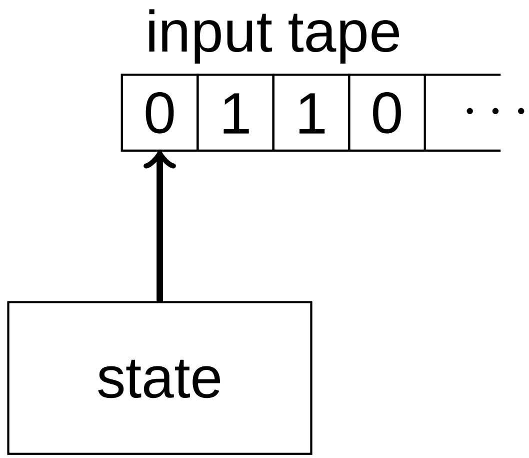
TDDD14/TDDD85
Slides for Lecture 2, 2017

Slides originally for TDDD65 by Gustav Nordh

Minor differences to Kozen:
- Kozen draws states as black dots, not circles.
- Kozen calls the start state s, not q_0.
- Kozen uses the recursive function $\widehat{\delta}$ to define when a DFA accepts a string. Both definitions give the same result.

Finite Automata

What can be computed with finite memory?



Finite Automata

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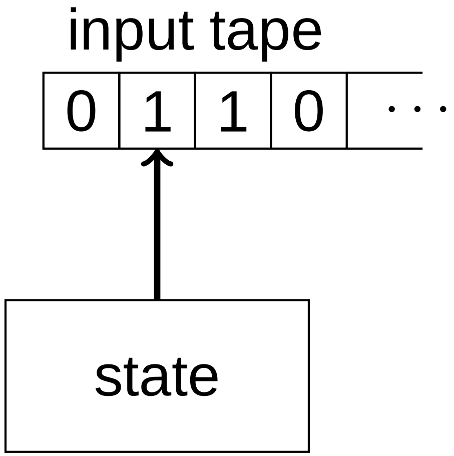

Finite Automata

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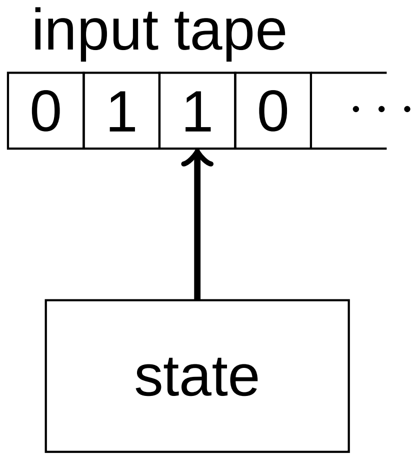
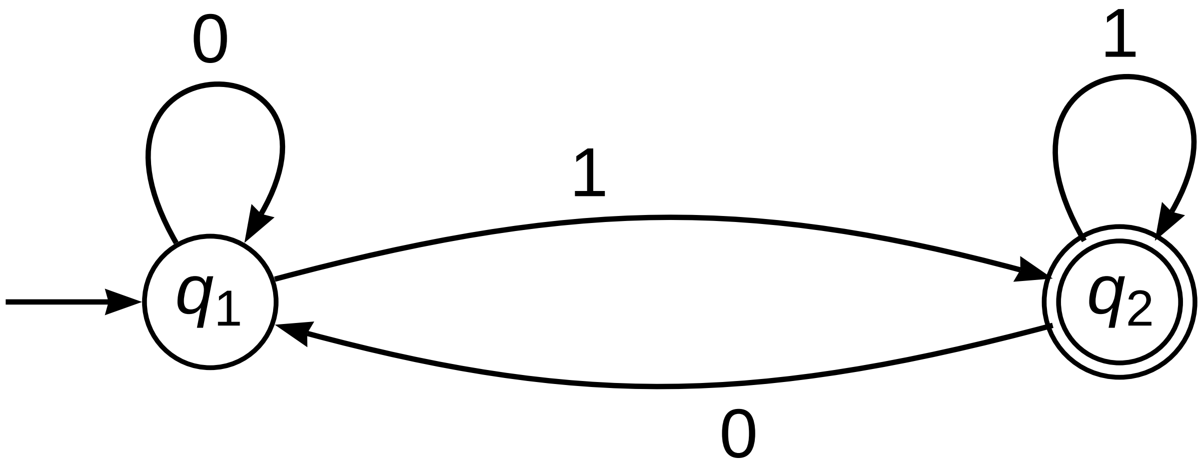


A “Wild” Finite Automaton

- The input is the numbers pressed by the user
- The correct code is 1234
- Actually the door is opened if $\{0, \ldots, 9\}^*1234\{0, \ldots, 9\}^*$ is entered
- The machine needs to remember if the input given so far contains the subsequence 1234

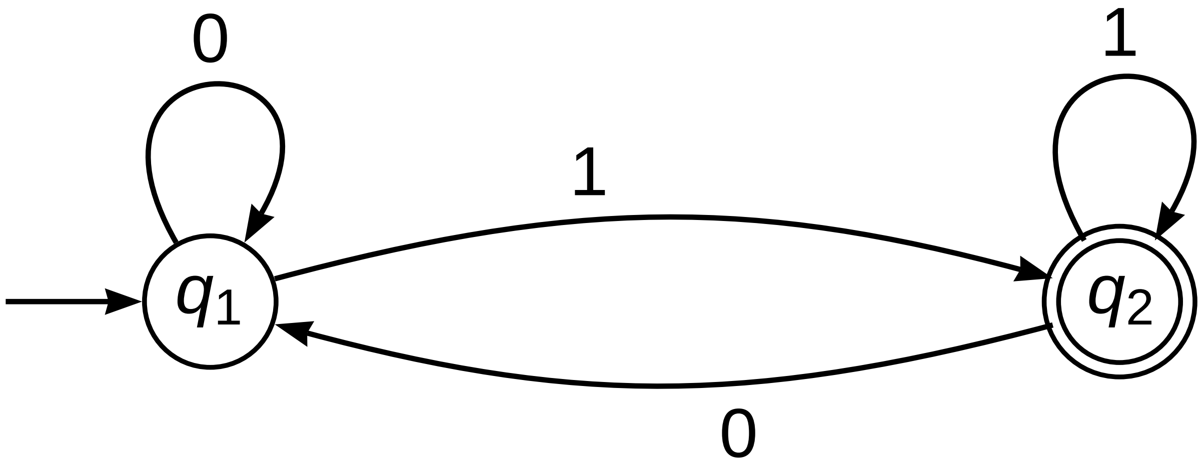
Finite Automata: Informal Definition



The machine M:

- States: q_1 and q_2
- Start state: q_1 (arrow from nowhere)
- Accept state: q_2 (double circle)
- State transitions: arrows

Finite Automata: Informal Definition

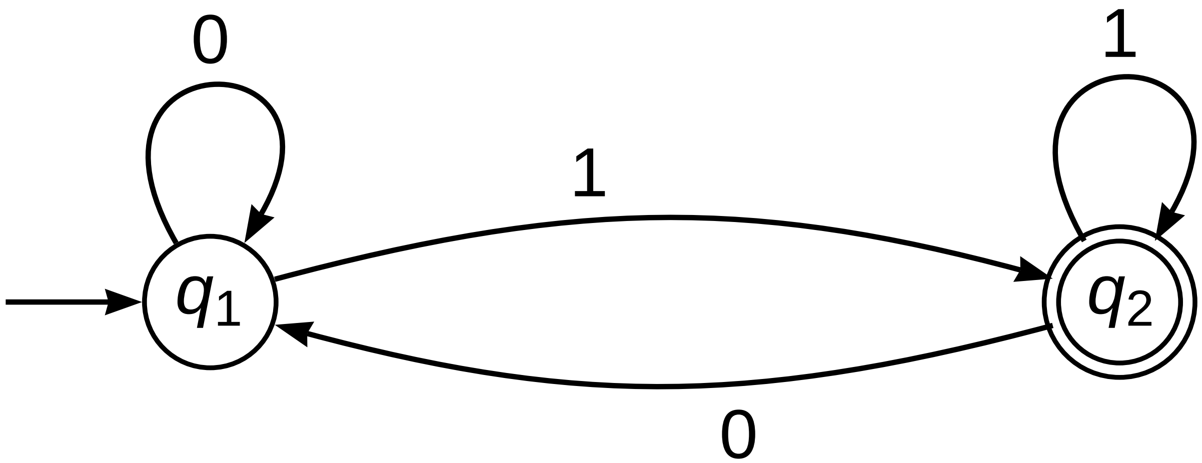


The machine M:

On input string $s = s_1 s_2 \cdots s_n$, M operates as follows:

- Begins in start state q_1 and reads the string s from left to right
- When reading symbol s_i it follows the transition labeled s_i from the current state
- After reading s_n, the last symbol in the string, it
  - accepts s if it is in an accept state
  - rejects s if it is not in an accept state

Finite Automata: Informal Definition

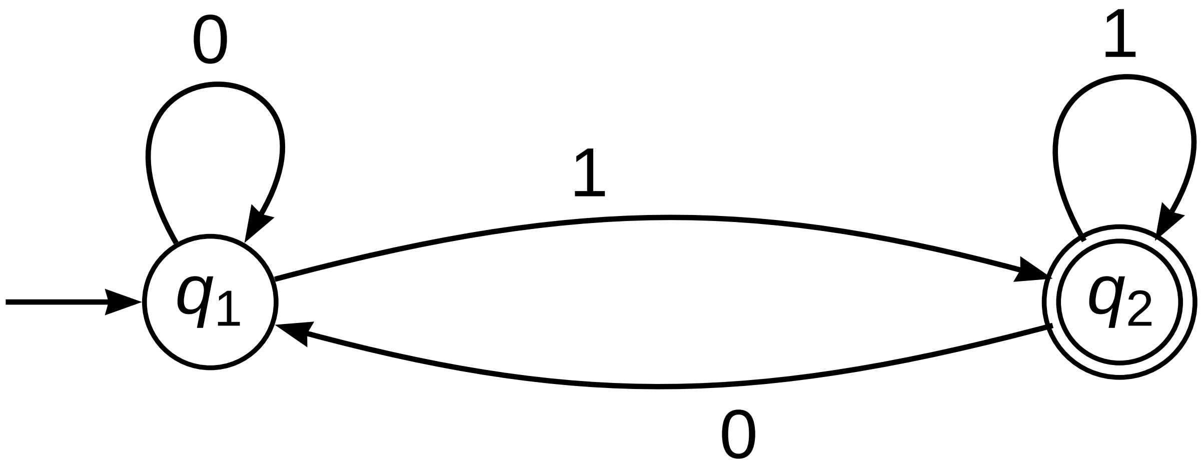


The machine M:

On input:

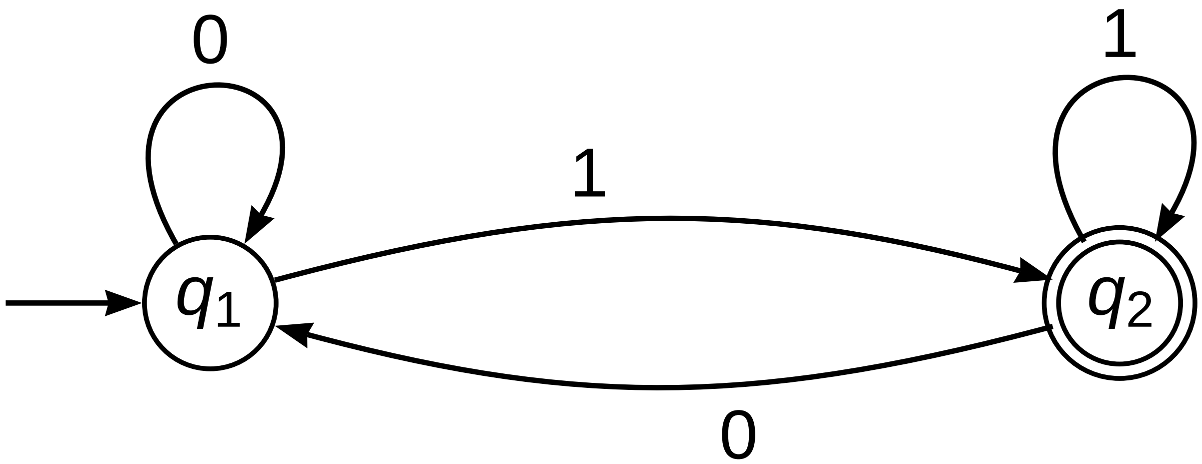
011

Finite Automata: Informal Definition



The machine M:
On input:
011 ACCEPT

Finite Automata: Informal Definition



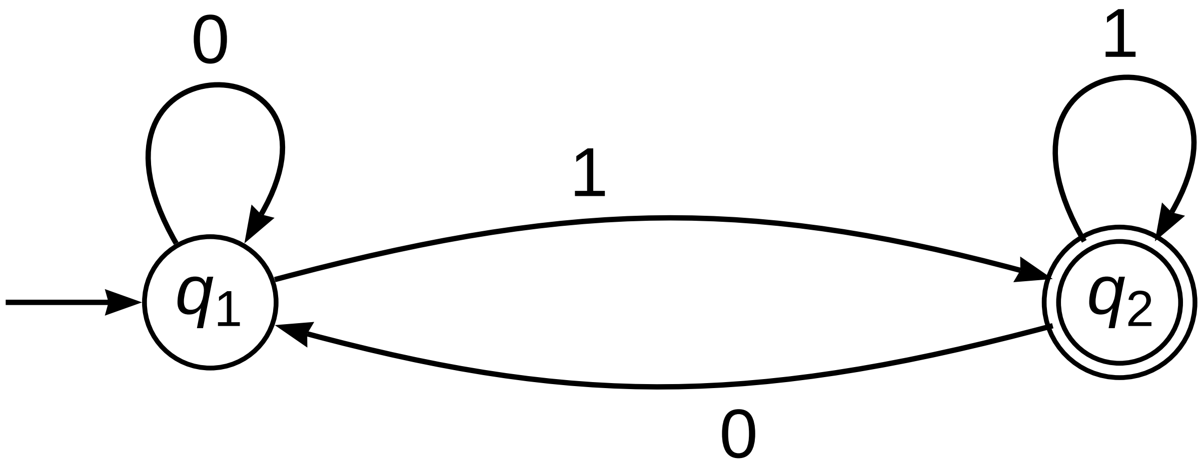
The machine M:

On input:

011 ACCEPT

10

Finite Automata: Informal Definition



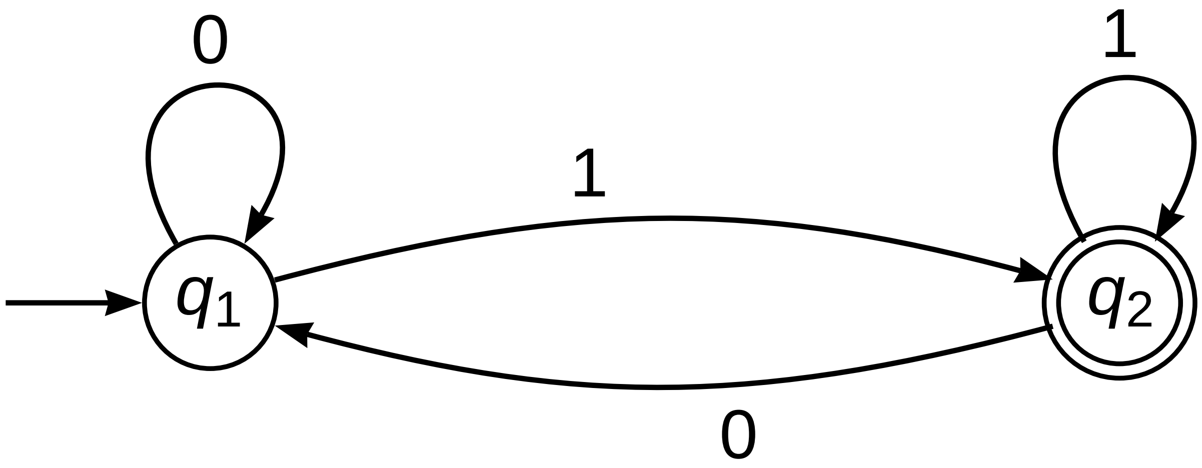
The machine M:

On input:

011 ACCEPT

10 REJECT

Finite Automata: Informal Definition



The machine *M*:

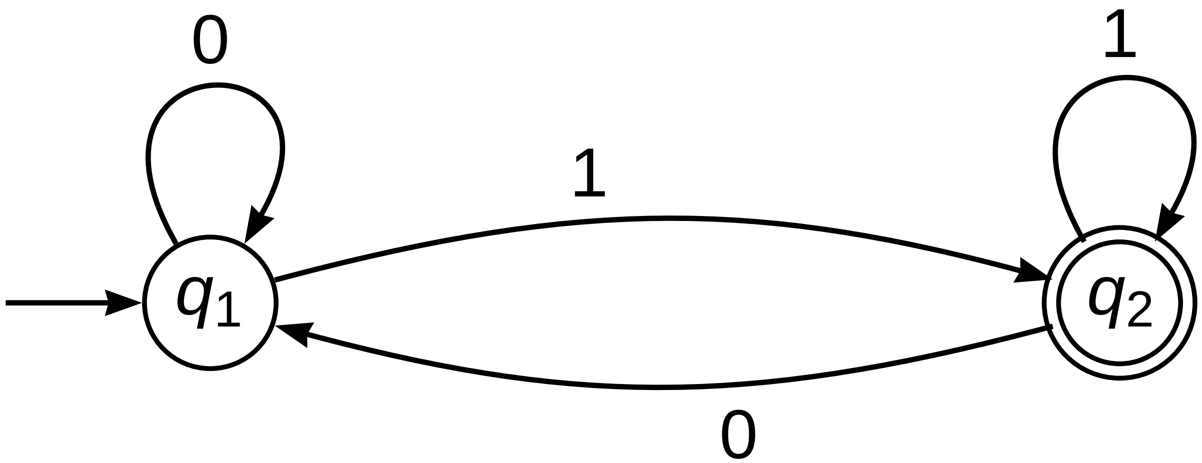
On input:

011 ACCEPT

10 REJECT

110

Finite Automata: Informal Definition



The machine M:

On input:

011 ACCEPT

10 REJECT

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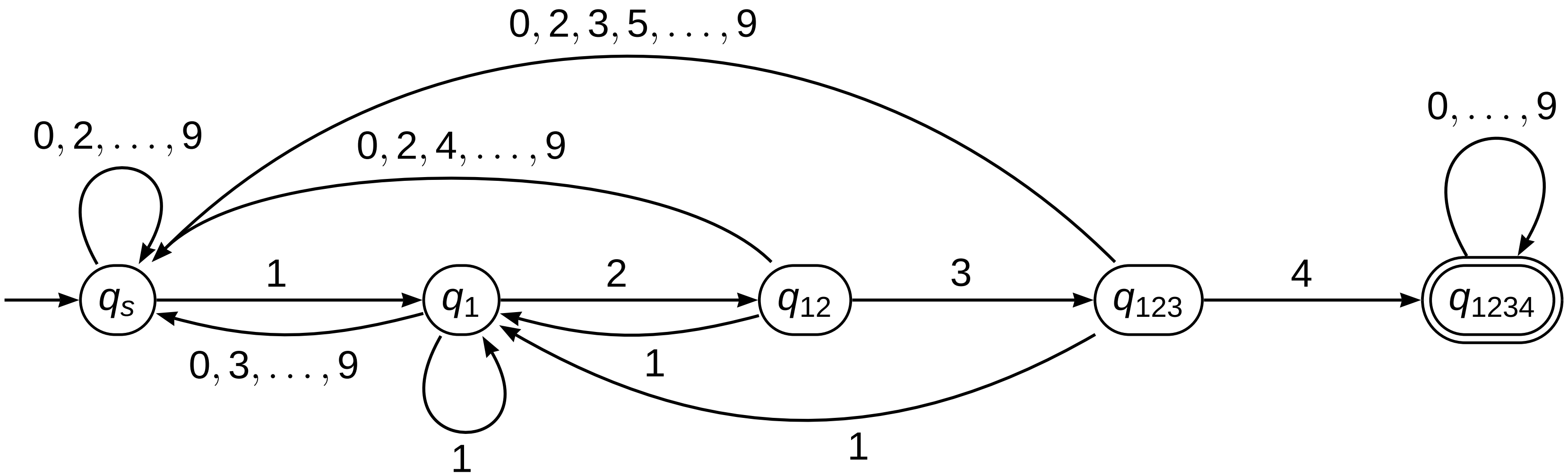
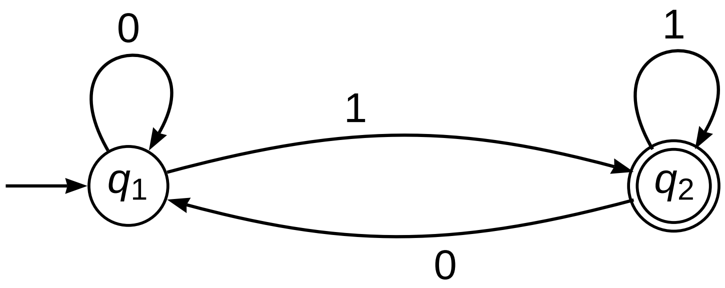


Representation of finite automata

- State diagram



- Transition table

	0	1
$\rightarrow q_1$	q_1	q_2
$F\ q_2$	q_1	q_2

Definition of DFAs

Definition

A deterministic finite automaton (DFA) is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$ where

- Q is a finite set called the states
- Σ is a finite set called the alphabet
- $\delta : Q \times \Sigma \to Q$ is the transition function
- $q_0 \in Q$ is the start state
- $F \subseteq Q$ is the set of accept states

The language recognized by a DFA

Definition

Let $M = (Q, \Sigma, \delta, q_0, F)$ be a DFA and $s = s_1 s_2 \cdots s_n$ a string over Σ. M accepts s if there is a sequence of states $r_0, r_1, \ldots r_n$ from Q such that

- $r_0 = q_0$,
- $\delta(r_i, s_{i+1}) = r_{i+1}$ ($i = 0, \ldots, n-1$), and
- $r_n \in F$

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Definition

- M recognizes language L if $L = \{s \mid M \textit{ accepts } s\}$
- $L(M)$ denotes the language recognized by M

Definition

A language is a **regular language** if some DFA recognizes it