3 Regular Expressions and Minimization of DFAs

3.1 Let $r = (1 + 00^*11)(0 + 1(0 + 10)^*11)^*$. Which of the following strings belong to $L(r)$?

a) 010001

b) 00111011

c) 1100110

d) 101100

e) 10011001

3.2 Give regular expressions for the following languages over the alphabet $\{0, 1\}$.

a) The set of all strings ending in 00.

b) The set of all strings in which the substring 00 occurs at most once.

3.3 Construct a NFA_ϵ which accepts the language defined by the regular expression $10 + (0 + 11)0^*1$.

3.4 Show that the equalities below hold for regular expressions. (r, s and t denote arbitrary regular expressions over some alphabet.)

a) $r + t = t + r$

b) $r(s + t) = rs + rt$

c) $(r + \epsilon)^* = r^*$

d) $r\emptyset = \emptyset r = \emptyset$

e) $\emptyset^* = \epsilon$

3.5 Give regular expressions that define

a) the language accepted by the DFA in figure 9,

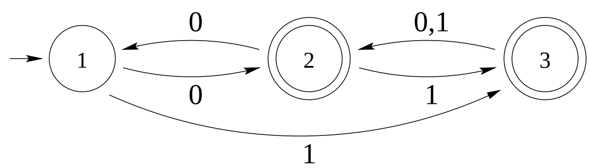

Figure 9: M_9

b) the language accepted by the DFA in figure 10.

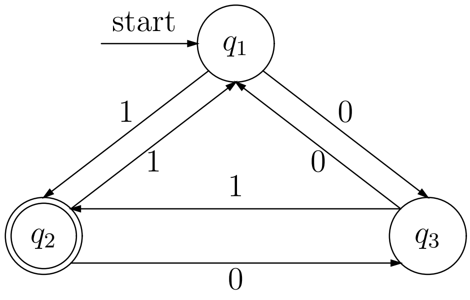

Figure 10: M_{10}

3.6 a) Minimize the DFA in figure 11

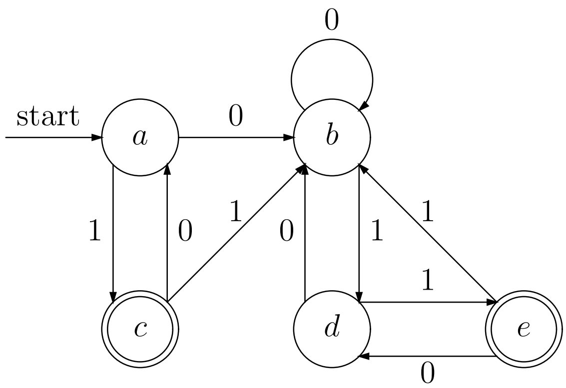

Figure 11: M_{11}

b) Minimize the DFA in figure 12

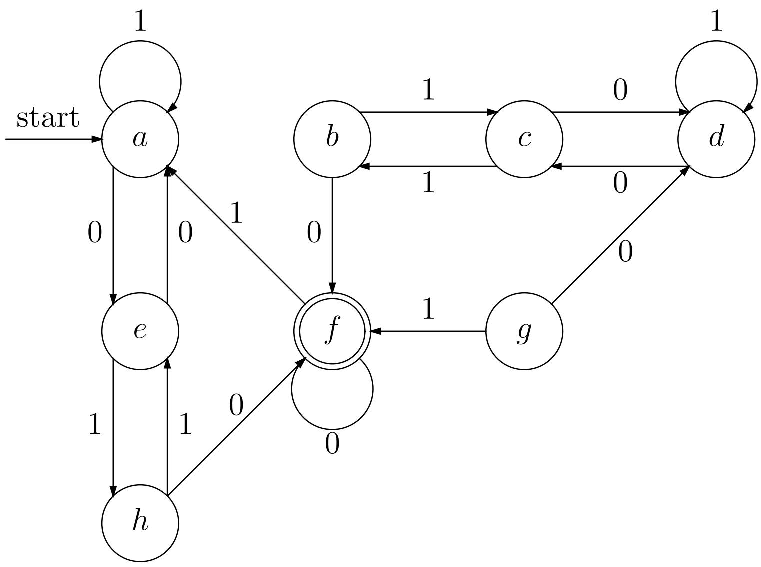

Figure 12: M_{12}

3.2 a) $(0+1)^*00$

b) $(1+01)^*(\epsilon+0+00)(1+10)^*$

3.3 By decomposing the regular expression syntactically according to the recursive definition of regular expressions, an NFA$_\epsilon$ can be constructed systematically in a bottom-up fashion by successively joining NFA$_\epsilon$s corresponding to subexpressions according to the regular operator (*, concatenation, +) in question. The resulting NFA$_\epsilon$ is shown in figure 23.

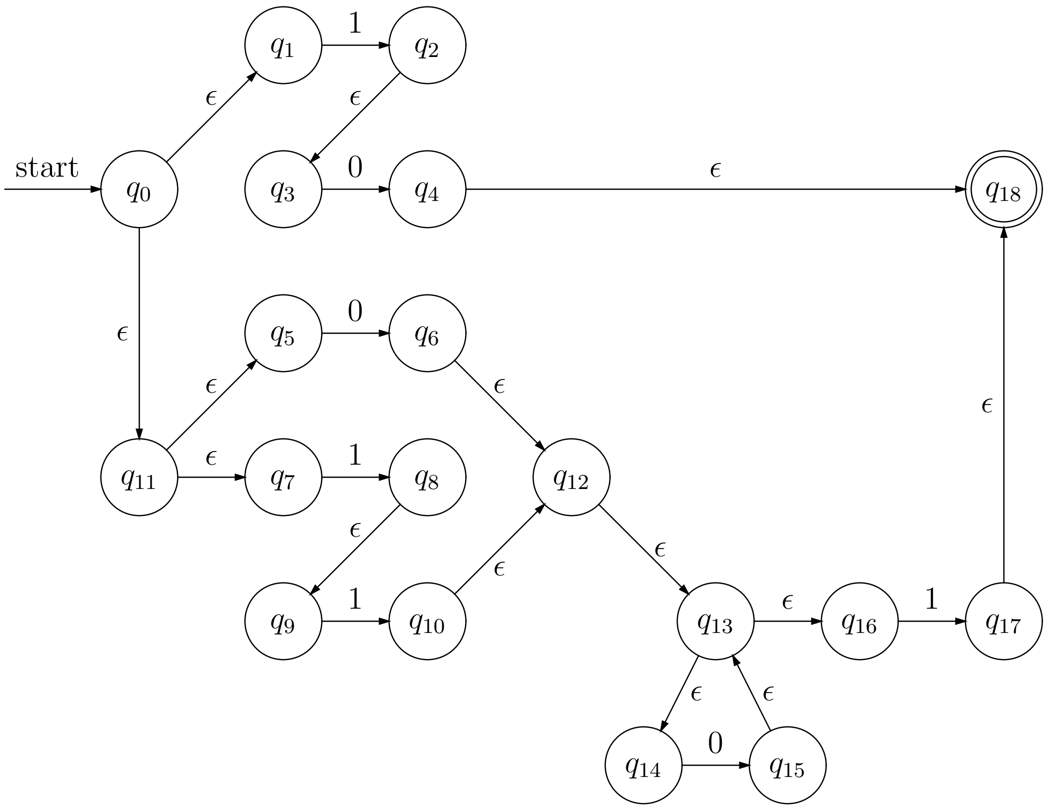

Figure 23: M_{23}

3.5 a) We apply the approach presented at the lecture. It is based on equation solving. The equation system corresponding to the automaton is:

$$\begin{aligned} A_1 &= 0A_2 + 1A_3 \\ A_2 &= 0A_1 + 1A_3 + \epsilon \\ A_3 &= 0A_2 + 1A_2 + \epsilon \end{aligned}$$

Each A_i stands for a regular expression giving the language of strings that move the automaton from state i to some final state. As 1 is the initial state, we have to find A_1.

The main principle is that the solution of the equation $X = AX + B$ is $X = A^*B$ (provided the language described by A does not contain ϵ). For details see the slides from the lecture.

First we eliminate A_2

$$\begin{aligned} A_1 &= 0(0A_1 + 1A_3 + \epsilon) + 1A_3 \\ A_3 &= 0(0A_1 + 1A_3 + \epsilon) + 1(0A_1 + 1A_3 + \epsilon) + \epsilon \end{aligned}$$

This is equivalent to

$$A_1 = 00A_1 + 01A_3 + 0 + 1A_3$$
$$A_3 = (0+1)(0A_1 + 1A_3 + \epsilon) + \epsilon$$

and to

$$A_1 = 00A_1 + (01+1)A_3 + 0$$
$$A_3 = (0+1)0A_1 + (0+1)1A_3 + (0+1+\epsilon)$$

Now we can eliminate A_3. The solution of the last equation is

$$A_3 = ((0+1)1)^*(((0+1)0)A_1 + (0+1+\epsilon))$$

Applying this to the first one we obtain

$$A_1 = 00A_1 + (01+1)((0+1)1)^* \Big(((0+1)0)A_1 + (0+1+\epsilon)\Big) + 0$$

Let us denote $(01+1)((0+1)1)^*$ by B. The previous equation is equivalent to

$$A_1 = 00A_1 + B((0+1)0)A_1 + B(0+1+\epsilon) + 0$$

The solution to this equation is

$$A_1 = \Big((00 + B((0+1)0))\Big)^* \Big(B(0+1+\epsilon) + 0\Big)$$

The latter regular expression defines the language accepted by M_9.

b) Here we use the method presented in [Hopcroft&Ullman]. It is similar to that from [Kozen]. r_{ij}^k denotes the same regular expression as $\alpha_{q_iq_j}^{\{1,\ldots,k\}}$ in the notation of [Kozen].

$L(M) = r_{1,2}^3$

$$r_{1,2}^3 = r_{1,3}^2(r_{3,3}^2)^* r_{3,2}^2 + r_{1,2}^2$$
$$r_{1,3}^2 = r_{1,2}^1(r_{2,2}^1)^* r_{2,3}^1 + r_{1,3}^1$$
$$r_{1,2}^1 = 1$$
$$r_{2,2}^1 = 11 + \epsilon$$
$$r_{2,3}^1 = 10 + 0$$
$$r_{1,3}^1 = 0$$
$$r_{1,3}^2 = 1(11+\epsilon)^*(10+0) + 0 = 1^*0$$
$$r_{3,3}^2 = r_{3,2}^1(r_{2,2}^1)^* r_{2,3}^1 + r_{3,3}^1$$
$$r_{3,2}^1 = 01 + 1$$
$$r_{3,3}^1 = 00 + \epsilon$$
$$r_{3,3}^2 = (01+1)(11+\epsilon)^*(10+0) + (00+\epsilon) = (0+1)1^*0 + \epsilon$$
$$r_{3,2}^2 = r_{3,2}^1(r_{2,2}^1)^* r_{2,2}^1 + r_{3,2}^1$$
$$r_{3,2}^2 = (01+1)(11+\epsilon)^*(11+\epsilon) + 01 + 1 = (01+1)(11)^*$$
$$r_{1,2}^2 = r_{1,2}^1(r_{2,2}^1)^* r_{2,2}^1 + r_{1,2}^1$$
$$r_{1,2}^2 = 1(11+\epsilon)^*(11+\epsilon) + 1 = 1(11)^*$$

$$r_{1,2}^3 = 1^*0((0+1)1^*0)^*(01+1)(11)^* + 1(11)^*$$

3.6 a) A minimal DFA is given in figure 24.

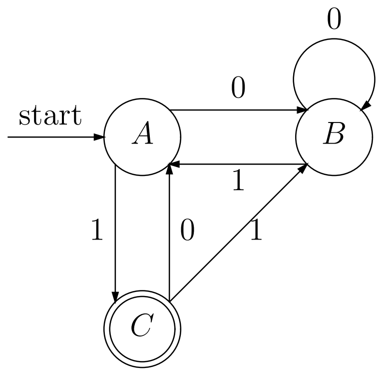

Figure 24: M_{24}

b) A minimal DFA is given in figure 25.

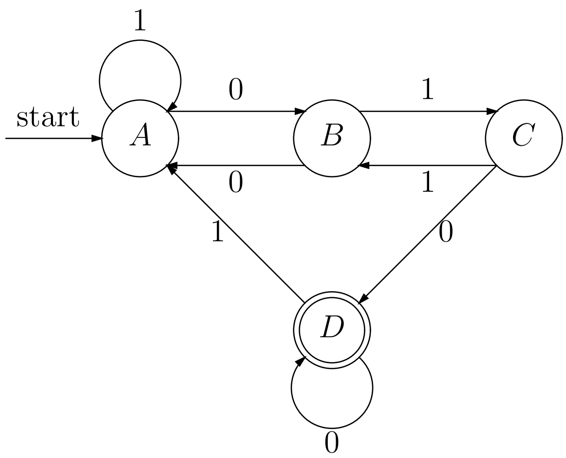

Figure 25: M_{25}