Throughput optimization (next) ...

- Max-flow optimization
  - Zongpeng's slides: 2-8

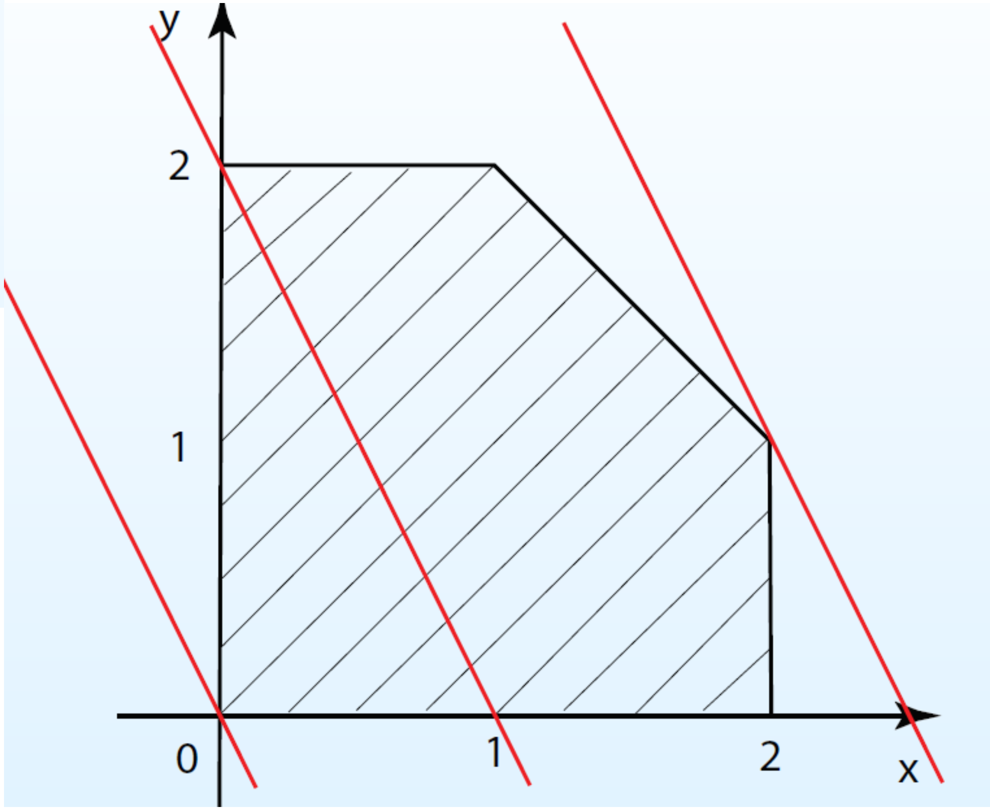
- Network coding
  - Butterfly example
  - Zongpeng's slides: 14-16

Linnear Programming: "Hello world" …

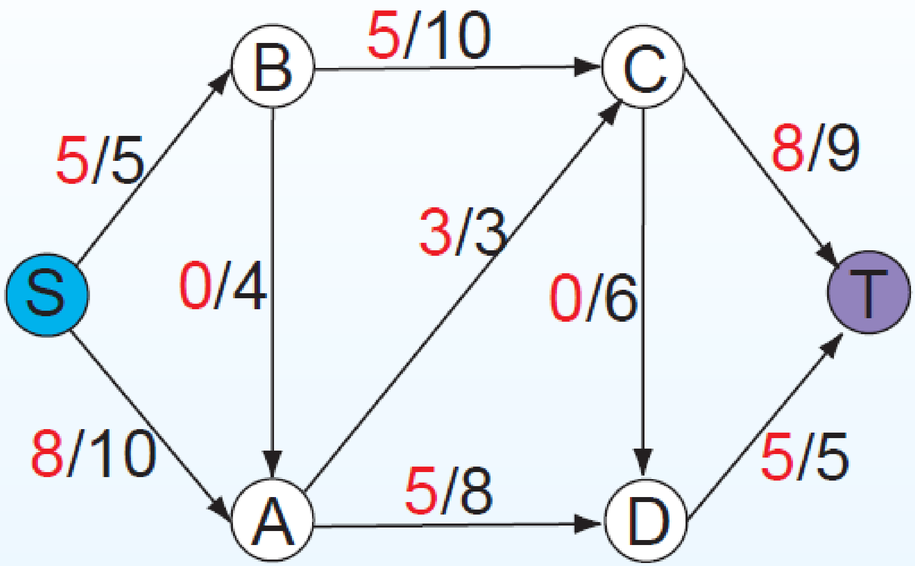
$$
\begin{aligned}
\text{maximize} \quad & 2x + y \\
\text{s.t.}: \quad & \\
& x \leq 2 \\
& y \leq 2 \\
& x + y \leq 3 \\
& \\
& x, y \geq 0
\end{aligned}
$$

Linnear Programming: "Hello world" …

$$\begin{aligned} \text{maximize} \quad & 2x + y \\ \text{s.t. :} \quad & \\ & x \leq 2 \\ & y \leq 2 \\ & x + y \leq 3 \\ & x, y \geq 0 \end{aligned}$$

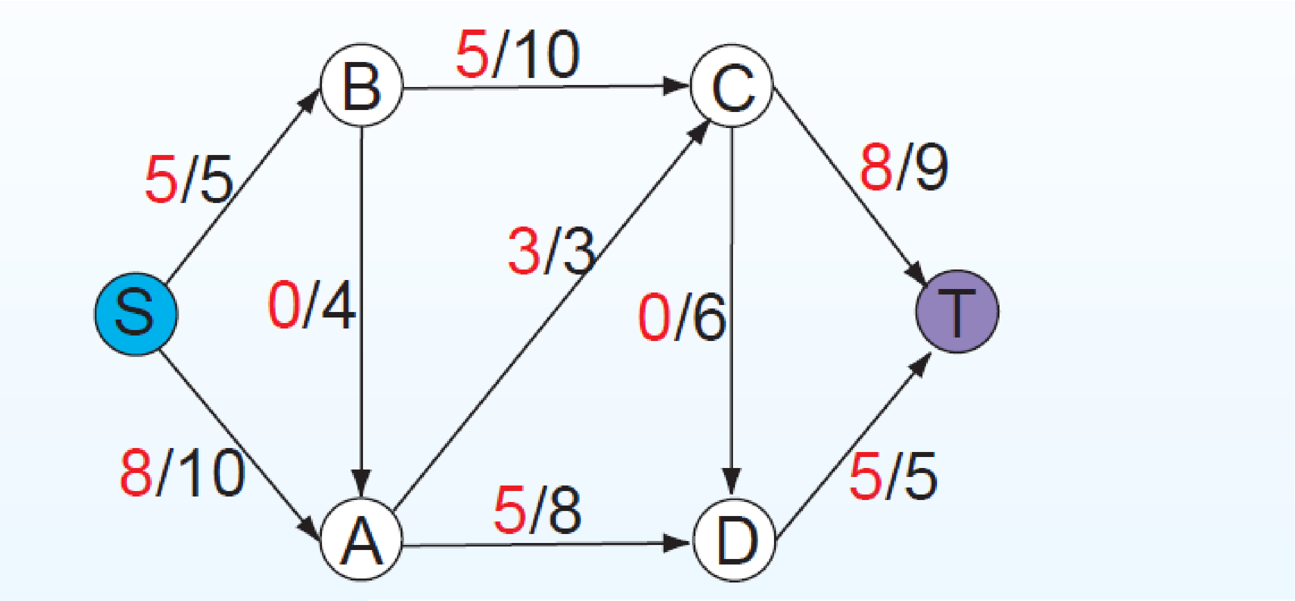


LP-model: Max flow



- Maximum rate we can push flows from S to T in a given capacitied flow network.
- flow-rate/link-capacity

LP-model: Max flow



Maximize $\chi = f(\overrightarrow{TS})$

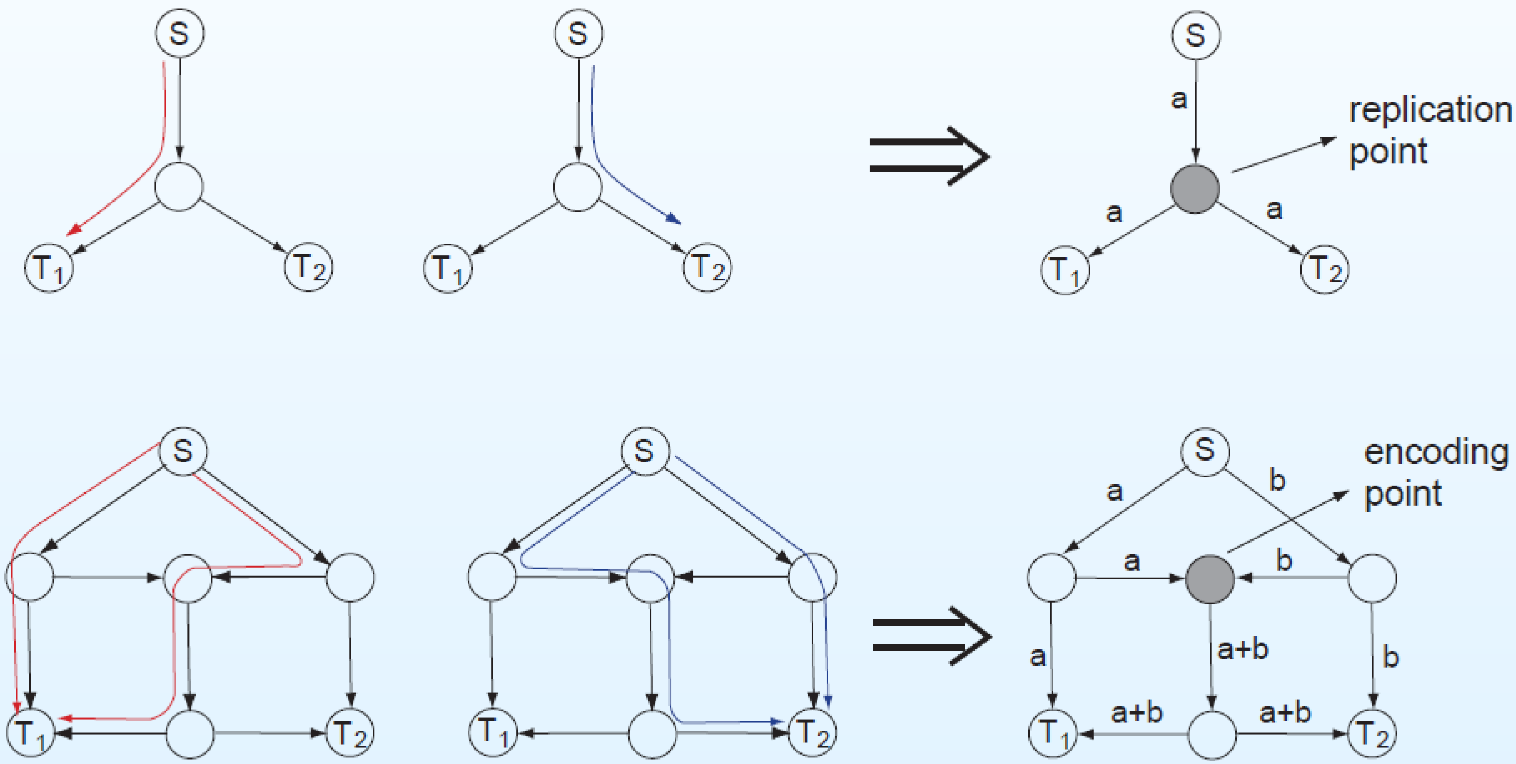
Subject to:

$$\begin{cases} f(\overrightarrow{uv}) \leq C(uv) & \forall\ \overrightarrow{uv} \neq \overrightarrow{TS} \\ \sum_{v \in N(u)} f(\overrightarrow{uv}) = \sum_{v \in N(u)} f(\overrightarrow{vu}) & \forall u \end{cases}$$

$$f(\overrightarrow{uv}) \geq 0 \qquad \forall\ \overrightarrow{uv}$$

Max-rate multicast with network coding

Given network coding, a multicast rate x is feasible in a directed network iff it is feasible as an independent unicast to every receiver. [Ahlswede et al. IT 2000][Koetter and Médard TON 2003]



Max-rate multicast with network coding

Maximize χ

Subject to:

$$\begin{cases} \chi \leq f_i(\overrightarrow{T_i S}) & \forall i & (1) \\ f_i(\overrightarrow{uv}) \leq c(\overrightarrow{uv}) & \forall i, \forall\ \overrightarrow{uv} \neq \overrightarrow{T_i S} & (2) \\ \sum_{v \in N(u)} f_i(\overrightarrow{uv}) = \sum_{v \in N(u)} f_i(\overrightarrow{vu}) & \forall i, \forall u & (3) \\ c(\overrightarrow{uv}) + c(\overrightarrow{vu}) \leq C(uv) & \forall uv \neq T_i S & (4) \end{cases}$$

$$c(\overrightarrow{uv}), f_i(\overrightarrow{uv}), \chi \geq 0 \qquad \forall i, \forall\ \overrightarrow{uv}$$

Max-rate multicast WITHOUT network coding

Minimize $\sum_t f(t)$

Subject to:

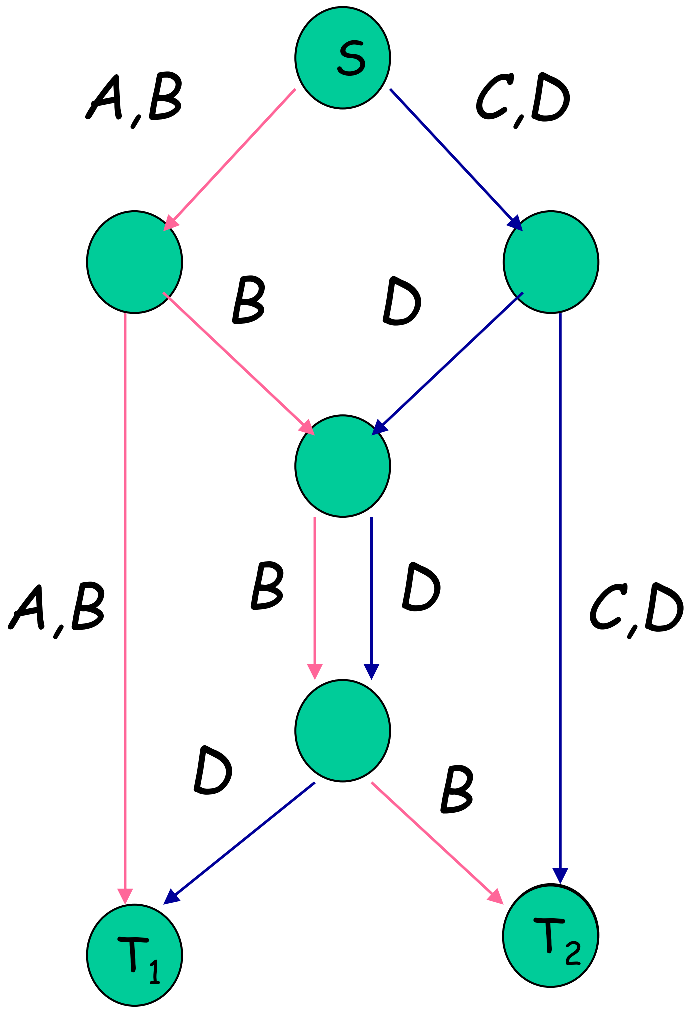
$$\sum_{t:e\in t} f(t) \leq c(e) \qquad \forall e$$

$$f(t) \geq 0 \qquad \forall t$$

- Don't be misguided by the seeming simplicity of the LP.
- It has exponentially many variables.
- We know a network instance with 16 nodes only, having ~ 50 million different trees.
- But, what else can we do? It's an NP-hard problem.

Let's start with throughput …

Without network coding …

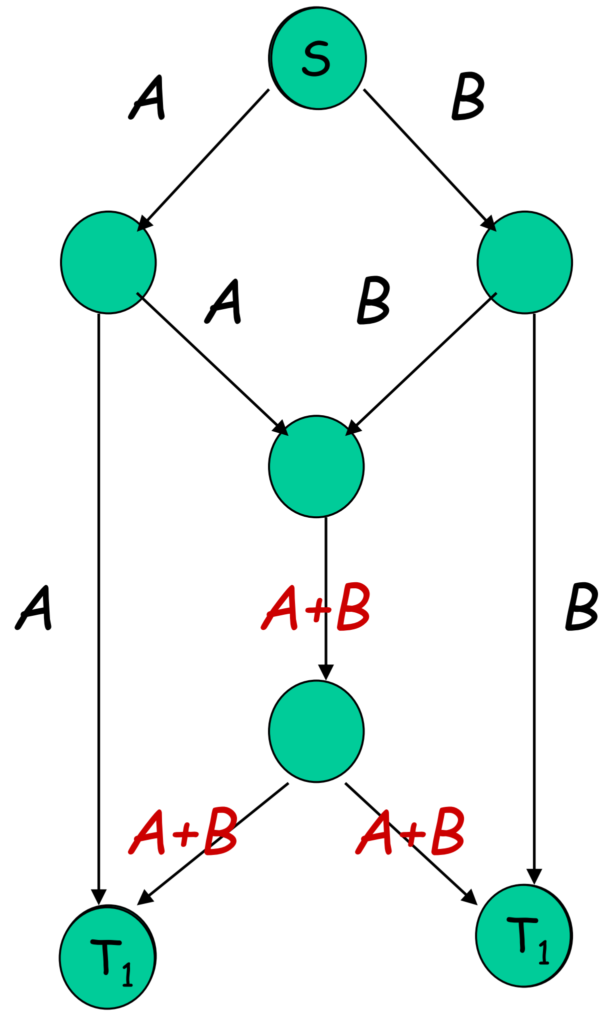


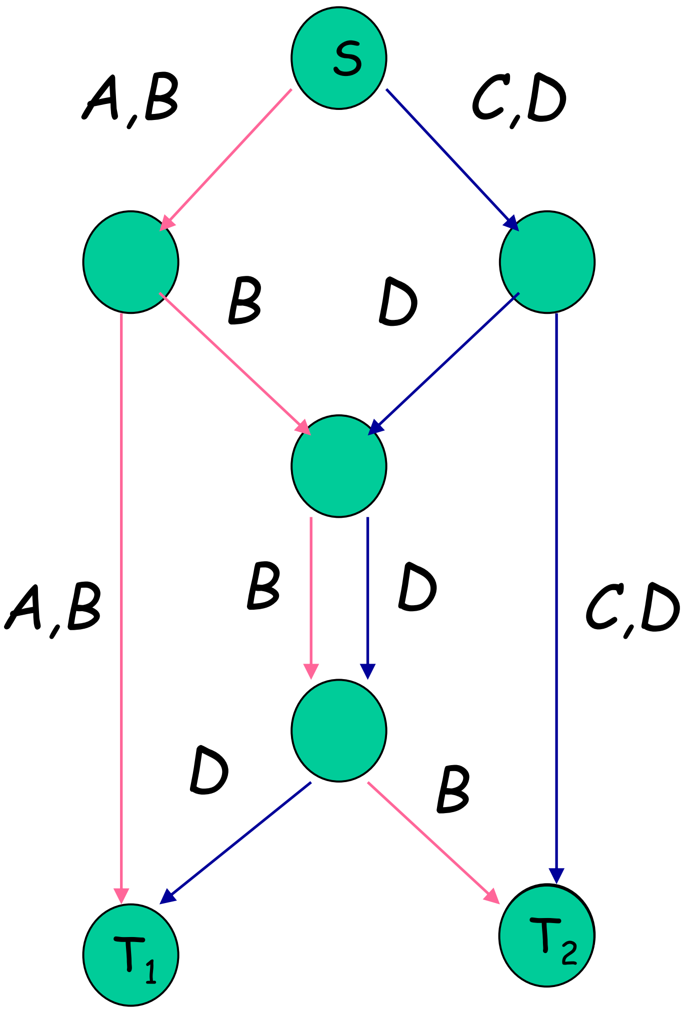
- T1 and T2 both get $\frac{3}{4}$ streams (75% of senders capacity)

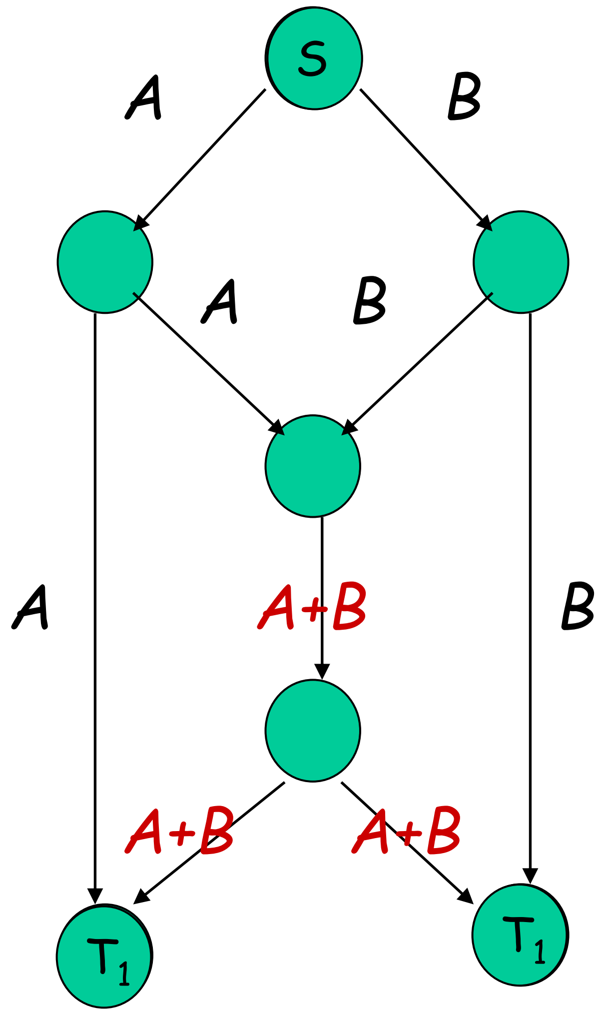
- Optimization problem equal to "packing of Steiner trees" (NP-hard problem)

With network coding ...

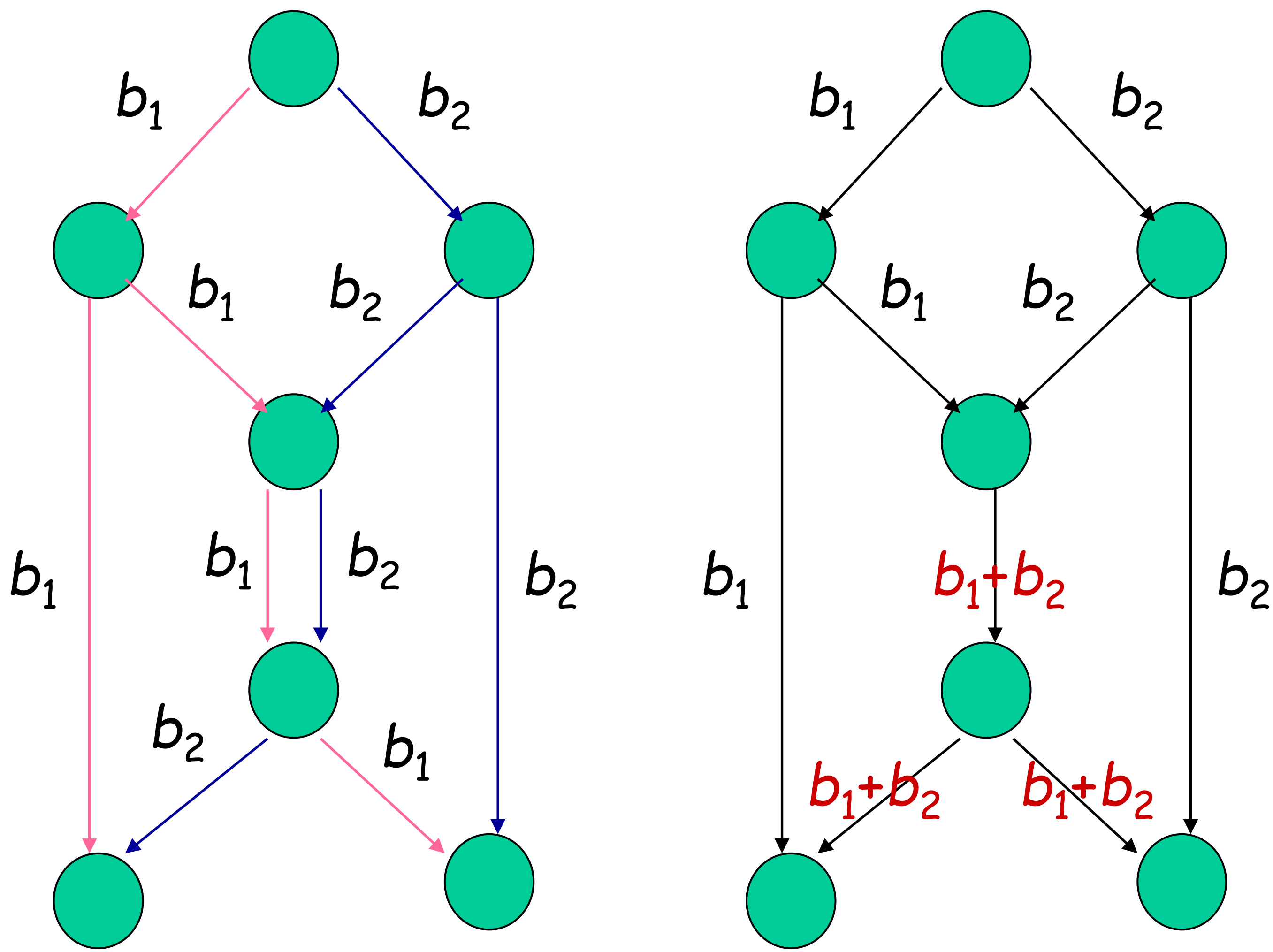
- T1 and T2 both get 2/2 streams (100% of senders capacity)
- Improvement by 33%




S
A,B
C,D
B
D
A,B
B
D
C,D
D
B
T_1
T_2


S
A
B
A
B
A
A+B
B
A+B
A+B
T_1
T_1

- Savings can also be in terms of “bandwidth”
  …



b_1
b_2
b_1
b_2
b_1
b_1
b_2
b_2
b_2
b_1
b_1
b_2
b_1
b_2
b_1
b_1+b_2
b_2
b_1+b_2
b_1+b_2

- …or “time” …

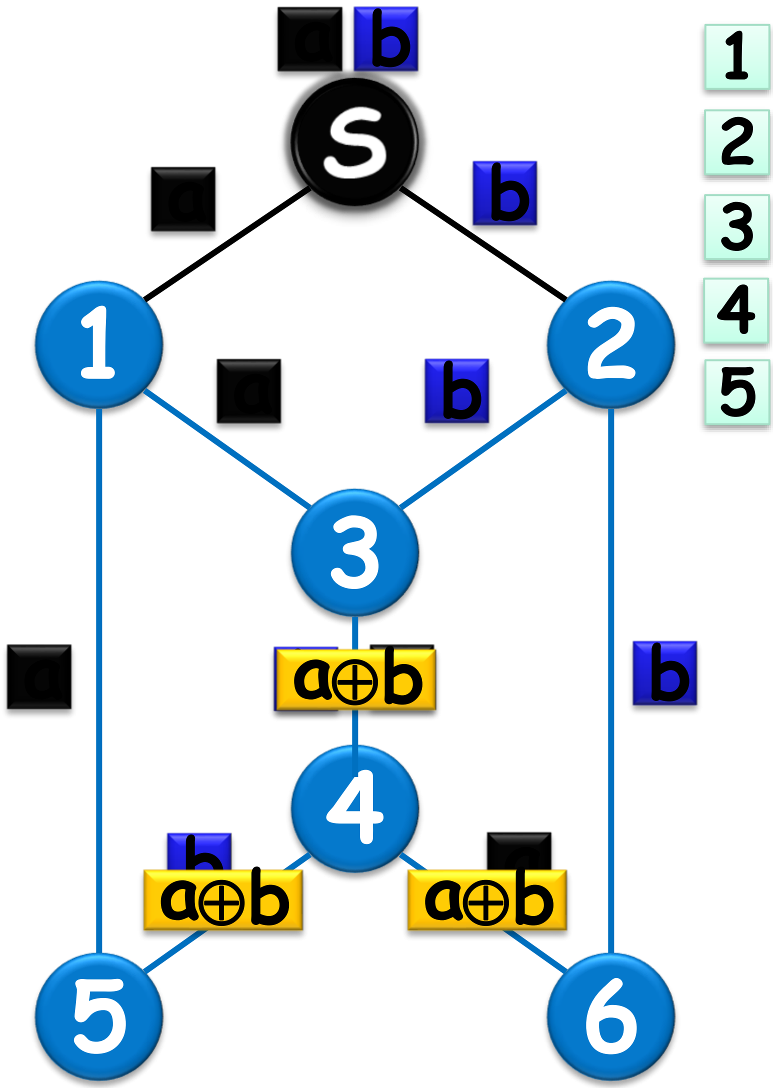
Network Coding

- A technique to improve:

  1. network throughput
  2. efficiency
  3. scalability

  …

- Information is coded at potentially every node



Without animation …

Network Coding

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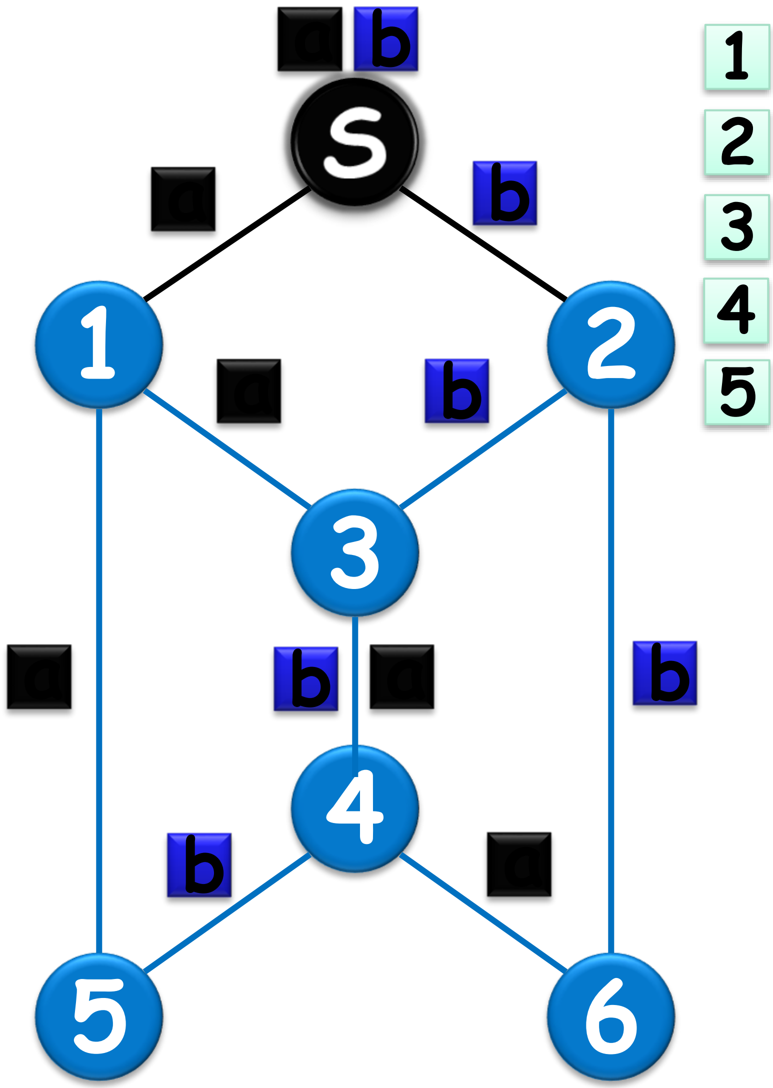

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