Assignment 1
for Formal Languages and Automata Theory
TDDD85

Deadline: Monday week 18 (28 April 2014), 12.00

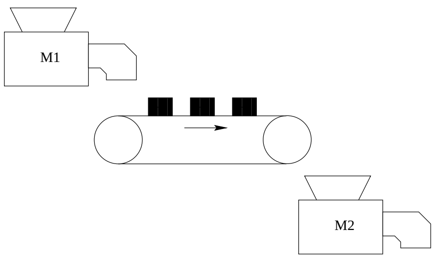
For all the problems below it is not sufficient just to give a solution. **Justify your answers**. In the final exam unexplained answers will be granted 0 points.

To pass a problem in a homework assignment (as opposed to an exam) the solution doesn't need to be complete or close to complete, but it must be clear that you have understood the concepts and done a serious attempt to solve each problem.

The solutions should be handed in to Jonas Wallgren or Johannes Schmidt. Use the "från studenter till lärare" postbox in front of "Java Cafe" in building B or hand them in at a lecture or a tutorial.

Your solutions can be written in Swedish or English.

1. In a factory, there is a conveyor connecting two machines M1 and M2. Some items are to be processed by M1 and then transported to M2. At most 3 items can be placed simultaneously on the conveyor.



   (a) Assume that a computer receives symbol b whenever an item is placed on the conveyor (by machine M1), and symbol c whenever an item is taken from the conveyor (by machine M2). Consider the language L_1 over $\{b, c\}^*$ of the sequences received by the computer. (Assume that at the beginning and at the end the conveyor is empty). So, for instance, all the strings in L_1 have the same number of b's and c's. Strings bc, $bbcbcc$, $bbbccc$ are in L_1, but any string containing a substring $bbbb$ or $bbbcbb$ is not.

   Provide a deterministic finite automaton for L_1. Using the equation solving approach presented at the lectures, construct from the automaton a regular expression for L_1. (You may begin with a common sense construction and then compare both results).

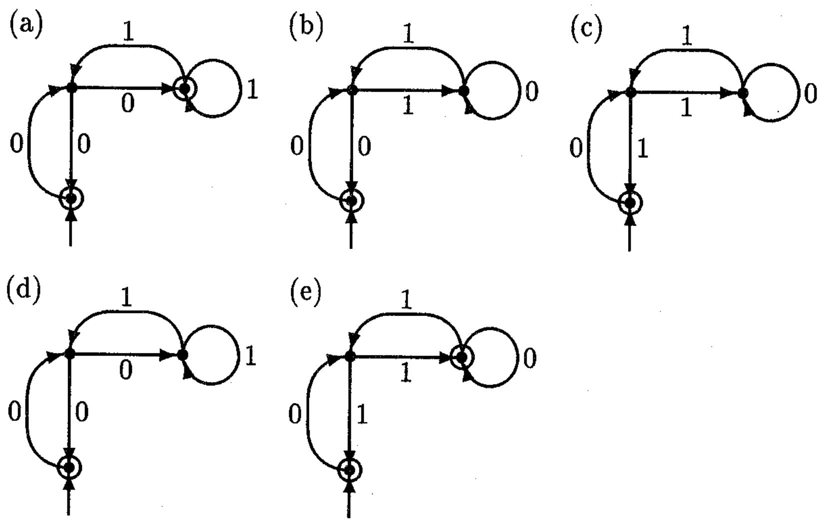
(b) Now assume that each machine can process at most one item at a time. Assume that the computer receives symbol a whenever an item enters machine M1. Consider the corresponding language L_2 over $\{a, b, c\}^*$, assuming that at the beginning and at the end both the conveyor and the machine is empty. Provide a DFA for L_2.

(c) Assume that symbol d is sent whenever an item leaves M2. Provide a DFA for the corresponding language L_3 over $\{a, b, c, d\}^*$. Hint: As in the last case the DFA may be rather big, do not proceed without finding some regular pattern in its diagram. Beginning with a conveyor of a smaller capacity may be helpful.

(d) Are you able to find out whether your automata are minimal? Hint: Straightforwardly applying the minimization algorithm probably results in too much work.

2. Match each NFA with an equivalent regular expression from the list below. Explain briefly your choice. Explain briefly why the remaining regular expressions are not equivalent to any of the automata.

The diagram is from the textbook by D. Kozen.



(i) $\epsilon + 1(10^*1 + 10)^*0$

(ii) $\epsilon + 0(01^*1 + 00)^*01^*$

(iii) $\epsilon + 0(10^*1 + 10)^*10^*$

(iv) $\epsilon + 0(10^*1 + 00)^*0$

(v) $\epsilon + 0(10^*1 + 10)^*0$

(vi) $\epsilon + 0(01^*1 + 00)^*0$

(vii) $\epsilon + 0(10^*1 + 10)^*1$

3. Construct a DFA equivalent to the following NFA-ϵ:

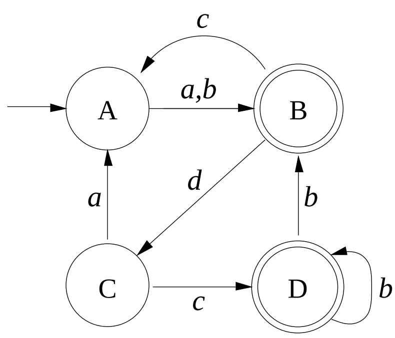
	a	b	ϵ
$\to$1	$\emptyset$	$\emptyset$	{2,5}
2 F	{3}	$\emptyset$	$\emptyset$
3	$\emptyset$	{4}	$\emptyset$
4	$\emptyset$	{2}	$\emptyset$
5	{5}	$\emptyset$	{6}
6 F	$\emptyset$	{7}	$\emptyset$
7	{6}	$\emptyset$	$\emptyset$

4. Write a regular expression for the language

$$L = \left\{ 0x \in \{0,1\}^* \;\middle|\; \begin{array}{l} 0x \text{ contains an equal number of occurrences} \\ \text{of the substrings 01 and 10} \end{array} \right\}.$$

E.g. $010 \in L$ since 010 contains a single 01 and a single 10 as substrings; but $01101 \notin L$ because 01101 contains two 01's and one 10.

5. Consider the finite automaton given by the diagram. Is the automaton deterministic? Using a standard method construct a regular expression for the language defined by the automaton.



6. Prove that the language

$$\{\, a^r b^s c^t \mid r, s, t \in \mathbb{N} \text{ and } s = r + t \,\}$$

is not regular. Use the pumping lemma or a reasoning similar to the proof of the lemma.

Hint: proceed similar as with the language $\{\, a^n b^n \mid n \in \mathbb{N} \,\}$.