2 DFA, NFA, and NFA$_\epsilon$

2.1 Determine which of the strings below belong to the language $L(M_1)$ (the DFA is given in figure 1). Also give $L(M_1)$ in set notation.

a) 001

b) 001011011011

c) 00101101001

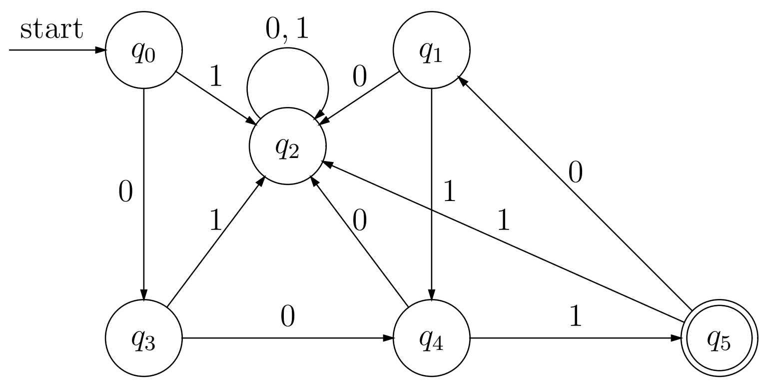

Figure 1: M_1

2.2 Determine which of the strings below belong to the language $L(M_2)$ (the NFA is given in figure 2). Also give $L(M_2)$ in set notation.

a) 11110101

b) 1111111

c) 101101101111

d) 10110010011

2.3 Determine which of the strings below belong to the language $L(M_3)$ (the NFA$_\epsilon$ is given in figure 3). Also give $L(M_3)$ in set notation.

a) 11111

b) 1101011

c) 1011100

d) 0101111

2.4 For each of the following languages, construct a DFA that accepts the language.

a) $L_1 = \{x \in \{0,1\}^* \mid x \text{ ends in } 00\}$

b) $L_2 = \{x \in \{0,1\}^* \mid x = (01)^n, n \geq 0\}$

c) $L_3 = \{x \in \{0,1\}^* \mid \text{every } 0 \text{ is immediately followed by } 1\}$

2.5 Two automata M and M' are equivalent if they accept the same language, i.e. $L(M) = L(M')$.

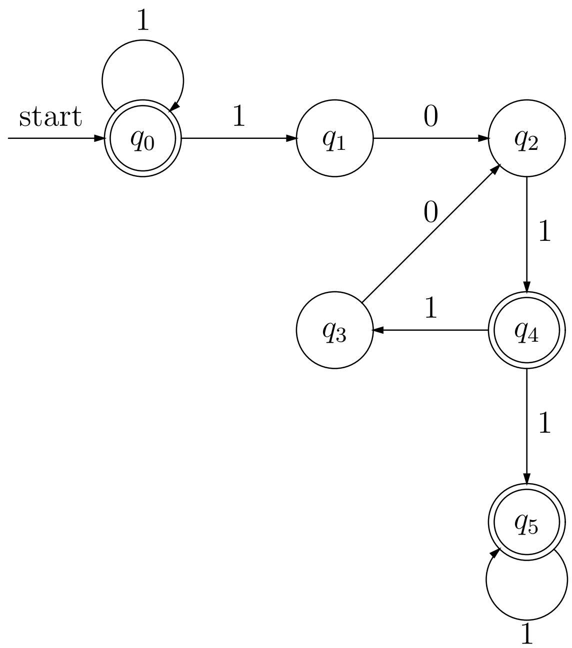

Figure 2: M_2

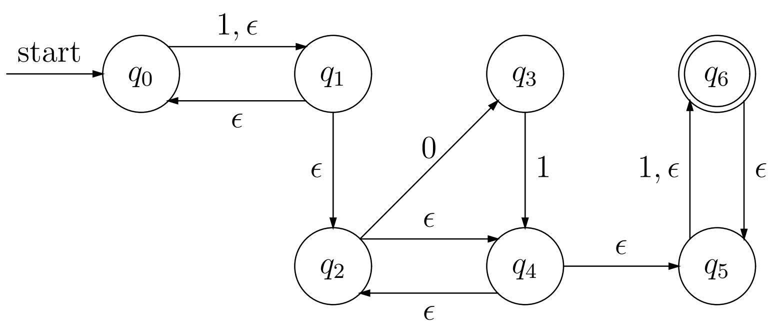

Figure 3: M_3

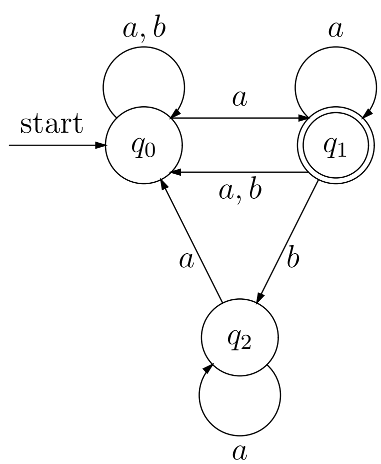

Figure 4: M_4

a) Given the NFA in figure 4, construct an equivalent DFA.
b) Given the NFA in figure 5, construct an equivalent DFA.
c) Given the NFA in figure 6, construct an equivalent DFA.

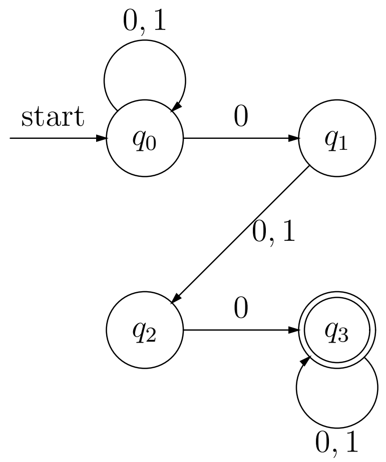

Figure 5: M_5

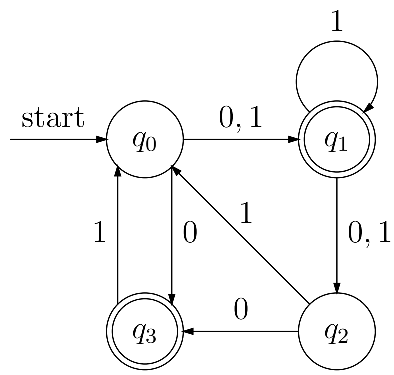

Figure 6: M_6

2.6 a) Given the NFA$_\epsilon$ in figure 7, construct an equivalent DFA.

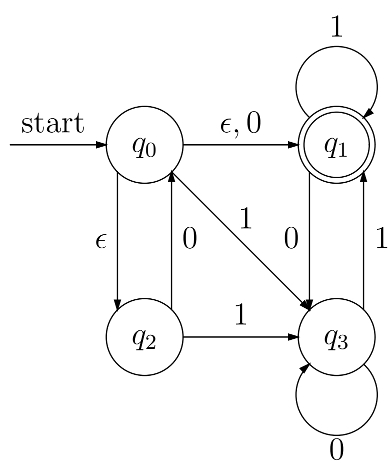

Figure 7: M_7

b) Given the NFA$_\epsilon$ in figure 8, construct an equivalent DFA.

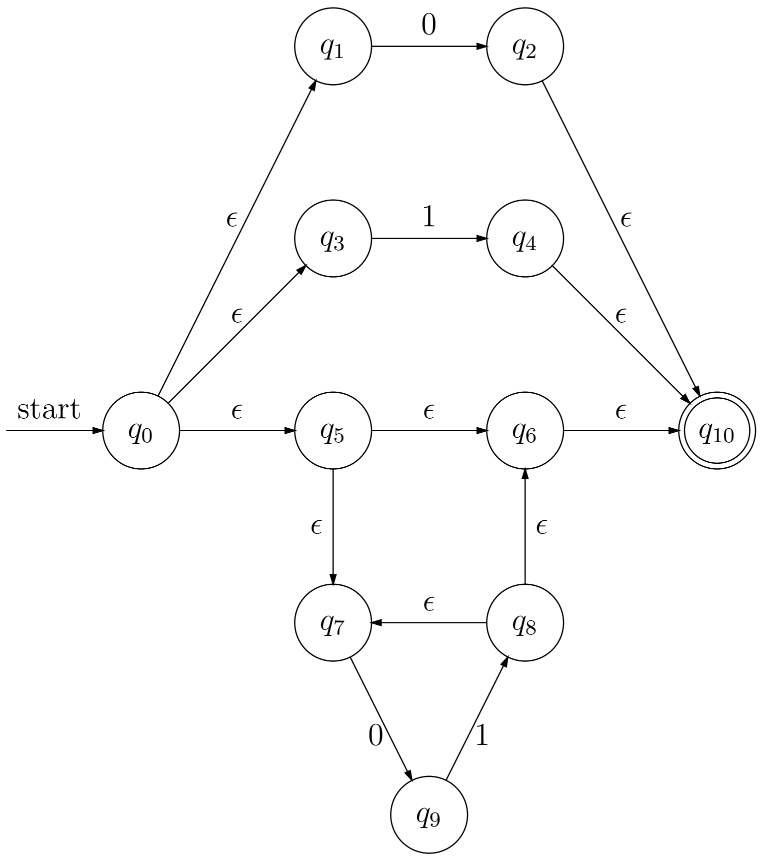

Figure 8: M_8

2.4 a) An example of a DFA M_{15} such that $L(M_{15}) = L_1$ is given in figure 15.

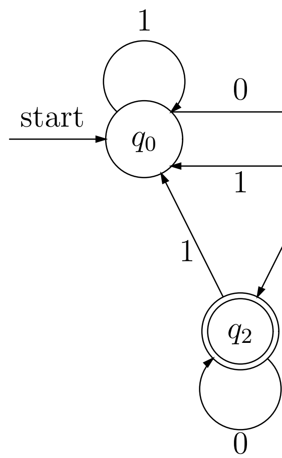

Figure 15: M_{15}

Specification of the states:

q_0**:** The last symbol read, if any, is 1.

q_1**:** The last symbol read is 0; the last but one, if any, is 1.

q_2**:** The last two symbols read are 00.

b) An example of a DFA M_{16} such that $L(M_{16}) = L_2$ is given in figure 16.

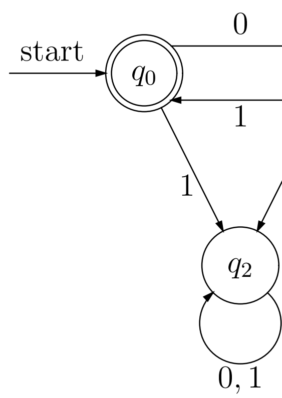

Figure 16: M_{16}

q_0**:** Any number (incl. 0) of 01:s read.

q_1**:** Any number (incl. 0) of 01:s followed by 0 read.

q_2**:** Something else read.

c) An example of a DFA M_{17} such that $L(M_{17}) = L_4$ is given in figure 17.

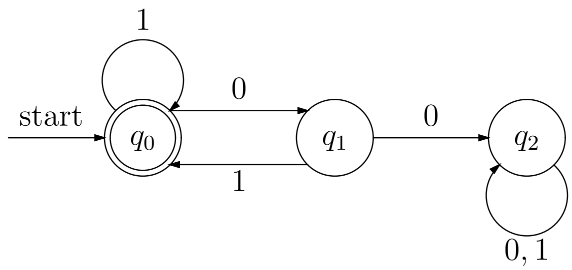

Figure 17: M_{17}

q_0**:** The last symbol read, if any, is 1; any previous 0 is immediately followed by 1.

q_1**:** The last symbol read is 0, any previous 0 is immediately followed by 1.

q_2**:** 00 has been read.

2.5 Following [Hopcroft&Ullman] we often use [ ] instead of { } to denote a set of states which is a state of a DFA.

a) δ_{18} is given in table 1.

State	*Input*	
	a	b
$[q_0]$	$[q_0, q_1]$	$[q_0]$
$[q_0, q_1]$	$[q_0, q_1]$	$[q_0, q_2]$
$[q_0, q_2]$	$[q_0, q_1, q_2]$	$[q_0]$
$[q_0, q_1, q_2]$	$[q_0, q_1, q_2]$	$[q_0, q_2]$

Table 1: δ_{18}

The new set of final states is $F' = \{x \in 2^Q \mid x \cap \{q_1\} \neq \emptyset\} = \{[q_0, q_1], [q_0, q_1, q_2]\}$. Let

$[q_0] = A$
$[q_0, q_1] = B$
$[q_0, q_2] = C$
$[q_0, q_1, q_2] = D$

The transition diagram for the DFA is given in figure 18.

b) δ_{19} is given in table 2.

The new set of final states is $F' = \{x \in 2^Q \mid x \cap \{q_3\} \neq \emptyset\} = \{[q_0, q_3], [q_0, q_1, q_3], [q_0, q_2, q_3], [q_0, q_1, q_2, q_3]\}$. Let

$[q_0] = A$
$[q_0, q_1] = B$
$[q_0, q_1, q_2] = C$
$[q_0, q_2] = D$
$[q_0, q_1, q_2, q_3] = E$

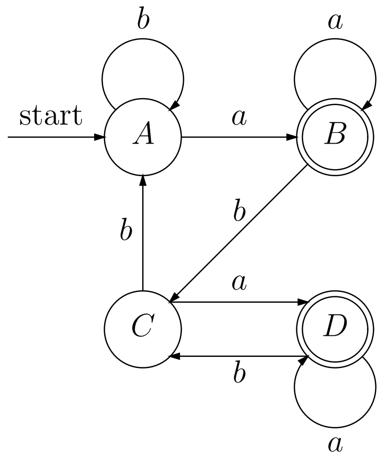

Figure 18: M_{18}

State	*Input*	
	0	1
$[q_0]$	$[q_0, q_1]$	$[q_0]$
$[q_0, q_1]$	$[q_0, q_1, q_2]$	$[q_0, q_2]$
$[q_0, q_1, q_2]$	$[q_0, q_1, q_2, q_3]$	$[q_0, q_2]$
$[q_0, q_2]$	$[q_0, q_1, q_3]$	$[q_0]$
$[q_0, q_1, q_2, q_3]$	$[q_0, q_1, q_2, q_3]$	$[q_0, q_2, q_3]$
$[q_0, q_2, q_3]$	$[q_0, q_1, q_3]$	$[q_0, q_3]$
$[q_0, q_1, q_3]$	$[q_0, q_1, q_2, q_3]$	$[q_0, q_2, q_3]$
$[q_0, q_3]$	$[q_0, q_1, q_3]$	$[q_0, q_3]$

Table 2: δ_{19}

$[q_0, q_2, q_3] = F$
$[q_0, q_1, q_3] = G$
$[q_0, q_3] = H$

The transition diagram for the DFA is given in figure 19.

c) δ_{20} is given in table 3.
The new set of final states is $F' = \{x \in 2^Q \mid x \cap \{q_1, q_3\} \neq \emptyset\} = \{[q_1, q_3], [q_1], [q_0, q_1, q_2], [q_1, q_2], [q_3], [q_1, q_2, q_3], [q_2, q_3]\}$ Let

$[q_0] = A$
$[q_1, q_3] = B$
$[q_1] = C$
$[q_2] = D$
$[q_0, q_1, q_2] = E$
$[q_1, q_2] = F$
$[q_3] = G$
$[q_1, q_2, q_3] = H$
$[q_2, q_3] = I$
$\emptyset = J$

The transition diagram for the DFA is given in figure 20.

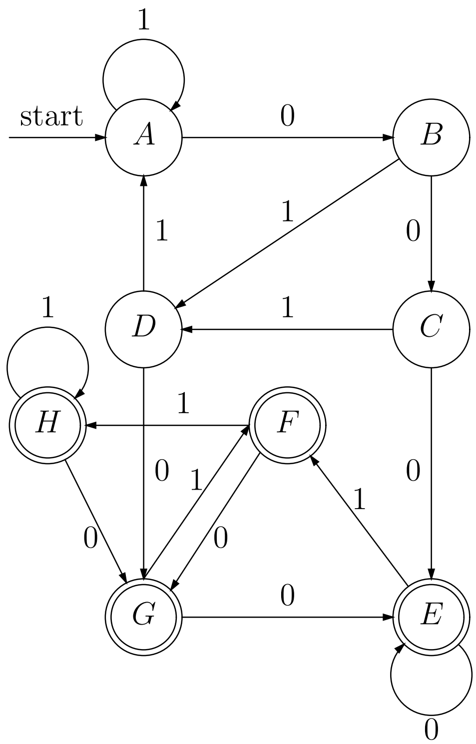

Figure 19: M_{19}

State	*Input*	
	0	1
$[q_0]$	$[q_1, q_3]$	$[q_1]$
$[q_1, q_3]$	$[q_2]$	$[q_0, q_1, q_2]$
$[q_1]$	$[q_2]$	$[q_1, q_2]$
$[q_2]$	$[q_3]$	$[q_0]$
$[q_0, q_1, q_2]$	$[q_1, q_2, q_3]$	$[q_0, q_1, q_2]$
$[q_1, q_2]$	$[q_2, q_3]$	$[q_0, q_1, q_2]$
$[q_3]$	$\emptyset$	$[q_0]$
$[q_1, q_2, q_3]$	$[q_2, q_3]$	$[q_0, q_1, q_2]$
$[q_2, q_3]$	$[q_3]$	$[q_0]$
$\emptyset$	$\emptyset$	$\emptyset$

Table 3: δ_{20}

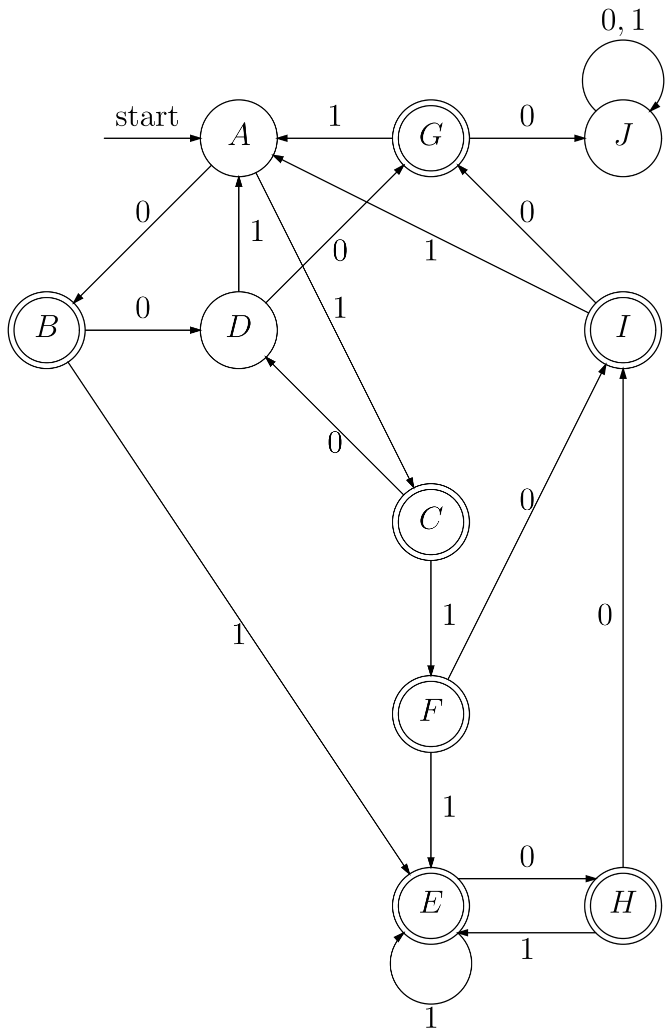

Figure 20: M_{20}

2.6 a) Here we apply the subset construction to an NFA$_\epsilon$. Whenever a state P of the contructed DFA contains a state q (of the NFA$_\epsilon$), all the states reachable from q by ϵ-transitions (in the NFA$_\epsilon$) are also in P.

Table 4 gives the transition function δ_{21} of a DFA corresponding to the NFA$_\epsilon$ from Figure 7. The initial state is $\{q_0, q_1, q_2\}$. All the states are final except $\{q_3\}$.

State	*Input*	
	0	1
$\{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2, q_3\}$	$\{q_1, q_3\}$
$\{q_1, q_3\}$	$\{q_3\}$	$\{q_1\}$
$\{q_1\}$	$\{q_3\}$	$\{q_1\}$
$\{q_3\}$	$\{q_3\}$	$\{q_1\}$
$\{q_0, q_1, q_2, q_3\}$	$\{q_0, q_1, q_2, q_3\}$	$\{q_1, q_3\}$

Table 4: δ_{21}

b) The subset construction results in an DFA with reachable states $\{q_0, q_1, q_3, q_5, q_6, q_7, q_{10}\}$, $\{q_2, q_9, q_{10}\}$, $\{q_4, q_{10}\}$, $\emptyset$, $\{q_6, q_7, q_8, q_{10}\}$, $\{q_9\}$. The initial state is $\{q_0, q_1, q_3, q_5, q_6, q_7, q_{10}\}$. The final states are those containing q_{10}.