Examination
Formal Languages and Automata Theory
TDDD14/TDDD85

(Formella Språk och Automatateori)

2023–06–01

1. You may answer in Swedish or English.
2. Allowed help materials
   - A sheet of notes - 2 sided A5 or 1 sided A4.
     The contents is up to you. Return the notes together with the exam. The notes should be signed in the same way as the exam sheets and returned together with the exam.
   - English dictionary
3. The maximum number of points is 28. The grades are as follows:

Grade	TDDD14	TDDD85
3	14–19	12–17
4	20–23	18–22
5	24–28	23–28

Make sure that you justify your answers! Unexplained answers will be granted 0 points. (For instance, if you are writing a grammar for a given language then you should also explain that the grammar indeed generates the language. If you apply some known method then you should explain each step. And so on.)

GOOD LUCK !

1. *(2p)* For each pair of regular expressions R_1 and R_2 below, answer whether they generate the same language ($L(R_1) = L(R_2)$). If no, give a string which belongs to one of the languages and does not belong to the other. If yes, show that they are equivalent, e.g., by (1) computing $L(R_1)$ and $L(R_2)$ as far as you can and (2) verifying that the two resulting sets are equal. For the last step, an informal explanation is sufficient.

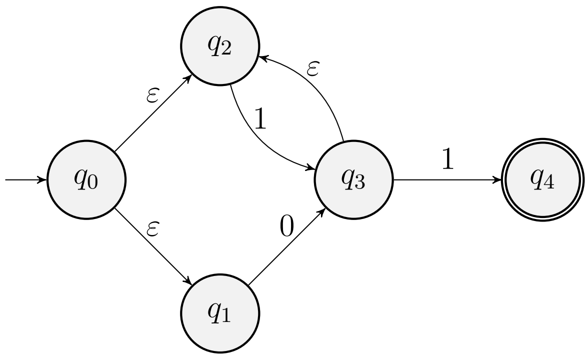
   (a) $a + \varepsilon$ and a.

   (b) $a + \emptyset$ and a.

   (c) $(a + b)b^*$ and $b^*(a + b)b^*$.

   (d) $a(ab)^*b^*$ and $(aab)^*b^*$.

2. *(4p)* Consider the following NFA M.



   (a) What is the language of M?

   (b) Using the subset construction method, construct an equivalent DFA.

Explain each step of the construction. A table/figure without any explanation will be given 0 points.

3. *(4p)* Which of the following claims are true?

   (a) A language is regular if and only if it is finite.

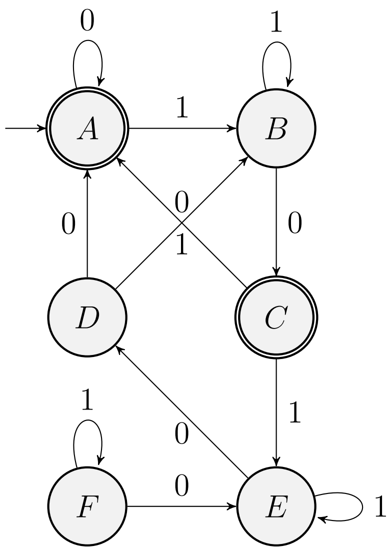
   (b) For any alphabet Σ the language Σ^* is regular.

   (c) Every regular language can be recognized by a pushdown automaton.

   (d) Every Σ^* has a homomorphism to $\emptyset^*$ (where Σ is an arbitrary alphabet).

   Answer each claim by first stating whether the claim is true or false, and then motivate your claim. A substantial motivation is necessary in order to get points.

4. *(4p)* Consider the following DFA.



   (a) Construct the minimal DFA equivalent to this DFA by using the marking algorithm (or a similar algorithm).

   Include all relevant calculations. In particular, if you in an iteration detect that two states are not equivalent, then you have to include the corresponding calculation.

   (b) Recall that $\hat{\delta}(q, x)$ for a string $x \in \{0, 1\}^*$ is the state resulting from "simulating" the DFA on the input string x, starting from the state $q \in \{A, B, C, D, E, F\}$. Define the binary relation R as $\{(q, q') \mid q, q' \in \{A, B, C, D, E, F\}, \hat{\delta}(q, x) \in \{A, C\} \Leftrightarrow \delta(q', x) \in \{A, C\}, x \in \{0, 1\}^*\}$. Describe the relation R by listing all its tuples. Hint: this question can likely answered by the result of the previous question.

5. *(6p)* For a Boolean string $w \in \{0,1\}^*$, let $\overline{w}$ denote the complement. For example, $\overline{010} = 101$. Consider the language $L = \{\, w\overline{w} \mid w \in \{0,1\}^* \}$.

   (a) Prove that L is not regular by using the pumping lemma for regular languages.

   (b) Prove that L is not regular by using the Myhill-Nerode theorem.

6. *(4p)* Is the language $\{\, a^i b^j c^{i+j} \mid i, j > 0 \,\}$ context-free? Either prove your claim by giving a context-free grammar for the language or prove that it is not context-free by using the pumping lemma for context-free languages.

7. *(4p)* For two languages A_1 and A_2 over the same alphabet Σ where $1, 2 \notin \Sigma$, define the *disjoint union* of A_1 and A_2 as $A_1 \coprod A_2 = \{(x1 \mid x \in A\} \cup \{x2 \mid x \in B\}$. For example, if $A_1 = \{aa, b\}$ and $A_2 = \{aa, bb\}$ then $A_1 \coprod A_2 = \{aa1, b1, aa2, bb2\}$.

   (a) Prove or disprove that $A_1 \coprod A_2$ is Turing-decidable whenever A_1 and A_2 are Turing-decidable.

   (b) Prove or disprove that $A_1 \coprod A_2$ is Turing-decidable whenever A_1 and A_2 are Turing-recognizable.