We want to answer whether the program is safe or not (i.e., has some erroneous reachable configurations or not):

- **Safe Program**
- **Unsafe Program**

- Finding all configurations or behaviours (and hence errors) of arbitrary computer programs can be easily reduced to the halting problem of a Turing machine.
- This problem is proven to be undecidable, i.e., there is no algorithm that is guaranteed to terminate and to give an exact answer to the problem.
- An algorithm is **sound** in the case where each time it reports the program is safe wrt. some errors, then the original program is indeed safe wrt. those errors.
- An algorithm is **complete** in the case where each time it is given a program that is safe wrt. some errors, then it does report it to be safe wrt. those errors.
The idea is then to come up with efficient approximations and algorithms to give correct answers in as many cases as possible.

A sound analysis cannot give false negatives
A complete analysis cannot give false positives

Two Lectures on Static Analysis

These two lectures on static program analysis briefly introduce different types of analysis:

- Previous lecture:
  - syntactic analysis: scalable but neither sound nor complete
  - abstract interpretation sound but not complete

- This lecture:
  - symbolic executions: complete but not sound
  - inductive methods: may require heavy human interaction in proving the program correct

First, What Are SMT Solvers?

- Stands for Satisfiability Modulo Theory
- Intuitively, these are constraint solvers that extend SAT solvers to richer theories
- Many solvers exist (Face’s, CVC, STP, OpenSMT), you will use Z3 http://z3.codeplex.com in the lab.
- SAT solvers find a satisfying assignment to a formula where all variables are booleans or establishes its unsatisfiability
- SMT solvers find satisfying assignments to first order formulas where some variables may range over other values than just booleans
- For instance, formulas can involve Linear real arithmetic, Linear integer arithmetic, uninterpreted functions, bit-vectors, etc.
- E.g., \( f(x) = z \land f(2y) = z \land x - y = y \) is unsat while \( f(x) = z \land f(2y) = z \land x + y = y \) is sat.
- Many applications in verification, testing, planning, theorem proving, etc.
Overview
Symbolic Execution
Hoare Triples and Deductive Reasoning

Symbolic Testing

- Main idea by JC. King in “Symbolic Execution and Program Testing” in the 70s
- Use symbolic values instead of concrete ones
- Along the path, maintain a **Patch Constraint** (**PC**) and a symbolic state (**σ**)
- **PC** collects constraints on variables' values along a path,
- **σ** associates variables to symbolic expressions,
- We get concrete values if **PC** is satisfiable
- The program can be run on these values
- Negate a condition in the path constraint to get another path

Symbolic Execution: a simple example

- Can we get to the ERROR? explore using SSA forms.
- Useful to check array out of bounds, assertion violations, etc.

```plaintext
foo(int x,y,z){
  x = y - z;
  if(x==z){
    z = z - 3;
    if(4*z < x + y) {
      if(25 > x + y) { ...
      }
      else{
        ERROR;
      }
    }
    else{
      PC10 = PC6 ∧ 25 ≤ x1 + y0      x → x1,y → y0,z → z1
    }
  }
}
```

**PC** = (x1 = y0 - z0 ∧ x1 = z0 ∧ z1 = z0 - 3 ∧ 4 * z1 < x1 + y0 ∧ 25 ≤ x1 + y0)
Check satisfiability with an SMT solver (e.g., http://rise4fun.com/Z3)
Symbolic execution today

- Leverages on the impressive advancements for SMT solvers
- Modern symbolic execution frameworks are not purely symbolic, and not necessarily static:
  - They can follow a concrete execution while collecting constraints along the way, or
  - They can treat some of the variables concretely, and some other symbolically
- This allows them to scale, to handle closed code or complex queries

Outline

Overview

Symbolic Execution

Hoare Triples and Deductive Reasoning

Function Specifications and Correctness

- Contract between the caller and the implementation. Total Correctness requires that:
  - if the pre-condition \((-100 <= x \&\& x <= 100)\) holds
  - then the implementation terminates,
  - after termination, the following post-condition holds
    \((x>=0 \&\& \text{result} == x || x<0 \&\& \text{result} == -x)\)
- Partial Correctness does not require termination

```c
/*@ requires -100 <= x && x <= 100;
@ ensures x>=0 && \text{result} == x || x<0 && \text{result} == -x;
*/
int abs(int x){
  if(x < 0)
    return -x;
  return x;
}
```

Hoare Triples and Partial Correctness

- a Hoare triple \(\{P\} \text{stmt} \{R\}\) consists in:
  - a predicate pre-condition \(P\)
  - an instruction \(\text{stmt}\),
  - a predicate post-condition \(R\)
- intuitively, \(\{P\} \text{stmt} \{R\}\) holds if whenever \(P\) holds and \(\text{stmt}\) is executed and terminates (partial correctness), then \(R\) holds after \(\text{stmt}\) terminates.
- For example:
  - \{true\} \(x = y \{x == y\}\)
  - \{(x == 1)\&\&(y == 2)\} \(x = y \{(x == 2)\}\)
  - \{(x > 1)\} \(y = 2 \{(x == 0)||\}(y <= 10)\)
  - \{(x > 1)\} \(\text{if}(y == 2) \text{then} x = 0 \{(x >= 0)\}\)
  - \{false\} \(x = 1 \{(x == 2)\}\)
Weakest Precondition of sequences

- Assume a sequence of two instructions \( stmt; stmt' \), for example \( x = 2 \ast y; y = x + 3 \ast y \);
- the the weakest precondition is given by:
  \[
  wp(stmt; stmt', R) = wp(stmt, wp(stmt', R)),
  \]
  \[
  wp(x = 2 \ast y, y = x + 3 \ast y, y > 10) = wp(x = 2 \ast y, y = x + 3 \ast y, y > 10) = wp(x = 2 \ast y, (y > 10)[y/x + 3 \ast y])
  \]
  \[
  = wp(x = 2 \ast y, x + 3 \ast y > 10) = (x + 3 \ast y > 10)[x/2 \ast y] = (2 \ast y + 3 \ast y > 10) = y > 2
  \]

Weakest Precondition of conditionals

- Assume a conditional (if\( B \) then \( stmt \) else \( stmt' \)), for example (if\( x > y \) then \( z = x \) else \( z = y \))
- The weakest precondition is given by:
  \[
  \begin{align*}
  wp((if(B) & \text{then } stmt \text{ else } stmt'), R) &= (B \Rightarrow wp(stmt, R)) \land (!B \Rightarrow wp(stmt', R)) \\
  wp((if(x > y) \text{ then } z = x \text{ else } z = y), R) &= (x > y \Rightarrow wp(z = x, z <= 10) \land (x <= y \Rightarrow wp(z = y, z <= 10))),
  \end{align*}
  \]
  \[
  = (x > y \Rightarrow x <= 10) \land (x <= y \Rightarrow y <= 10)
  \]

Weakest Precondition of assignments

- \( wp(x = E, R) = R[x/E], \) i.e., replace each occurrence of \( x \) in \( R \) by \( E \).
- For instance:
  \[
  \begin{align*}
  wp(x = 3, x == 5) &= (x == 5)[x/3] = (3 == 5) = false \\
  wp(x = 3, x > 0) &= (x > 0)[x/3] = (3 > 0) = true \\
  wp(x = y + 5, x > 0) &= (x > 0)[x/y + 5] = (y + 5 > 0) \\
  wp(x = 5 * y + 2 * z, x + y > 0) &= (x + y > 0)[x/5 * y + 2 * z] = (6 * y + 2 * z > 0)
  \end{align*}
  \]
In order to establish \( \{P\} \{\text{while}(B)\{\text{do} \{\text{stmt}\}\}\}\{R\} \), you will need to find an invariant \( \text{Inv} \) such that:

- \( P \Rightarrow \text{Inv} \)
- \( \{\text{Inv} \& \& B\}\ \{\text{stmt}\}\ \{\text{Inv}\} \)
- \( (\text{Inv} \& \& \neg B) \Rightarrow R \)

For example, \( \{i == j == 0\} \{\text{while}(i < 10)\{i = i + 1; j = j + 1\}\}\{j == 10\} \), we need to find \( \text{Inv} \) such that:

- \( (i == j == 0) \Rightarrow \text{Inv} \)
- \( \{\text{Inv} \& \& (i < 10)\} \ i = i + 1; j = j + 1\ \{\text{Inv}\} \)
- \( (\text{Inv} \& \& i >= 10) \Rightarrow j == 10 \)

Partial correctness: if we start from \( P \) and \( (\text{while}(B)\{\text{do} \{\text{stmt}\}\}) \) terminates, then \( R \) terminates.

- \( P \Rightarrow \text{Inv} \)
- \( \{\text{Inv} \& \& B\}\ \{\text{stmt}\}\ \{\text{Inv}\} \)
- \( (\text{Inv} \& \& \neg B) \Rightarrow R \)

Total correctness: the loop does terminate: find a variant function \( \nu \) such that:

- \( (\text{Inv} \& \& B) \Rightarrow (\nu > 0) \)
- \( \{\text{Inv} \& \& B \& \& \nu = \nu_0\} \ \{\text{stmt}\}\ \{\nu < \nu_0\} \)

For example, \( \{\text{while}(i < 10)\{i = i + 1; j = j + 1\}\} \) can be shown to terminate with \( \nu = (10 - i) \) and \( \text{Inv} = (i <= 10) \).