# Functional Dependencies and Normalization 

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* slides kindly provided by Vaida Jakonienė


## Overview



## Good Design

- Can we be sure that a translation from EER-diagram to relational tables results in good database design?
- Confronted with a deployed database, how can we be sure that it is well-designed?
- What is good database design?
$\square$ Four informal measures
$\square$ Formal measure: normalization


## Informal design guideline

- Easy to explain semantics of the relation schema
- Reducing redundant information in tuples

Redundancy causes update anomalies:
$\square$ Insertion anomalies
$\square$ Deletion anomalies
$\square$ Modification anomalies

| EMP( | EMPID, | EMPNAME, | DEPTNAME, | DEPTMGR) |
| :---: | :--- | :--- | :--- | :---: |
| 123 | Smith | Research | 999 |  |
| 333 | Wong | Research | 999 |  |
| 888 | Borg | Administration | null |  |

## Informal design guideline

- Reducing NULL values in tuples

Why
$\square$ Efficient use of space
$\square$ Avoid costly outer joins
$\square$ Ambiguous interpretation (unknown vs. doesn't apply).

- Disallow the possibility of generating spurious tuples
$\square$ Figures 10.5 and 10.6: cartesian product results in incorrect tuples
$\square$ Only join on foreign key/primary key-attributes
$\square$ Lossless join property: guarantees that the spurious tuple generation problem does not occur


## Functional dependencies (FD)

- Let $R$ be a relational schema with the attributes $A_{1}, \ldots, A_{n}$ and let $X$ and $Y$ be subsets of $\left\{A_{1}, \ldots, A_{n}\right\}$.
- Let $r(R)$ denote a relation in relational schema $R$.

We say that X functionally determines Y ,

$$
\mathrm{X} \rightarrow \mathrm{Y}
$$

if for each pair of tuples $t_{1}, t_{2} \in r(R)$ and for all relations in $r(R)$ : If $\mathrm{t}_{1}[\mathrm{X}]=\mathrm{t}_{2}[\mathrm{X}]$ then we must also have $\mathrm{t}_{1}[\mathrm{Y}]=\mathrm{t}_{2}[\mathrm{Y}]$

- Despite the mathematical definition an FD cannot be determined automatically. It is a property of the semantics of attributes.


## Inference rules

1. If $X \supseteq Y$ then $X \rightarrow Y$, or $X \rightarrow X$ (reflexive rule)
2. $X \rightarrow Y \mid=X Z \rightarrow Y Z$ (augmentation rule)
3. $X \rightarrow Y, Y \rightarrow Z \mid=X \rightarrow Z$ (transitive rule)
4. $\mathrm{X} \rightarrow \mathrm{YZ} \mid=\mathrm{X} \rightarrow \mathrm{Y}$ (decomposition rule)
5. $X \rightarrow Y, X \rightarrow Z \mid=X \rightarrow Y Z$ (union or additive rule)
6. $\mathrm{X} \rightarrow \mathrm{Y}, \mathrm{WY} \rightarrow \mathrm{Z} \mid=\mathrm{WX} \rightarrow \mathrm{Z}$ (pseudotransitive rule)

## Inference rules

- Textbook, page 341:
$" \ldots X \rightarrow A$, and $Y \rightarrow B$ does not imply that $X Y \rightarrow A B$."
Prove that this statement is wrong.
- Prove inference rules 4,5 and 6 by using only inference rules 1, 2 and 3 .


## Definitions

- Superkey: a set of attributes uniquely (but not minimally!) identifying a tuple of a relation.
- Key: A set of attributes that uniquely and minimally identifies a tuple of a relation.
- Candidate key: If there is more than one key in a relation, the keys are called candidate keys.
- Primary key: One candidate key is chosen to be the primary key.
- Prime attribute: An attribute $\boldsymbol{A}$ that is part of a candidate key $\boldsymbol{X}$ (vs. nonprime attribute)


## Normal Forms

- 1NF, 2NF, 3NF, BCNF (4NF, 5NF)
- Minimize redundancy


## - Minimize update anomalies

- Normal form $\uparrow=$ redundancy and update anomalies $\downarrow$ and relations become smaller.
- Join operation to recover original relations.


## 1NF

- 1NF: The relation should have no non-atomic values.

| $\mathbf{R}_{\text {non1NF }}$ |  |  |
| :--- | :--- | :--- |
| $\underline{I D}$ | Name | LivesIn |
| $\underline{100}$ | Pettersson | $\{$ Stockholm, Linköping $\}$ |
| $\underline{101}$ | Andersson | $\{$ Linköping $\}$ |
| $\underline{102}$ | Svensson | $\{$ Ystad, Hjo, Berlin $\}$ |


| $\mathbf{R 1}_{\text {1NF }}$ |
| :--- |
| $\underline{\text { ID }}$ |
| Name |
| 100 |



## 2NF

- 2NF: no nonprime attribute should be functionally dependent on a part of a candidate key (= partial dependency).
$\mathrm{R}_{\text {non2NF }}$

| EmpID | Dept | Work\% | EmpName |
| :--- | :--- | :--- | :--- |
| $\underline{100}$ | Dev | 50 | Baker |
| $\underline{100}$ | $\underline{\text { Support }}$ | 50 | Baker |
| $\underline{200}$ | $\underline{\text { Dev }}$ | 80 | Miller |



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## 2NF

- No 2NF: A part of a candidate key can have repeated values in the relation and, thus, so can have the nonprime attribute, i.e. redundancy + insertion and modification anomalies.
- An FD $X \rightarrow Y$ is a full functional dependency (FFD) if removal of any attribute $A_{i}$ from $X$ means that the dependency does not hold any more.
- 2NF: Every nonprime attribute is fully functionally dependent on every candidate key.


## 3NF

- 3NF: 2NF + no nonprime attribute should be functionally dependent on a set of nonprime attributes

| $\mathbf{R}_{\text {non3NF }}$ |  |  |  |
| :--- | :--- | :--- | :--- |
| $\underline{\text { ID }}$ | Name | Zip | City |
| $\underline{100}$ | Andersson | 58214 | Linköping |
| $\underline{101}$ | Björk | 10223 | Stockholm |
| $\underline{102}$ | Carlsson | 58214 | Linköping |


| $\Rightarrow$ | R1 ${ }_{\text {3NF }}$ |  |  | R2 ${ }_{3 \text { 3F }}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Normalization | ID | Name | Zip | Zip | City |
|  | 100 | Andersson | 58214 | 58214 | Linköping |
|  | $\underline{101}$ | Björk | 10223 | 10223 | Stockholm |
|  | 102 | Carlsson | 58214 |  |  |

## 3NF

- No 3NF (but 2NF): A set of nonprime attributes can have repeated values in the relation and, thus, so can have the nonprime attribute, i.e. redundancy + insertion and modification anomalies.
- An FD $X \rightarrow Y$ is a transitive dependency if there is a set of nonprime attributes $Z$ such that both $X \rightarrow Z$ and $Z \rightarrow Y$ hold.
- 3NF: 2NF + no nonprime attribute is transitively dependent on any candidate key.


## Little summary

## $X \rightarrow A$

2 NF and 3 NF do nothing if A is prime.
Assume A is nonprime.
$2 N F=$ decompose if $X$ is part of a candidate key.
3NF = decompose if $X$ is part of a candidate key
or X is nonprime, i.e. if $\mathrm{X} \rightarrow \mathrm{A}$ is partial or
transitive.
$3 \mathrm{NF}=\mathrm{X}$ is a superkey or A is prime.
Should $A$ be discriminated for being prime ?

## Boyce-Codd Normal Form

- BCNF: Every determinant is a superkey (in practice: every determinant is a candidate key)
- BCNF = decompose if $X \rightarrow A$ is such that $X$ is not a superkey and $A$ is a prime attribute.
- Example: Given $R(\underline{A}, \mathrm{~B}, \mathrm{C}, \mathrm{D})$ and $A B \rightarrow C D, C \rightarrow B$. Then $R$ is in 3NF but not in BCNF
$\square \mathrm{C}$ is a determinant but not a superkey (tuples are not uniquely identified in R)


## BCNF: Example

At a gym, an instructor is leading an activity in a certain room at a certain time.
$\mathrm{R}_{\text {nonbcin }}$

| Time | Room | Instructor | Activity |
| :--- | :--- | :--- | :--- |
| Mon 17.00 | Gym | Tina | IronWoman |
| Mon 17.00 | Mirrors | Anna | Aerobics |
| Tue 17.00 | Gym | Tina | Intro |
| Tue 17.00 | Mirrors | Anna | Aerobics |
| Wed 18.00 | Gym | Anna | IronWoman |

Normalization: Example

Given universal relation
R(PID, PersonNamn, Land, Kontinent, KontinentYta, AntalBesökILandet)

■ Functional dependencies?

- Keys?


## Properties of decomposition

- Keep all attributes from the universal relation R.
- Preserve the identified functional dependencies.
- Lossless join
$\square$ It must be possible to join the smaller tables to arrive at composite information without spurious tuples.


## Normalization: Example

PID $\rightarrow$ PersonNamn
PID, Land $\rightarrow$ AntalBesökILande $\dagger$
Land $\rightarrow$ Kontinent
Kontinent $\rightarrow$ KontinentYta

- Based on FDs, what are keys for R?
- Use inference rules


## Normalization: Example

```
Land }->\mathrm{ Kontinent, Kontinent }->\mathrm{ KontinentYta,
    then
Land }->\mathrm{ Kontinent, KontinentYta (transitive rule)
PID, Land }->\mathrm{ Kontinent, KontinentYta (augmentation rule),
PID, Land }->\mathrm{ PersonNamn (augmentation rule),
PID, Land }->\mathrm{ AntalBesökILandet,
    then
PID, Land }->\mathrm{ Kontinent, KontinentYta, PersonNamn,
    AntalBesökILandet (additive rule)
```

Person, Land is the key for R.

## Normalization: Example

Is
R (PID, Land, Kontinent, KontinentYta, PersonNamn, AntalBesökILandet) in 2NF?
No, PersonNamn depends on a part of the key (PID), then R1(PID, PersonNamn)
R2(PID, Land, Kontinent, KontinentYta, AntalBesökILandet)

Is R2 in 2NF?
No, Kontinent and Kontinentyta depend on a part of the key (Land), then R1(PID, PersonNamn)
R21(Land, Kontinent, KontinentYta) R22(PID, Land, AntalBesökILandet) $\rightarrow$ R1, R21, R22 are in 2NF

## Are R1, R21, R22 in 3NF?

R22(PID, Land, AntalBesökILandet),
R1 (PID, PersonNamn):
Yes, a single nonprime attribute, no transitive dependencies.

R21(Land, Kontinent, KontinentYta):
No, Kontinent defines KontinentYta, then
R211(Land, Kontinent)
R212(Kontinent, KontinentYta)
$\rightarrow$ R1, R22, R211, R212 are in 3NF

Are R1, R22, R211, R212 in BCNF?

## BCNF: Every determinant is a superkey

R22(PID, Land, AntalBesökILandet),
R1(PID, PersonNamn):
R211(Land, Kontinent)
R212(Kontinent, KontinentYta)
$\rightarrow$ Yes (don't be confused by candidate keys!)
Can the universal relation $R$ be reproduced from $R 1, R 22$,
R211 and R212 without spurious tuples?

## Summary and onen issues

- Good desigr relations
- Functiona forms, are world knol automated
- Are high nomish aito alecioces when it comes to pefforratim
$\square$ No, denormalization may be required.


## 1. Which normal form?

- The database contains data about cars, their owners and when the car was registered for that owner.

| PersonID | FirstName | LastName | LicensePlate | RegistrationDate | Birthdate |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1000 | Ann | Anderson | ABC123 | $2004-10-12$ | $1981-04-04$ |
| 1010 | Ben | Benson | DEF234 | $2003-02-12$ | $1945-12-12$ |
| 1000 | Ann | Anderson | ABC123 | $2001-04-23$ | $1981-04-04$ |

## 2. Which normal form?

- A database contains data about registered cars and their make (type).

| LicensePlate | Type | Maker |
| :---: | :---: | :---: |
| ABC123 | C70 | Volvo |
| DEF234 | S40 | Volvo |
| FGH345 | Corolla | Toyota |

## 3. Which normal form?

- The database contains data about flights, aircrafts and their pilots. Flights use different aircrafts depending on the number of booked passengers.

| Date | Flight | Aircraft | Pilot |
| :---: | :---: | :---: | :---: |
| 13-Jan-2005 | TGU7 | Airbus 300 | John |
| 14-Jan-2005 | TGU7 | Boeing 747 | Daniel |
| 12-Jan-2005 | SKX6 | Airbus 300 | John |
| 13-Jan-2005 | SKX6 | Boeing 747 | Ann |
| 14-Jan-2005 | SKX6 | Fokker 50 | Mary |

