Static Analysis: Symbolic Execution and Inductive Verification Methods

TDDC90: Software Security

Ahmed Rezine

IDA, Linköpings Universitet

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Outline

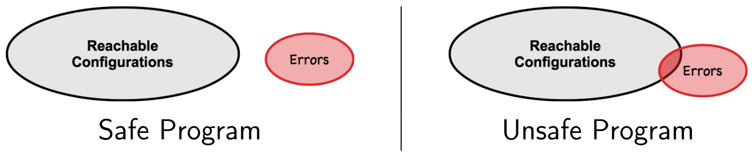
Overview

Symbolic Execution

Hoare Triples and Deductive Reasoning

Static Program Analysis and Approximations

We want to answer whether the program is **safe** or not (i.e., has some erroneous reachable configurations or not):

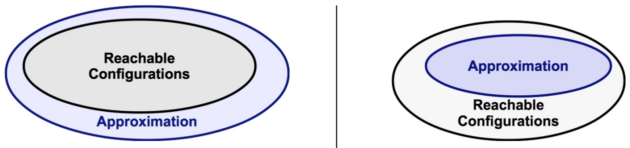


Static Program Analysis and Approximations

- Finding all configurations or behaviours (and hence errors) of arbitrary computer programs can be easily reduced to the halting problem of a Turing machine.
- This problem is proven to be undecidable, i.e., there is no algorithm that is guaranteed to terminate and to give an exact answer to the problem.
- An algorithm is **sound** in the case where each time it reports the program is safe wrt. some errors, then the original program is indeed safe wrt. those errors
- An algorithm is **complete** in the case where each time it is given a program that is safe wrt. some errors, then it does report it to be safe wrt. those errors

Static Program Analysis and Approximations

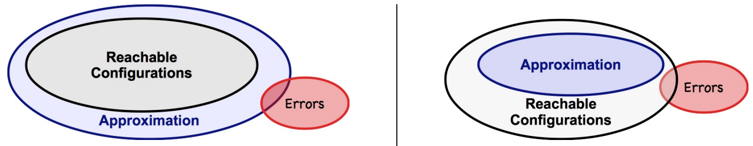
- The idea is then to come up with efficient approximations and algorithms to give correct answers in as many cases as possible.



Over-approximation | Under-approximation

Static Program Analysis and Approximations

- A sound analysis cannot give **false negatives**
- A complete analysis cannot give **false positives**



False Positive | False Negative

Two Lectures on Static Analysis

These two lectures on static program analysis briefly introduce different types of analysis:

- Previous lecture:
  - syntactic analysis: scalable but neither sound nor complete
  - abstract interpretation sound but not complete
- This lecture:
  - symbolic executions: complete but not sound
  - inductive methods: may require heavy human interaction in proving the program correct

First, What Are SMT Solvers?

- Stands for *Satisfiability Modulo Theory*
- Intuitively, these are constraint solvers that extend *SAT solvers* to richer theories
- Many solvers exist (Face's, CVC, STP, OpenSMT), you will use Z3 `http://z3.codeplex.com` in the lab.
- SAT solvers find a satisfying assignment to a formula where all variables are booleans or establishes its unsatisfiability
- SMT solvers find satisfying assignments to first order formulas where some variables may range over other values than just booleans
- For instance, formulas can involve Linear real arithmetic, Linear integer arithmetic, uninterpreted functions, bit-vectors, etc.
- E.g., $f(x)! = z \wedge f(2y) = z \wedge x - y = y$ is unsat while $f(x)! = z \wedge f(2y) = z \wedge x + y = y$ is sat.
- Many applications in verification, testing, planning, theorem proving, etc.

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Testing

- Most common form of software validation
- Explores only one possible execution at a time
- For each new value, run a new test.
- On a 32 bit machine, `if(i==2014) bug()` would require 2^{32} different values to make sure there is no bug.
- The idea in symbolic testing is to associate **symbolic values** to the variables

Symbolic Testing

- Main idea by JC. King in "Symbolic Execution and Program Testing" in the 70s
- Use symbolic values instead of concrete ones
- Along the path, maintain a *Patch Constraint* (PC) and a symbolic state (σ)
- PC collects constraints on variables' values along a path,
- σ associates variables to symbolic expressions,
- We get concrete values if PC is satisfiable
- The program can be run on these values
- Negate a condition in the path constraint to get another path

Symbolic Execution: a simple example

- Can we get to the ERROR? explore using SSA forms.
- Useful to check array out of bounds, assertion violations, etc.

Code	PC	σ
`foo(int x,y,z){`	$PC_1 = true$	
`  x = y - z;`	$PC_2 = PC_1$	$x \mapsto x_0, y \mapsto y_0, z \mapsto z_0$
`  if(x==z){`	$PC_3 = PC_2 \wedge x_1 = y_0 - z_0$	$x \mapsto x_1, y \mapsto y_0, z \mapsto z_0$
`    z = z - 3;`	$PC_4 = PC_3 \wedge x_1 = z_0$	$x \mapsto x_1, y \mapsto y_0, z \mapsto z_0$
`    if(4*z < x + y){`	$PC_5 = PC_4 \wedge z_1 = z_0 - 3$	$x \mapsto x_1, y \mapsto y_0, z \mapsto z_1$
`      if(25 > x + y) {`	$PC_6 = PC_5 \wedge 4 * z_1 < x_1 + y_0$	$x \mapsto x_1, y \mapsto y_0, z \mapsto z_1$
`        ...`		
`      }`		
`      else{`		
`        ERROR;`	$PC_{10} = PC_6 \wedge 25 \leq x_1 + y_0$	$x \mapsto x_1, y \mapsto y_0, z \mapsto z_1$
`      }`		
`    }`		
`  }`		
`  ...`		

$PC = (x_1 = y_0 - z_0 \wedge x_1 = z_0 \wedge z_1 = z_0 - 3 \wedge 4 * z_1 < x_1 + y_0 \wedge 25 \leq x_1 + y_0)$

Check satisfiability with an SMT solver (e.g., `http://rise4fun.com/Z3`)

Symbolic execution today

- Leverages on the impressive advancements for SMT solvers
- Modern symbolic execution frameworks are not purely symbolic, and not necessarily static:
  - They can follow a concrete execution while collecting constraints along the way, or
  - They can treat some of the variables concretely, and some other symbolically
- This allows them to scale, to handle closed code or complex queries

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Function Specifications and Correctness

- Contract between the caller and the implementation. **Total Correctness** requires that:
  - if the pre-condition (`-100 <= x && x <= 100`) holds
  - then the implementation terminates,
  - after termination, the following post-condition holds (`x>=0 && \result == x || x<0 && \result == -x`)
- **Partial Correctness** does not require termination

```
/*@ requires -100 <= x && x <= 100;
@ ensures x>=0 && \result == x || x<0 && \result == -x;
*/
int abs(int x){
  if(x < 0)
  return -x;
  return x;
}
```

Hoare Triples and Partial Correctness

- a Hoare triple $\{P\}$ *stmt* $\{R\}$ consists in:
  - a predicate pre-condition P
  - an instruction *stmt*,
  - a predicate post-condition R
- intuitively, $\{P\}$ *stmt* $\{R\}$ holds if whenever P holds and *stmt* is executed and terminates (**partial correctness**), then R holds after *stmt* terminates.
- For example:
  - $\{true\}\ x = y\ \{(x == y)\}$
  - $\{(x == 1)\&\&(y == 2)\}\ x = y\ \{(x == 2)\}$
  - $\{(x >= 1)\}\ y = 2\ \{(x == 0)||(y <= 10)\}$
  - $\{(x >= 1)\}\ (\text{if}(y == 2) \text{ then } x = 0)\ \{(x >= 0)\}$
  - $\{false\}\ x = 1\ \{(x == 2)\}$

Weakest Precondition

- if $\{P\}$ *stmt* $\{R\}$ and $P' \Rightarrow P$ for any P' s.t. $\{P'\}$ *stmt* $\{R\}$, then P is the **weakest precondition** of R wrt. *stmt*, written $wp(stmt, R)$
- $wp(x = x + 1, x >= 1) = (x >= 0)$. $(x >= 5), (x = 6), (x >= 0 \&\& y = 8)$ are all valid preconditions, but they are not weaker than $x >= 0$.
- Intuitively $wp(stmt, R)$ is the weakest predicate P for which $\{P\}$ *stmt* $\{R\}$ holds

Weakest Precondition of assignments

- $wp(x = E, R) = R[x/E]$, i.e., replace each occurrence of x in R by E.
- For instance:
  - $wp(x = 3, x == 5) = (x == 5)[x/3] = (3 == 5) = false$
  - $wp(x = 3, x >= 0) = (x >= 0)[x/3] = (3 >= 0) = true$
  - $wp(x = y + 5, x >= 0) = (x >= 0)[x/y + 5] = (y + 5 >= 0)$
  - $wp(x = 5 * y + 2 * z, x + y >= 0) = (x + y >= 0)[x/5 * y + 2 * z] = (6 * y + 2 * z >= 0)$

Weakest Precondition of sequences

- Assume a sequence of two instructions $stmt; stmt'$;, for example $x = 2 * y; y = x + 3 * y$;
- the the weakest precondition is given by: $wp(stmt; stmt', R) = wp(stmt, wp(stmt', R))$,

$$\begin{aligned} & wp(x = 2 * y; y = x + 3 * y, y > 10) \\ = \; & wp(x = 2 * y, wp(y = x + 3 * y, y > 10)) \\ = \; & wp(x = 2 * y, (y > 10)[y/x + 3 * y]) \\ = \; & wp(x = 2 * y, x + 3 * y > 10) \\ = \; & (x + 3 * y > 10)[x/2 * y] \\ = \; & (2 * y + 3 * y > 10) \\ = \; & y > 2 \end{aligned}$$

Weakest Precondition of conditionals

- Assume a conditional (if(B) then *stmt* else *stmt'*), for example (if$(x > y)$ then $z = x$ else $z = y$)
- The weakest precondition is given by:

$$\left(\begin{aligned} & wp((\text{if}(B) \text{ then } stmt \text{ else } stmt'), R) \\ = \; & (B \Rightarrow wp(stmt, R)) \&\& (!B \Rightarrow wp(stmt', R)) \end{aligned} \right)$$

- For example,

$$\begin{aligned} & wp((\text{if}(x > y) \text{ then } z = x \text{ else } z = y), z <= 10) \\ = \; & (x > y \Rightarrow wp(z = x, z <= 10)) \\ & \&\& (x <= y \Rightarrow wp(z = y, z <= 10)) \\ = \; & (x > y \Rightarrow x <= 10) \&\& (x <= y \Rightarrow y <= 10) \end{aligned},$$

Hoare Triples for Loops, Partial Correctness

- In order to establish $\{P\}$ ($\texttt{while}(B)\texttt{do}\{stmt\}$) $\{R\}$, you will need to find an invariant *Inv* such that:
  - $P \Rightarrow Inv$
  - $\{Inv \&\& B\}$ $stmt$ $\{Inv\}$
  - $(Inv \&\& !B) \Rightarrow R$
- For example $\{i == j == 0\}$ ($\texttt{while}(i < 10)\texttt{do}\{i = i + 1; j = j + 1\}$) $\{j == 10\}$, we need to find *Inv* such that:
  - $(i == j == 0) \Rightarrow Inv$
  - $\{Inv \&\& (i < 10)\}$ $i = i + 1; j = j + 1$ $\{Inv\}$
  - $(Inv \&\& i >= 10) \Rightarrow j == 10$

Hoare Triples for Loops, Total Correctness

- $\{P\}$ ($\texttt{while}(B)\texttt{do}\{stmt\}$) $\{R\}$
- Partial correctness: if we start from P and ($\texttt{while}(B)\texttt{do}\{stmt\}$) terminates, then R terminates.
  - $P \Rightarrow Inv$
  - $\{Inv \&\& B\}$ $stmt$ $\{Inv\}$
  - $(Inv \&\& !B) \Rightarrow R$
- Total correctness: the loop does terminate: find a **variant function** v such that:
  - $(Inv \&\& B) \Rightarrow (v > 0)$
  - $\{Inv \&\& B \&\& v = v_0\}$ $stmt$ $\{v < v_0\}$
- For example ($\texttt{while}(i < 10)\texttt{do}\{i = i + 1; j = j + 1\}$) can be shown to terminate with $v = (10 - i)$ and $Inv = (i <= 10)$