features of the Homotopy Algorithm

Sensitivity to the Homotopy Parameters

Small changes to the homotopy parameters can lead to leaving the correct path:



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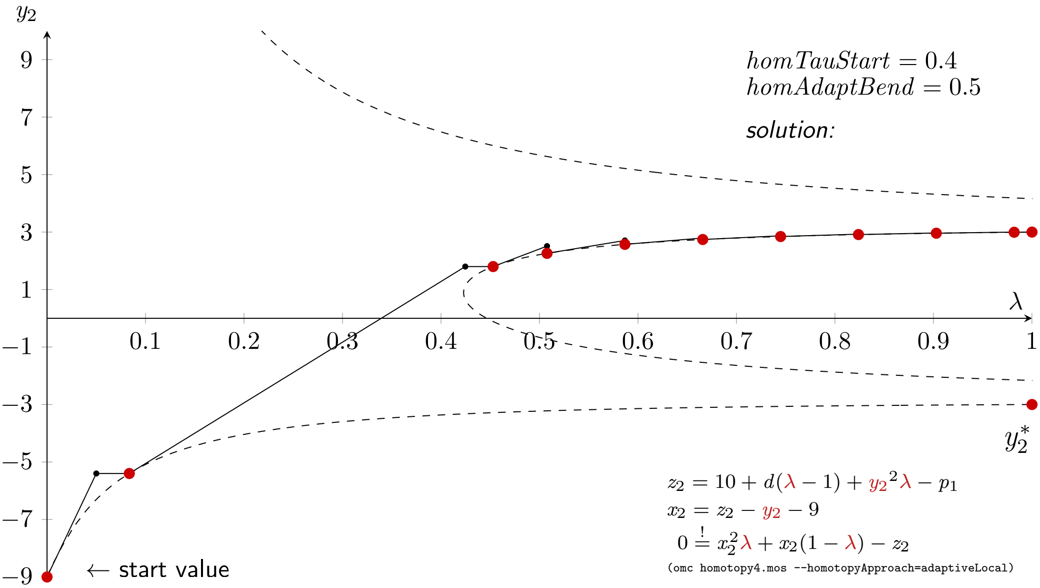

$homTauStart = 0.4$
$homAdaptBend = 0.5$

solution:

$$z_2 = 10 + d(\lambda - 1) + {y_2}^2\lambda - p_1$$
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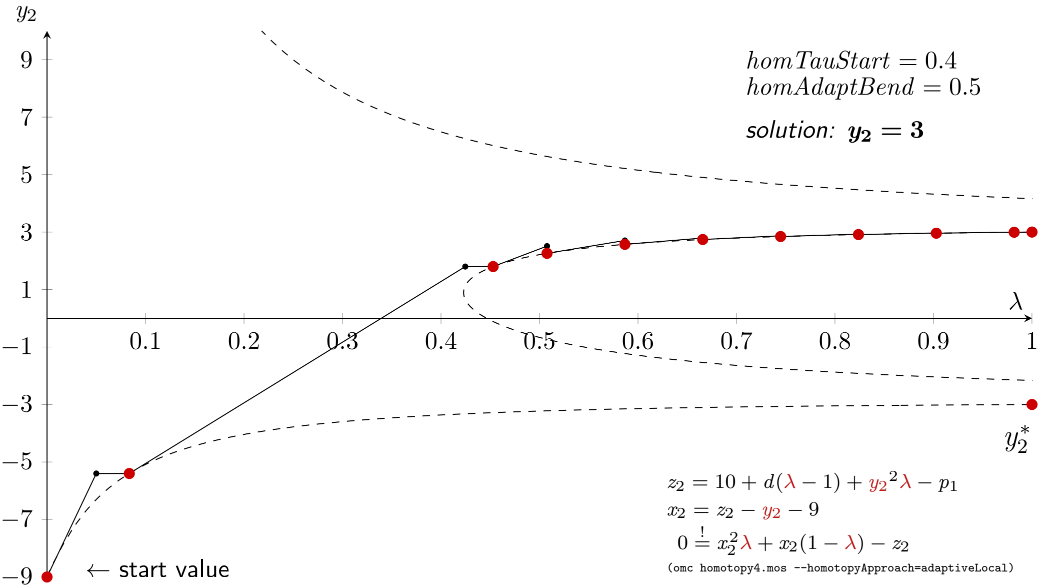

$homTauStart = 0.4$
$homAdaptBend = 0.5$

solution: $\boldsymbol{y_2 = 3}$

$$z_2 = 10 + d(\lambda - 1) + {y_2}^2\lambda - p_1$$

$$x_2 = z_2 - y_2 - 9$$

$$0 \stackrel{!}{=} x_2^2\lambda + x_2(1-\lambda) - z_2$$

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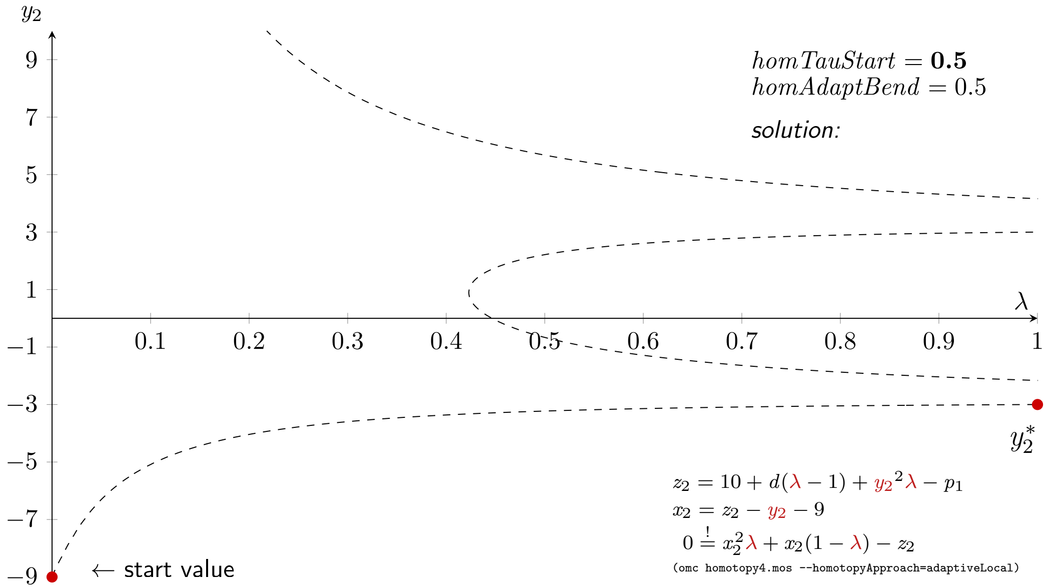

$homTauStart = \mathbf{0.5}$
$homAdaptBend = 0.5$

solution:

$$z_2 = 10 + d(\lambda - 1) + {y_2}^2\lambda - p_1$$
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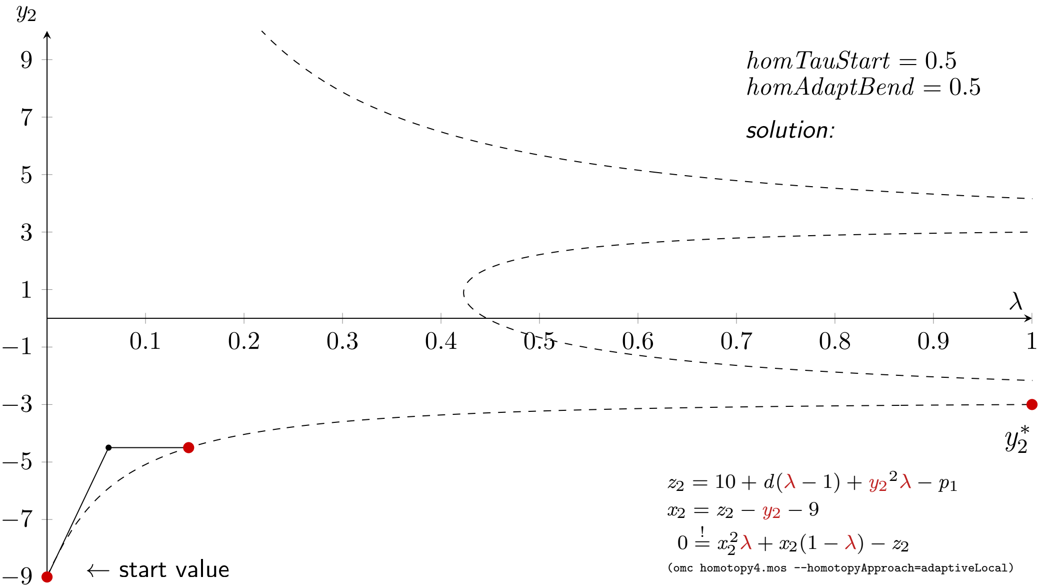

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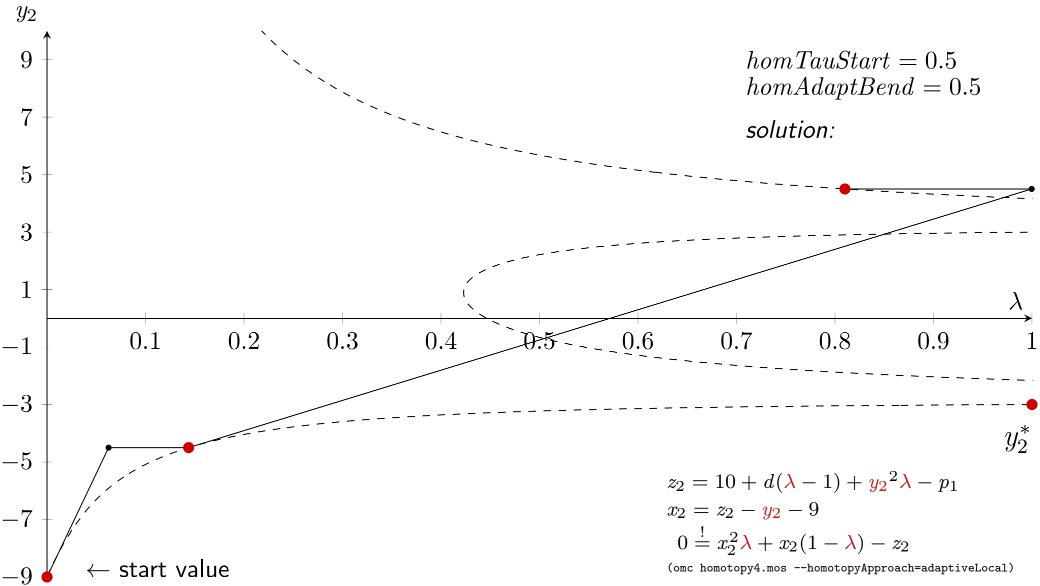

$homTauStart = 0.5$
$homAdaptBend = 0.5$

solution:

$$z_2 = 10 + d(\lambda - 1) + {y_2}^2\lambda - p_1$$
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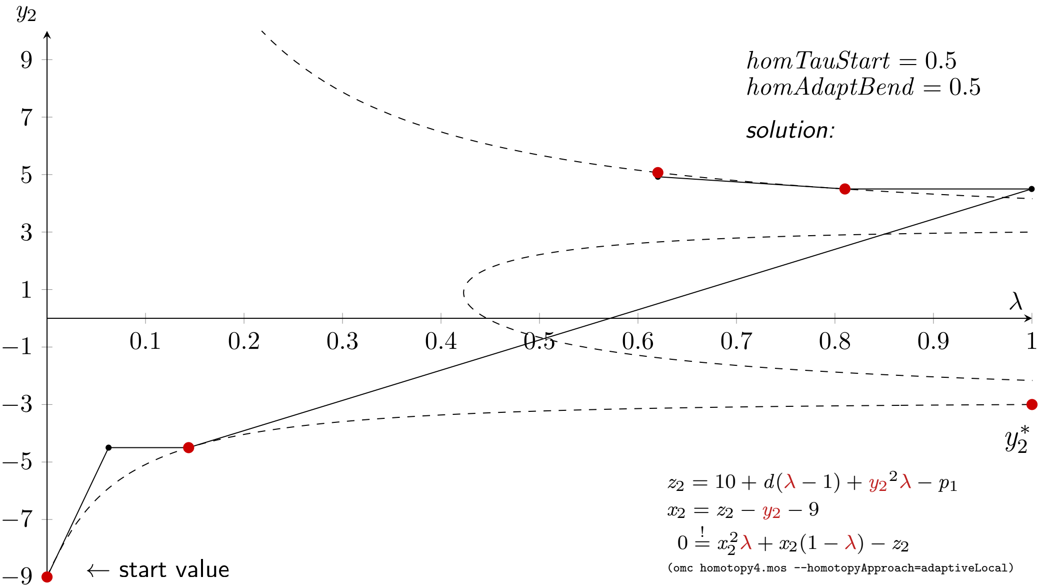

$homTauStart = 0.5$
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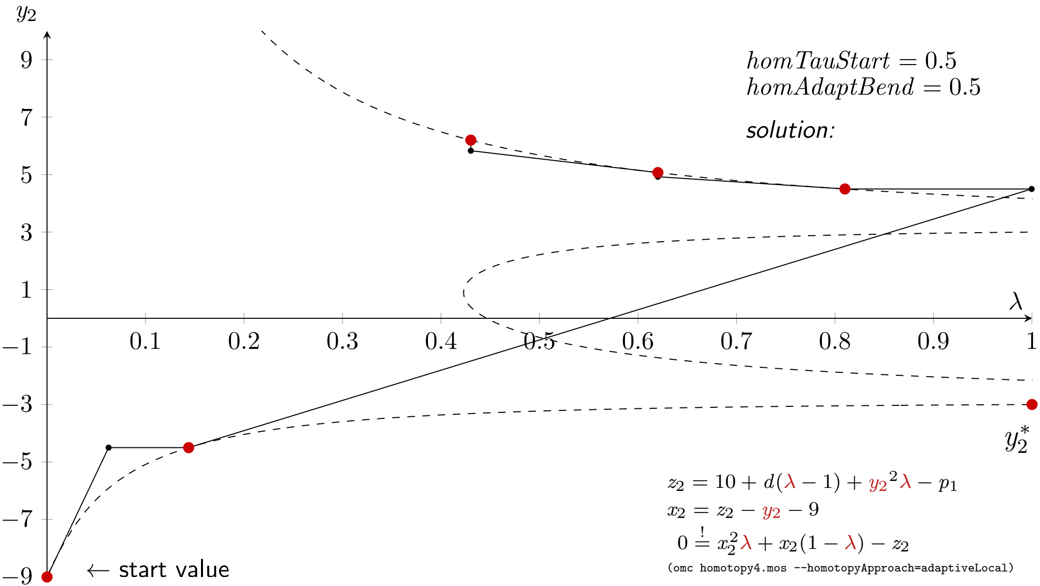

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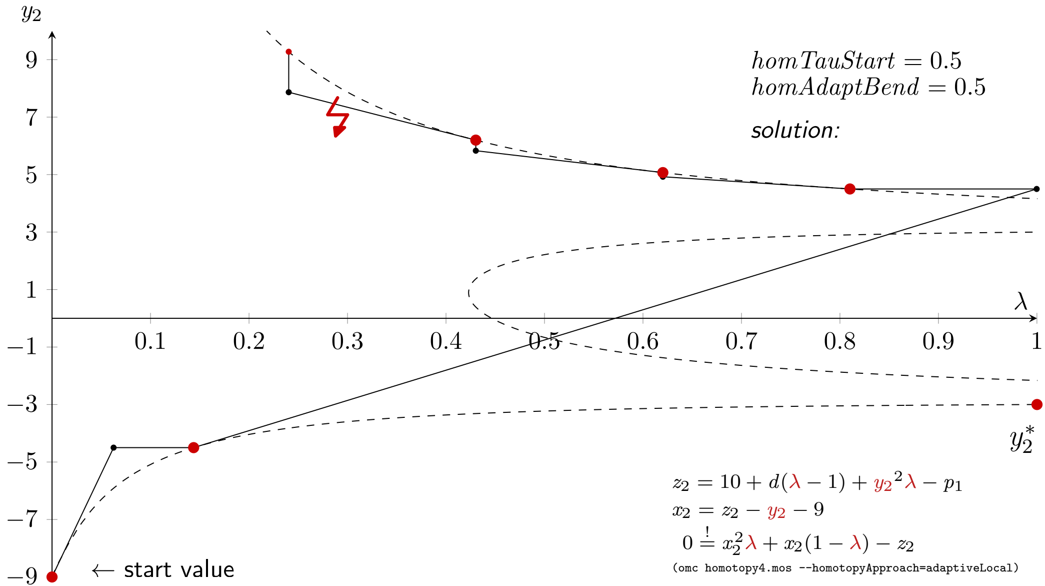

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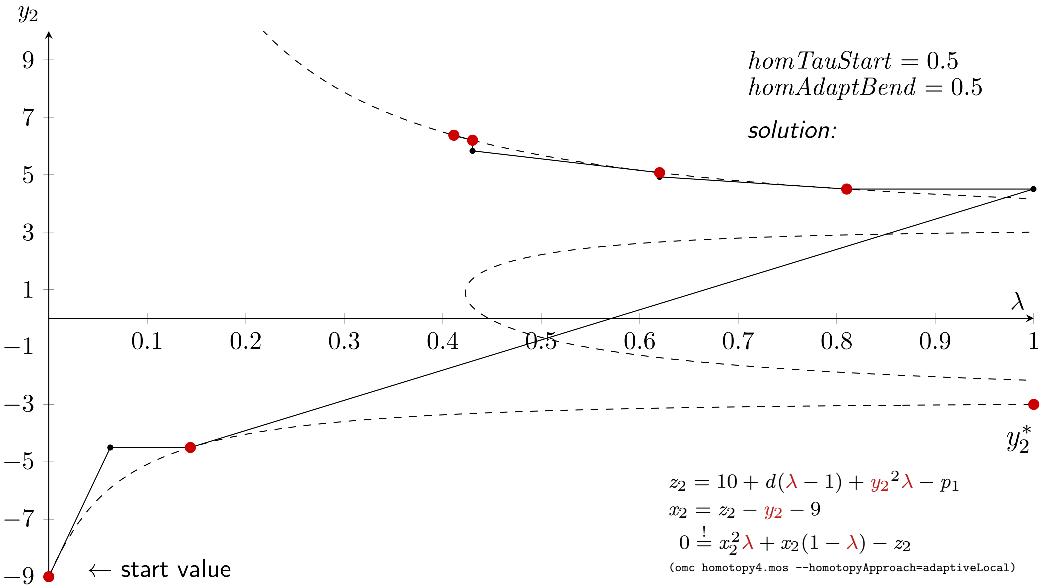

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$$z_2 = 10 + d(\lambda - 1) + {y_2}^2\lambda - p_1$$
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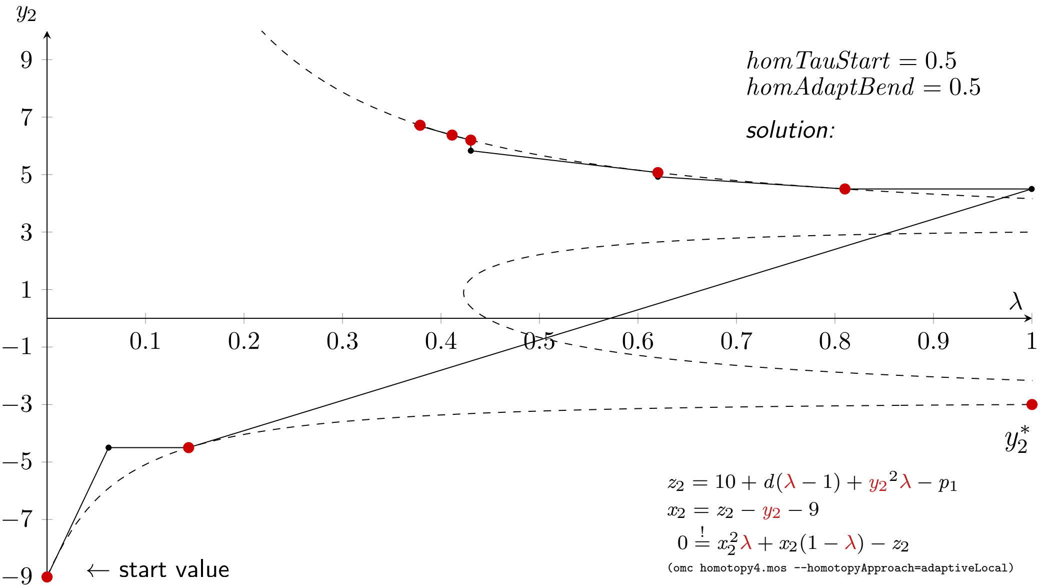

$homTauStart = 0.5$
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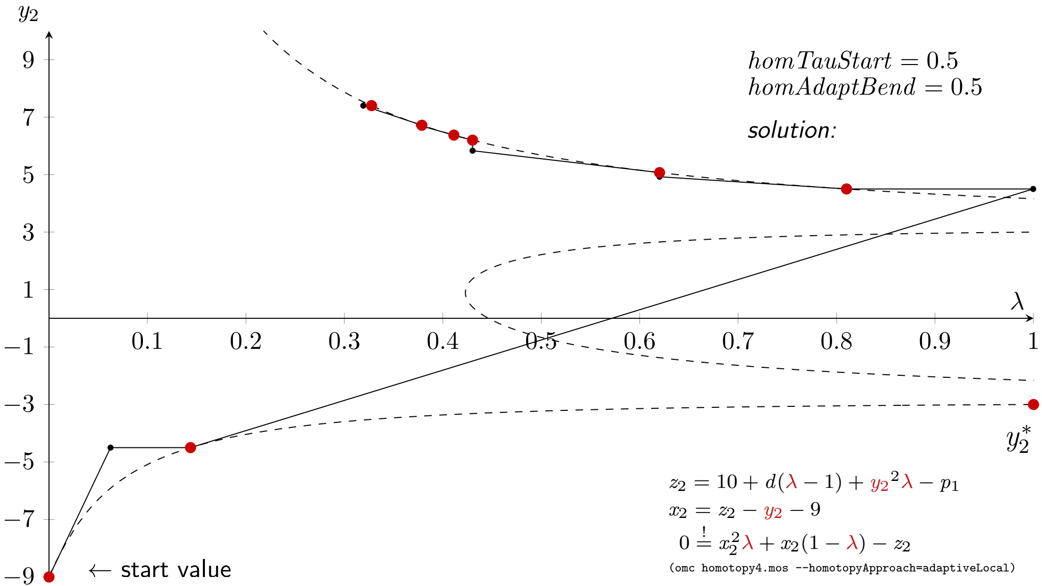

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$$x_2 = z_2 - y_2 - 9$$

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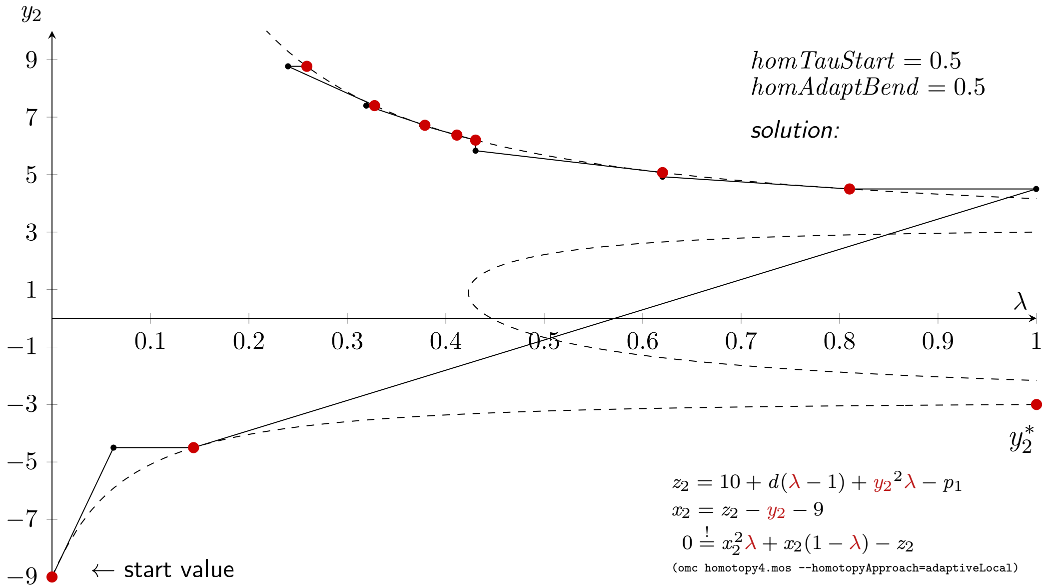

$homTauStart = 0.5$
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$z_2 = 10 + d(\lambda - 1) + {y_2}^2\lambda - p_1$

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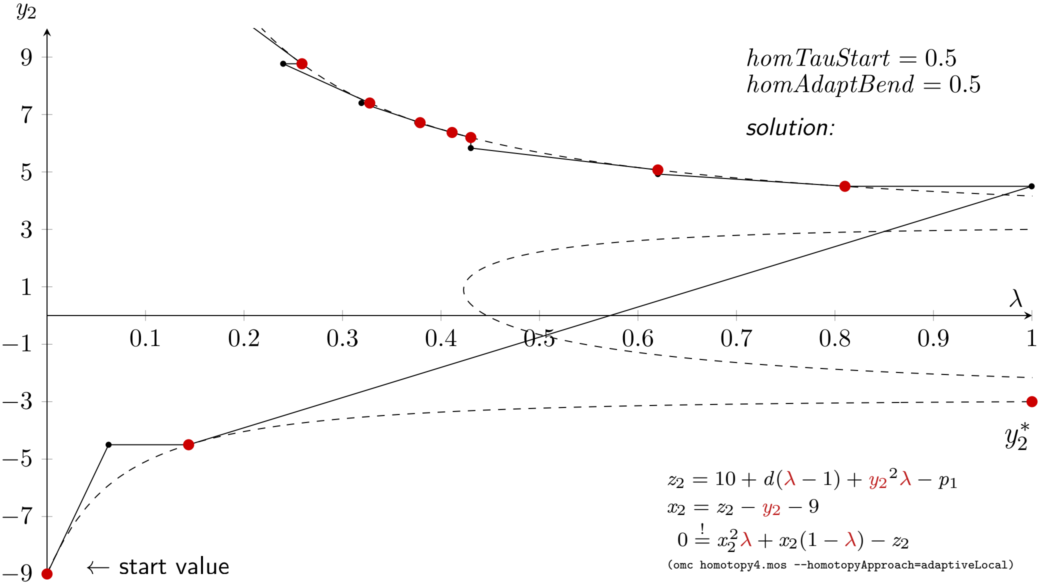

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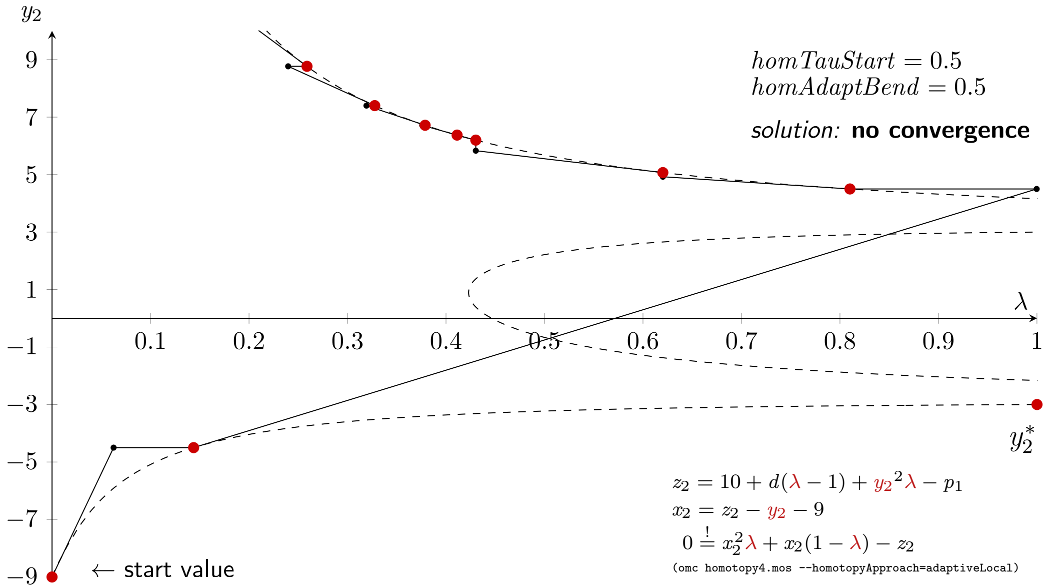

$homTauStart = 0.5$
$homAdaptBend = 0.5$

solution: **no convergence**

$$z_2 = 10 + d(\lambda - 1) + {y_2}^2\lambda - p_1$$
$$x_2 = z_2 - y_2 - 9$$
$$0 \overset{!}{=} x_2^2\lambda + x_2(1 - \lambda) - z_2$$

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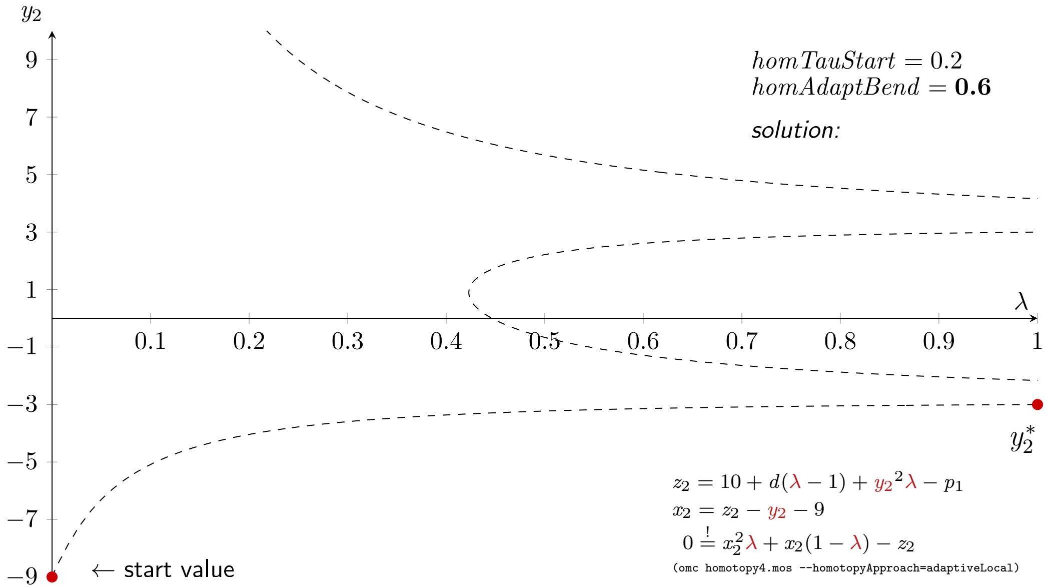

$$homTauStart = 0.2$$
$$homAdaptBend = \mathbf{0.6}$$

solution:

$$z_2 = 10 + d(\lambda - 1) + {y_2}^2\lambda - p_1$$
$$x_2 = z_2 - y_2 - 9$$
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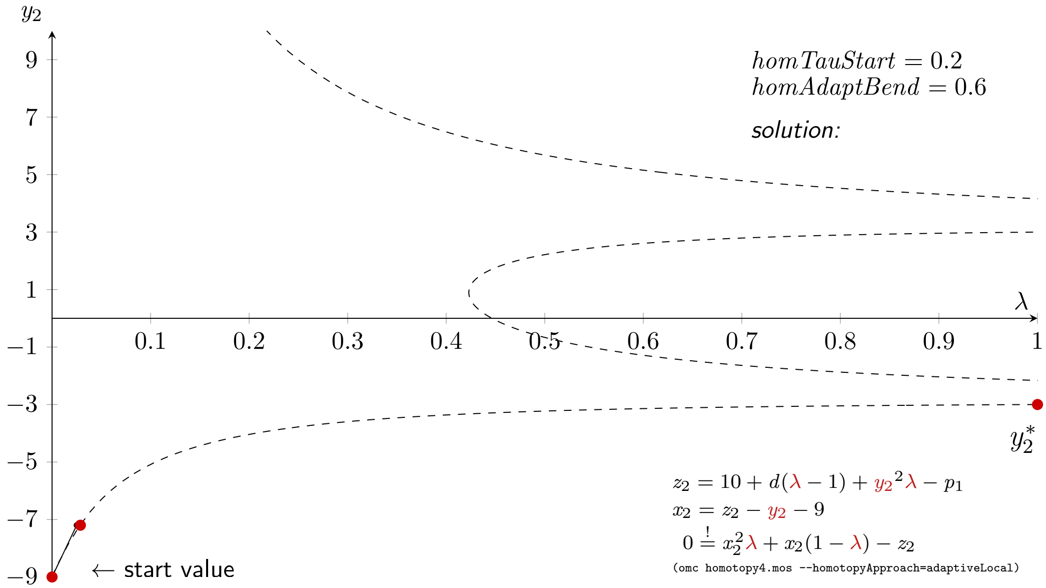

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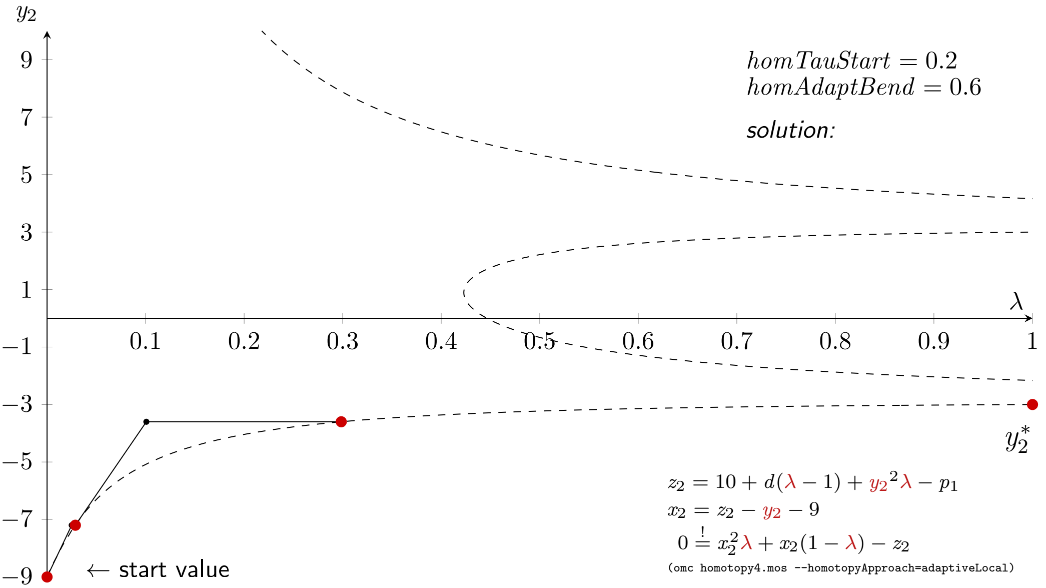

$homTauStart = 0.2$
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solution:

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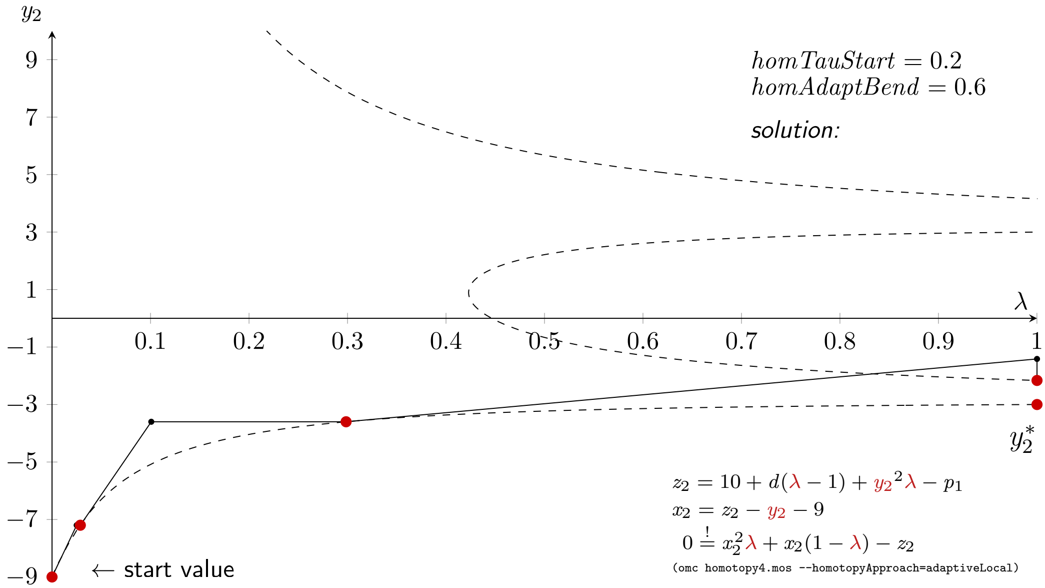

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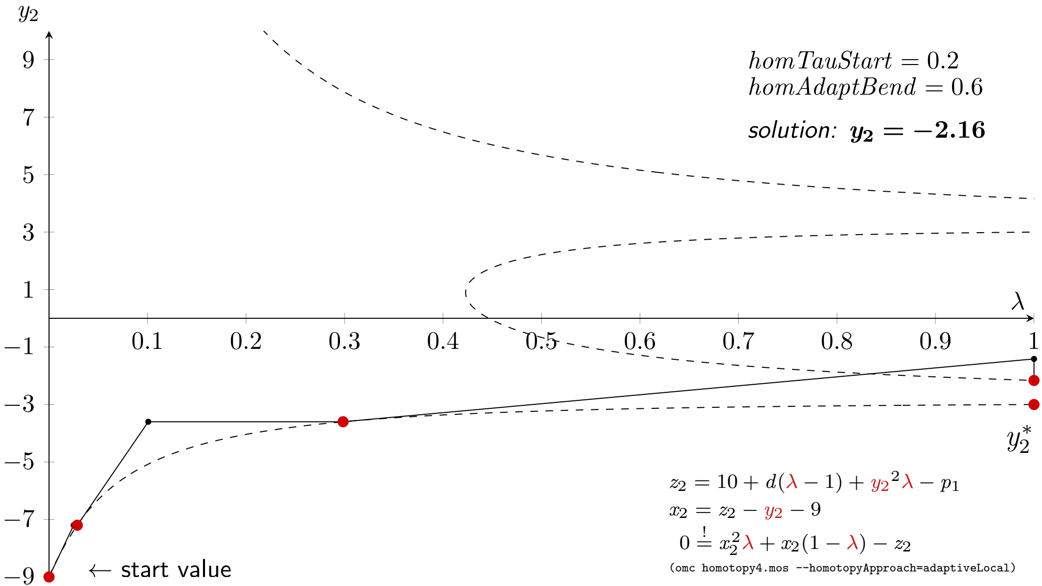

$homTauStart = 0.2$
$homAdaptBend = 0.6$

solution: $\mathbf{y_2 = -2.16}$

$$z_2 = 10 + d(\lambda - 1) + {y_2}^2\lambda - p_1$$
$$x_2 = z_2 - y_2 - 9$$
$$0 \overset{!}{=} x_2^2\lambda + x_2(1-\lambda) - z_2$$

(`omc homotopy4.mos --homotopyApproach=adaptiveLocal`)

Characteristic Features of the Homotopy Algorithm

Impact of Tearing

Tearing can influence the homotopy path calculation:

Tearing Set 1 (omc homotopy4.mos --homotopyApproach=adaptiveGlobal)

Iteration variables: z_1, y_2, z_3

$$
\begin{aligned}
c &= b \cdot (1-\lambda) + b^2 \cdot \lambda \\
x_1 &= z_1 + c - 5 \\
y_1 &= 9 + x_1 - z_1 \\
d &= x_1 + y_1 + z_1 \\
z_2 &= 10 + d \cdot (\lambda - 1) + y_2^2 \cdot \lambda - p_1 \\
x_2 &= z_2 - (9 + y_2) \\
e &= \sin(d \cdot x_2) \\
x_3 &= z_3 + e - 5 \\
y_3 &= 9 + x_3 - z_3 \\
0 &\overset{!}{=} -10 + p_1 + z_1 + y_1 \\
0 &\overset{!}{=} x_2^2 \cdot \lambda + x_2 \cdot (1-\lambda) - z_2 \\
0 &\overset{!}{=} -10 + p_1 + y_3 \cdot (1 + z_3)
\end{aligned}
$$

Tearing Set 2 (omc homotopy4.mos --homotopyApproach=adaptiveGlobal)

Iteration variables: z_1, y_2, x_3

$$
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c &= b \cdot (1-\lambda) + b^2 \cdot \lambda \\
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z_2 &= 10 + d \cdot (\lambda - 1) + y_2^2 \cdot \lambda - p_1 \\
x_2 &= z_2 - (9 + y_2) \\
e &= sin(d \cdot x_2) \\
z_3 &= 5 + x_3 - e \\
y_3 &= 9 + x_3 - z_3 \\
0 &\overset{!}{=} -10 + p_1 + z_1 + y_1 \\
0 &\overset{!}{=} x_2^2 \cdot \lambda + x_2 \cdot (1-\lambda) - z_2 \\
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Characteristic Features of the Homotopy Algorithm

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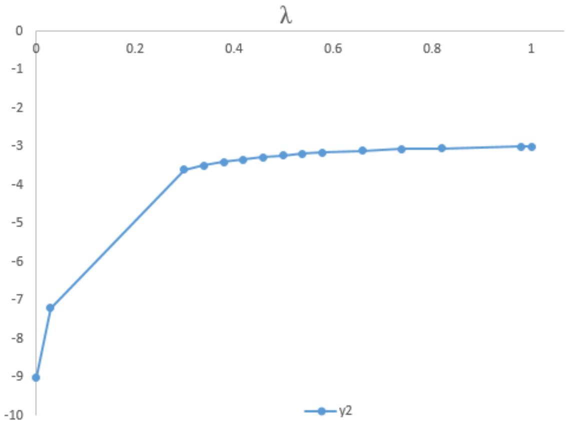
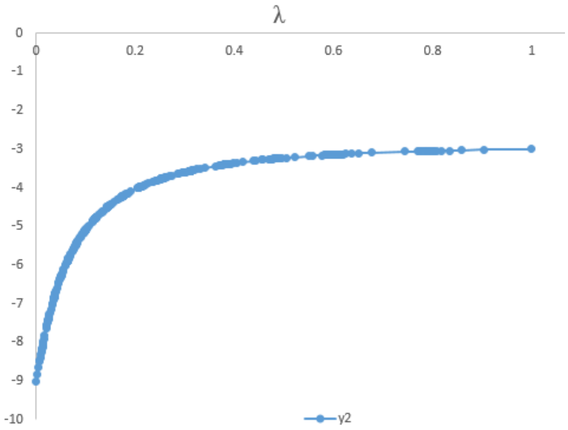
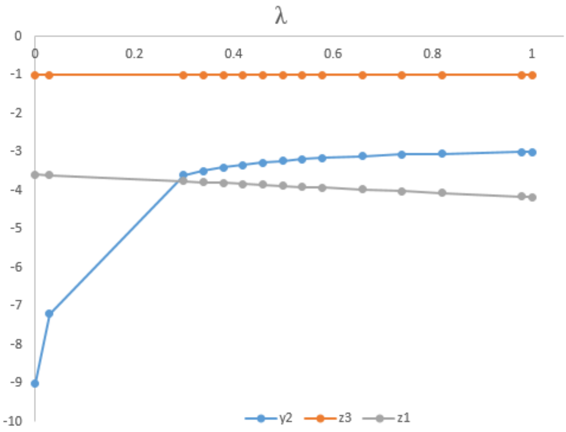




15 homotopy steps

426 homotopy steps

Tearing can influence the homotopy path calculation:



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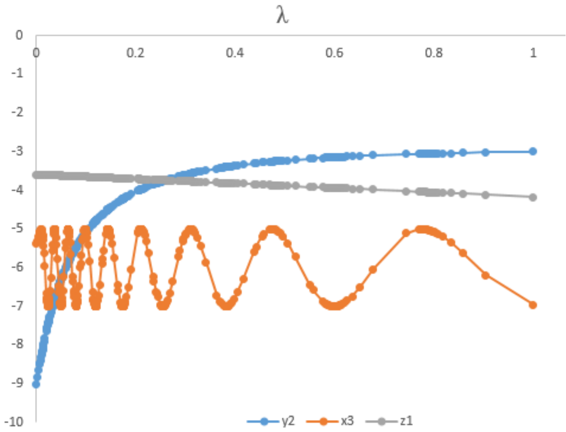

426 homotopy steps

Debugging

Shell

- `-lv=LOG_INIT`
  - General information about initialization and the homotopy process
- `-lv=LOG_NLS_V`
  - General information about the solving process of nonlinear systems
- `-lv=LOG_NLS_JAC`
  - To print the Jacobians
- `-lv=LOG_NLS_HOMOTOPY`
  - Detailed information about the solving process of the homotopy solver

Homotopy Path

- `-lv=LOG_INIT` creates a csv-file with the homotopy path,
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→ Visualize with OMPlot, Excel, ...

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Conclusion

Summary

- OpenModelica provides several options for the initialization with homotopy
  - local/global, equidistant/adaptive
- Some expertise in homotopy-based initialization is needed to ...
  - guarantee a sensible usage of the homotopy operator.
  - create a unique homotopy path wherever possible.
  - choose the appropriate method.
  - set the homotopy parameters properly.
  - promote the convergence of the method (Tearing, ...).

Outlook

- Adding a homotopy library with well-conceived homotopy examples to the OpenModelica coverage test
  - → Find good default values for the homotopy parameters
- Activate one of the adaptive methods as the default fallback method for initialization

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