Static Analysis: Symbolic Execution and Inductive Verification Methods

TDDC90: Software Security

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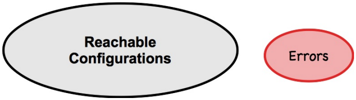
Outline

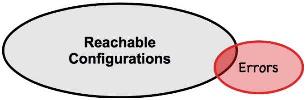
Outline

Static Program Analysis and Approximations

We want to answer whether the program is **safe** or not (i.e., has some erroneous reachable configurations or not):



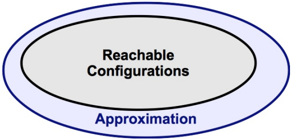
Safe Program



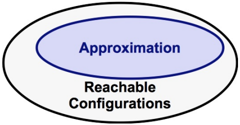
Unsafe Program

Static Program Analysis and Approximations

- The idea is then to come up with efficient approximations and algorithms to give correct answers in as many cases as possible.



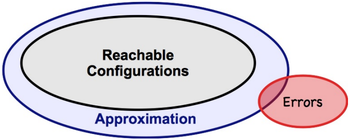
Over-approximation



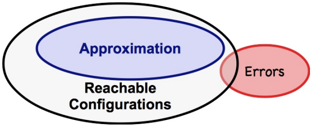
Under-approximation

Static Program Analysis and Approximations

- A sound analysis cannot give **false negatives**
- A complete analysis cannot give **false positives**



False Positive



False Negative

Two Lectures on Static Analysis

These two lectures on static program analysis briefly introduce different types of analysis:

- Previous lecture:
  - syntactic analysis: scalable but neither sound nor complete
  - abstract interpretation sound but not complete
- This lecture:
  - symbolic executions: complete but not sound
  - inductive methods: may require heavy human interaction in proving the program correct

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First, What are SMT Solvers?

- Stands for *Satisfiability Modulo Theory*
- Intuitively, these are constraint solvers that extend *SAT solvers* to richer theories
- Many solvers exist (Yices, CVC, STP, OpenSMT, Princess, Z3, etc),
- You will be using Z3 `https://github.com/Z3Prover/z3` in the lab z3
- SAT solvers find a satisfying assignment to a formula where all variables are booleans or establishes its unsatisfiability
- SMT solvers find satisfying assignments to first order formulas where some variables may range over other values than just booleans

Introduction

Originates from automating proof-search for first order logic.

- Variables: $x, y, z, \dots$
- Constants: $a, b, c, \dots$
- N-ary functions: $f, g, h, \dots$
- N-ary predicates: $p, q, r, \dots$
- Atoms: $\bot, \top, p(t_1, \dots, t_n)$
- Literals: atoms or their negation
- A FOL formula is a literal, boolean combinations of formulas, or quantified ($\exists$, $\forall$) formulas.

Evaluation of formula φ, with respect to interpretation I over non-empty (possibly infinite) domains for variables and constants gives true or false (resp. $I \models \varphi$ or $I \not\models \varphi$)

Satisfiability and Validity

A formula φ is:

- satisfiable if $I \models \varphi$ for **some** interpretation I
- valid if $I \models \varphi$ for **all** interpretations I

Satisfiability of FOL is undecidable. Instead, target decidable or domain-specific fragments.

Introduction

Given a quantifier free FOL formula and a combination of theories, is there an interpretation to the free variables that makes the formula true?

$$\varphi \quad \triangleq \quad g(a) = c \wedge (f(g(a)) \neq f(c) \vee g(a) = d) \wedge c \neq d$$

- EUF: Equality over Uninterpreted functions
- Satisfiable?

Introduction

Given a quantifier free FOL formula and a combination of theories, is there an interpretation to the free variables that makes the formula true?

$$\begin{aligned}\varphi \quad \triangleq \quad & (x_1 \geq 0) \wedge (x_1 < 1) \\ & \wedge((f(x_1) = f(0)) \Rightarrow (rd(wr(P, x_2, x_3), x_2 + x_1) = x_3 + 1)\end{aligned}$$

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- Linear Integer Arithmetic (LIA)

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- Linear Integer Arithmetic (LIA)
- Equality over Uninterpreted functions (EUF)
- Arrays (A)

Introduction

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- LIA: $x_1 = 0$
- EUF: $f(x_1) = f(0)$
- A: $rd(wr(P, x_2, x_3), x_2) = x_3$
- Bool: $rd(wr(P, x_2, x_3), x_2) = x_3 + 1$
- LIA: $\perp$

Introduction

- Sometimes more natural to express in logics other than propositional logic
- SMT decide satisfiablity of ground FO formulas wrt. background theories
- Many applications: Model checking, predicate abstraction, symbolic execution, scheduling, test generation, ...

Outline

Testing

- Most common form of software validation
- Explores only one possible execution at a time
- For each new value, run a new test.
- On a 32 bit machine, `if(i==2021) bug()` would require 2^{32} different values to make sure there is no bug.
- The idea in symbolic testing is to associate **symbolic values** to the variables

Symbolic Testing

- Use symbolic values instead of concrete ones
- Along the path, maintain a *Path Constraint* (PC) and a symbolic state (σ)
- PC collects constraints on variables' values along a path,
- σ associates variables to symbolic expressions,
- We get concrete values if PC is satisfiable
- The program can be run on these values
- Negate a condition in the path constraint to get another path

Symbolic Execution: a simple example

- Can we get to the ERROR? explore using SSA forms.
- Useful to check array out of bounds, assertion violations, etc.

```
foo(int x,y,z){
  x = y - z;
  if(x==z){
    z = z - 3;
    if(4*z < x + y){
      if(25 > x + y) {
        ...
      }
      else{
        ERROR;
      }
    }
  }
  ...
```

$PC_1 = true$

$PC_2 = PC_1$

$PC_3 = PC_2 \wedge x_1 = y_0 - z_0$

$PC_4 = PC_3 \wedge x_1 = z_0$

$PC_5 = PC_4 \wedge z_1 = z_0 - 3$

$PC_6 = PC_5 \wedge 4 * z_1 < x_1 + y_0$

$PC_{10} = PC_6 \wedge \neg(25 > x_1 + y_0)$

$PC = (x_1 = y_0 - z_0 \wedge x_1 = z_0 \wedge z_1 = z_0 - 3 \wedge 4 * z_1 < x_1 + y_0 \wedge \neg(25 > x_1 + y_0))$

Check satisfiability with a solver (e.g., Alt-Ergo, Boolector, CVC4, MathSAT5, OpenSMT2, STP, Yices2, Z3)

Symbolic execution today

- Leverages on the impressive advancements of SMT solvers
- Modern symbolic execution frameworks are not purely symbolic, and not necessarily purely static:
  - They can follow a concrete execution while collecting constraints along the way, or
  - They can treat some of the variables concretely, and some other symbolically
- This allows them to scale, to handle closed code or complex queries

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Function Specifications and Correctness

- Contract between the caller and the implementation. **Total Correctness** requires that:
  - if the pre-condition (`-100 <= x && x <= 100`) holds
  - then the implementation terminates,
  - after termination, the following post-condition holds (`x>=0 && \result == x || x<0 && \result == -x`)
- **Partial Correctness** does not require termination

```
/*@ requires -100 <= x && x <= 100;
@ ensures x>=0 && \result == x || x<0 && \result == -x;
*/
int abs(int x){
  if(x < 0){
    return -x;
  }
  return x;
}
```

Hoare Triples and Partial Correctness

- a Hoare triple $\{P\}$ *stmt* $\{R\}$ consists in:
  - a predicate pre-condition P
  - an instruction *stmt*,
  - a predicate post-condition R
- intuitively, $\{P\}$ *stmt* $\{R\}$ holds if whenever P holds and *stmt* is executed and terminates (**partial correctness**), then R holds after *stmt* terminates.
- For example:
  - $\{true\}\ x := y\ \{(x = y)\}$
  - $\{(x = 1) \wedge (y = 2)\}\ x := y\ \{(x = 2)\}$
  - $\{(x \geq 1)\}\ y := 2\ \{(x = 0) \vee (y \leq 10)\}$
  - $\{(x \geq 1)\}\ (\mathrm{if}(y == 2)\ \mathrm{then}\ x := 0)\ \{(x \geq 0)\}$
  - $\{false\}\ x := 1\ \{(x = 2)\}$

Weakest Precondition

- if $\{P\}$ *stmt* $\{R\}$ and $P' \Rightarrow P$ for any P' s.t. $\{P'\}$ *stmt* $\{R\}$, then P is the **weakest precondition** of R wrt. *stmt*, written $wp(stmt, R)$
- $wp(x := x + 1, x \geq 1) = (x \geq 0)$.
  $(x \geq 5), (x = 6), (x \geq 0 \wedge y = 8)$ are all valid preconditions, but they are not weaker than $x \geq 0$.
- Intuitively $wp(stmt, R)$ is the weakest predicate P for which $\{P\}$ *stmt* $\{R\}$ holds

Weakest Precondition of assignments

- $wp(x = E, R) = R[x/E]$, i.e., replace each occurrence of x in R by E.
- For instance:
  - $wp(x := 3, x == 5) = (x == 5)[x/3] = (3 == 5) = \mathit{false}$
  - $wp(x := 3, x >= 0) = (x >= 0)[x/3] = (3 >= 0) = \mathit{true}$
  - $wp(x := y + 5, x >= 0) = (x >= 0)[x/y + 5] = (y + 5 >= 0)$
  - $wp(x := 5 * y + 2 * z, x + y >= 0) = (x + y >= 0)[x/5 * y + 2 * z] = (6 * y + 2 * z >= 0)$

Weakest Precondition of sequences

- Assume a sequence of two instructions $stmt; stmt';$, for example $x := 2 * y; y := x + 3 * y;$
- the the weakest precondition is given by: $wp(stmt; stmt', R) = wp(stmt, wp(stmt', R))$,

$$
\begin{array}{rl}
 & wp(x := 2 * y; y := x + 3 * y, y > 10) \\
= & wp(x := 2 * y, wp(y := x + 3 * y, y > 10)) \\
= & wp(x := 2 * y, (y > 10)[y/x + 3 * y]) \\
= & wp(x := 2 * y, x + 3 * y > 10) \\
= & (x + 3 * y > 10)[x/2 * y] \\
= & (2 * y + 3 * y > 10) \\
= & y > 2
\end{array}
$$

Weakest Precondition of conditionals

- Assume a conditional (if(B) then $stmt$ else $stmt'$), for example (if($x > y$) then $z := x$ else $z := y$)
- The weakest precondition is given by:
$$\left(\begin{array}{rl} & wp((\text{if}(B) \text{ then } stmt \text{ else } stmt'), R) \\ = & (B \Rightarrow wp(stmt, R))\&\&(!B \Rightarrow wp(stmt', R)) \end{array}\right)$$
- For example,
$$\begin{array}{rl} & wp((\text{if}(x > y) \text{ then } z := x \text{ else } z := y), z <= 10) \\ = & (x > y \Rightarrow wp(z := x, z <= 10)) \\ & \quad \&\&(x <= y \Rightarrow wp(z := y, z <= 10)) \\ = & (x > y \Rightarrow x <= 10)\&\&(x <= y \Rightarrow y <= 10) \end{array},$$

Hoare Triples for Loops, Partial Correctness

- In order to establish $\{P\}$ $(\texttt{while}(B)\texttt{do}\{stmt\})$ $\{R\}$, you will need to find an invariant *Inv* such that:
  - $P \Rightarrow Inv$
  - $\{Inv\&\&B\}$ $stmt$ $\{Inv\}$
  - $(Inv\&\&!B) \Rightarrow R$
- For example $\{i == j == 0\}$ $(\texttt{while}(i < 10)\texttt{do}\{i := i + 1; j := j + 1\})$ $\{j == 10\}$, we need to find *Inv* such that:
  - $(i == j == 0) \Rightarrow Inv$
  - $\{Inv\&\&(i < 10)\}$ $i = i + 1; j = j + 1$ $\{Inv\}$
  - $(Inv\&\&i >= 10) \Rightarrow j == 10$

Hoare Triples for Loops, Total Correctness

- $\{P\}$ $(\texttt{while}(B)\texttt{do}\{stmt\})$ $\{R\}$
- Partial correctness: if we start from P and $(\texttt{while}(B)\texttt{do}\{stmt\})$ terminates, then R terminates.
  - $P \Rightarrow Inv$
  - $\{Inv\&\&B\}$ $stmt$ $\{Inv\}$
  - $(Inv\&\&!B) \Rightarrow R$
- Total correctness: the loop does terminate: find a **variant function** v such that:
  - $(Inv\&\&B) \Rightarrow (v > 0)$
  - $\{Inv\&\&B\&\&v = v_0\}$ $stmt$ $\{v < v_0\}$
- For example $(\texttt{while}(i < 10)\texttt{do}\{i := i + 1; j := j + 1\})$ can be shown to terminate with $v = (10 - i)$ and $Inv = (i <= 10)$