TDDD14/TDDD85

Slides for Lecture 6

Homomorphisms

Christer Bäckström, 2017

Let Σ and Γ be two alphabets.

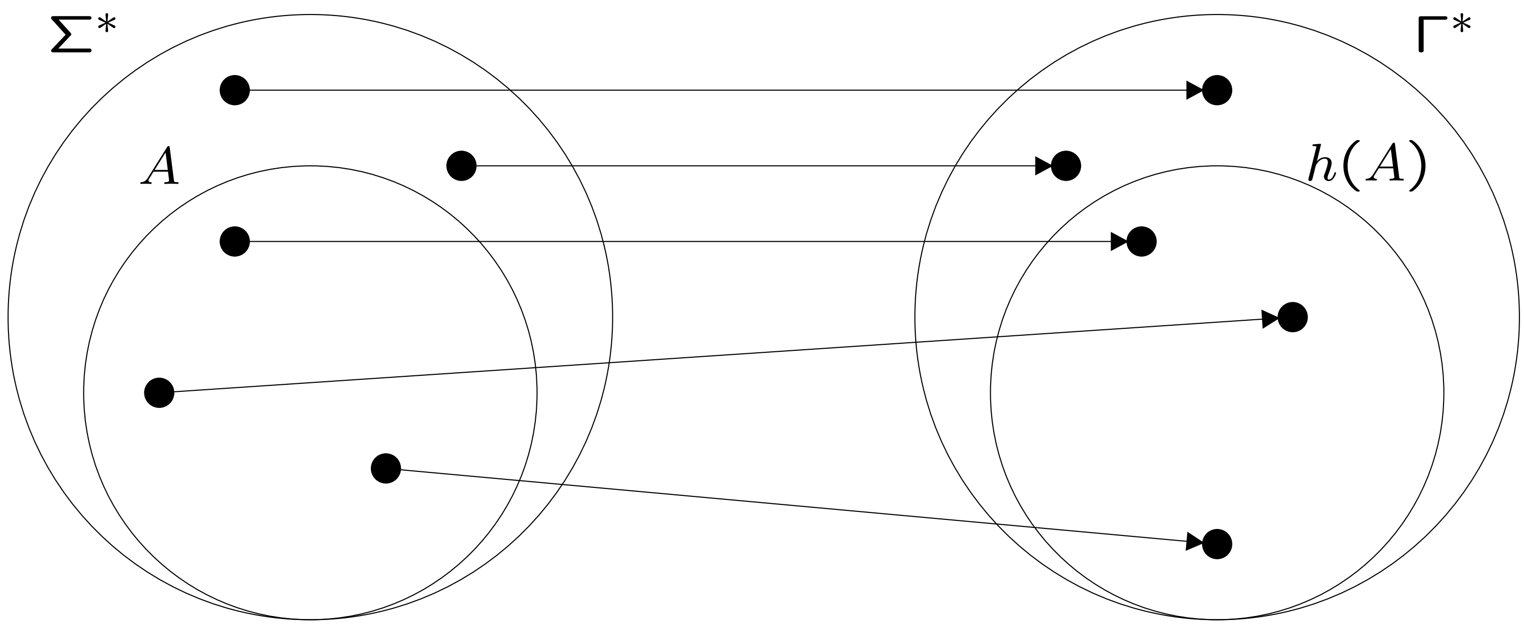
A function $h : \Sigma^* \to \Gamma^*$ is a *homomorphisms* if

1. $h(xy) = h(x)h(y)$ for all $x, y \in \Sigma^*$
2. $h(\varepsilon) = \varepsilon$.

Example: Let $\Sigma = \Gamma = \{a, b, c\}$ and define h as $h(a) = bab$, $h(b) = cbc$ and $h(c) = a$.

Then $h(abc) = h(a)h(b)h(c) = babcbca$.

Homomorphisms are extended to sets in the usual way, i.e.
if $A \subseteq \Sigma^*$, then $h(A) = \{h(x) \mid x \in A\}$.
$h(A)$ is the *image* of A under h.



If A is regular, then $h(A)$ is regular.

Intuition for why $h(A)$ is regular if A is regular.

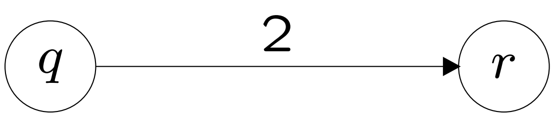
Let $\Sigma = \{0, 1, 2, 3\}$ and let $\Gamma = \{0, 1\}$.

Define h as a binary encoding of Σ, i.e.
$h(0) = 00$, $h(1) = 01$, $h(2) = 10$ and $h(3) = 11$.

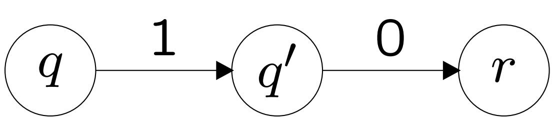
If $A \subseteq \Sigma^*$ is regular, then there is some DFA M for A.

Start with a DFA M for A. Construct a DFA for $h(A)$ as follows:

Given a transition with label 2 in M. Use that $h(2) = 10$

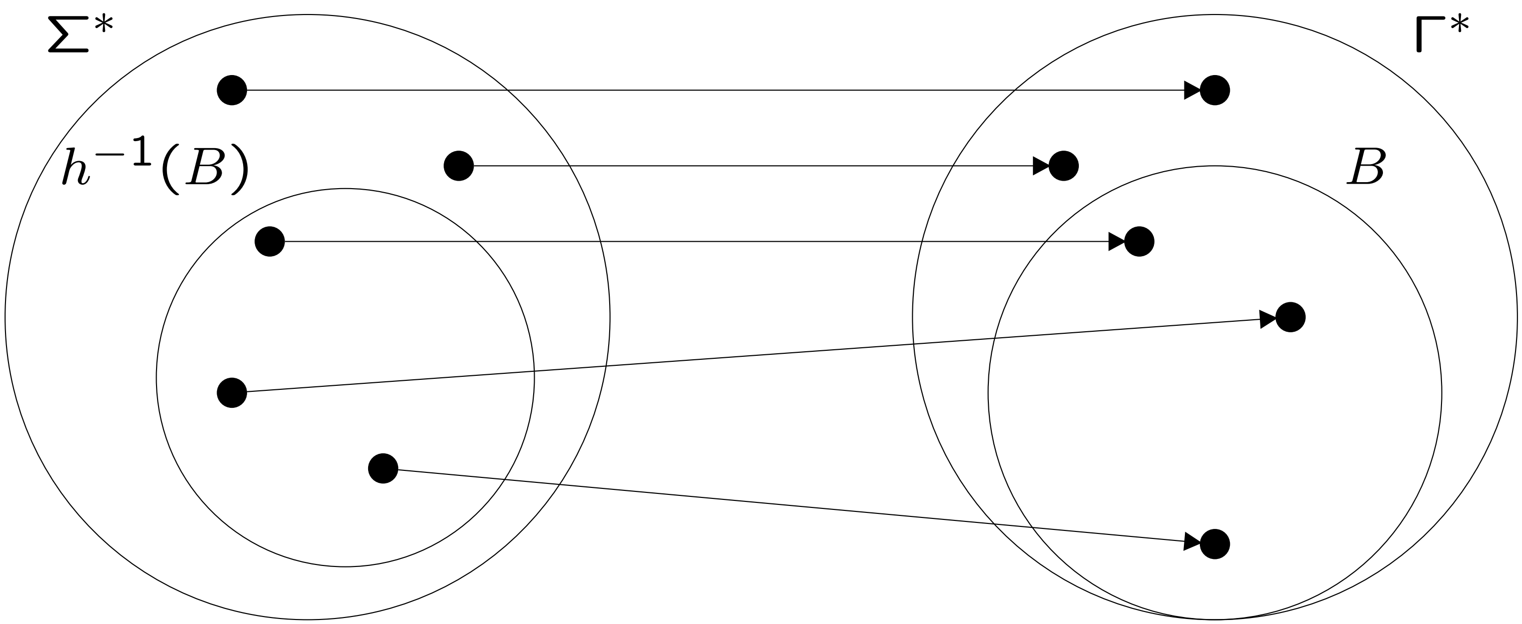


Add a new state and replace with two transitions in N.



This is still a DFA.

We also define the backwards direction:
if $B \subseteq \Gamma^*$, then $h^{-1}(B) = \{x \in \Sigma^* \mid h(x) \in B\}$.
$h^{-1}(B)$ is the *preimage* of B under h.



If B is regular, then $h^{-1}(B)$ is regular.