

TDDD55 Compilers and Interpreters

TDDB44 Compiler Construction

Code Generation

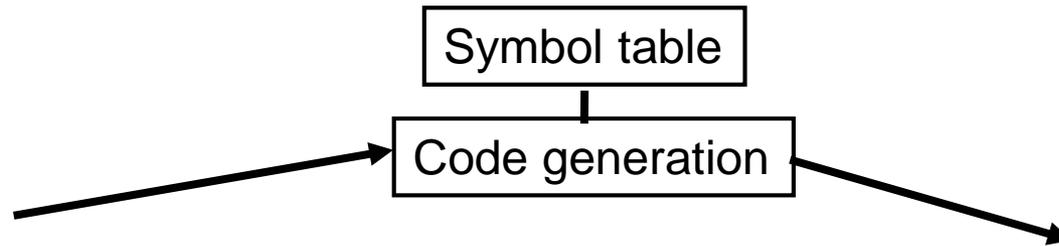
Requirements for code generation

- Correctness
- High code quality
- Efficient use of the resources of the target machine
- Quick code generation (for interactive use)
- Retargetability (parameterization in target machine spec.)

In practice:

- Difficult to generate good code
- Simple to generate bad code
- There are code generator generators ...

Intermediate Code vs. Target Code



Intermediate code

- High-level IR (Intermediate Repr)
e.g. Abstract Syntax Tree (AST)
- Medium-level IR
e.g. Control flow graph of complex operations (calls, array refs left)
- Low-level IR
e.g. Quadruples, DAGs

Target code

- Very low-level IR
(using target instructions only)
- Assembler code / Object code
 - Absolute machine code
 - Relocatable machine code(often generate asm (text) code and use an assembler tool to convert this to binary (object) code – easier, but slower compile

Lowering the IR

-
- Code for abstract stack machine
e.g. Postfix code

- Code for concrete stack machine
e.g. JVM byte code

Absolute vs. Relocatable Target Code

Absolute code

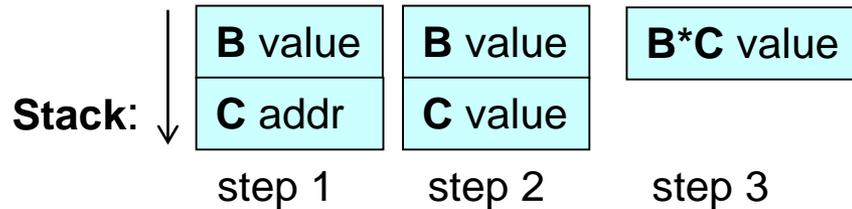
- Final memory area for program is statically known
- Hard-coded addresses
- Sufficient for very simple (typically, embedded) systems
- ☺ fast
- ☹ no separate compilation
- ☹ cannot call modules from other languages/compilers

Relocatable code

- Needs relocation table and relocating linker + loader or run-time relocation in MMU (memory management unit)
- ☺ most flexible

Stack Machines vs. Register Machines

Generate code for C assignment



On a stack machine:

```
PUSH _A // static address of A
PUSH _B // static address of B
LOAD    // dereference _B
PUSH fp // stack frame ptr reg
ADD #4 // C at stack adr. FP+4
        (step1 above)
LOAD    // load C value (step 2)
MUL    // multiply two stack values
        (step 3 above)
STORE  // store via address of A
```

$A = B * C;$

where A, B *global*, C *local* var.

On a register machine:

```
LDCONST _A, R1
LDCONST _B, R2
LOAD (R2), R2 // dereference _B
ADD FP, #4, R3 // addr. of C
LOAD (R3), R3 // dereference &C
MUL R2, R3, R2
STORE R2, (R1)
```

3 Main Tasks in Code Generation

■ Instruction Selection

- Choose set of instructions equivalent to IR code
- Minimize (locally) execution time, # used registers, code size
- Example: INCRM #4(fp) vs. LOAD #4(fp), R1
ADD R1, #1, R1
STORE R1, #4(fp)

■ Instruction Scheduling

- Reorder instructions to better utilize processor architecture
- Minimize temporary space (#registers, #stack locations) used, execution time, or energy consumption

■ Register Allocation

- Keep frequently used values in registers (limited resource!)
 - ▶ Some registers are reserved, e.g. sp, fp, pc, sr, retval ...
- Minimize #loads and #stores (which are expensive instructions!)
- **Register Allocation:** Which variables to keep when in some register?
- **Register Assignment:** In which particular register to keep each?

Machine Model (here: a simple register machine)

■ Register set

- E.g. 32 general-purpose registers R0, R1, R2, ...
some of them reserved (sp, fp, pc, sr, retval, par1, par2 ...)

■ Instruction set with different addressing modes

- Cost (usually, time / latency)
depends on the operation and the addressing mode
- Example: PDP-11 (CISC), instruction format *OP src, dest*

Source operand	Destination address	Cost
register	register	1
register	memory	2
memory	register	2
memory	memory	3



TDDB44: Code Generation

Two Example Machine Models

- Simple CISC machine model (CISC = Complex Instruction Set Computer). src, dest can be either memory or register
 - MOVE src, dest (or LOAD src, reg; STORE reg, dest)
 - OP src, dest

- Simple RISC machine model (RISC = Reduced Instruction Set Computer)
 - LOAD reg, mem
 - STORE mem, reg
 - OP reg, reg, reg // **Operations only between registers**

Example: $A = B + C;$

1. MOVE `_B, R0` ; 2
ADD `_C, R0` ; 2
MOVE `R0, _A` ; 2 → total cost = 6

2. MOVE `_B, _A` ; 3
ADD `_C, _A` ; 3 → total cost = 6

3. (B already in R2, C already in R3, C in R3 not used later)

ADD `R2, R3` ; 1
MOVE `R3, _A` ; 2 → total cost = 3

There is a lot to be gained with good register allocation!

4. (B already in R2, C in R3 and not needed later, A will be kept in R3)

ADD `R2, R3` ; 1 → total cost = 1

Some Code Generation Algorithms

- Macro-expansion of IR operations (quadruples)
- "Simple code generation algorithm" (textbook Section 8.6)
- Code generation for expression trees (textbook Section 8.10)
 - Labeling algorithm [Ershov 1958] [Sethi, Ullman 1970]
- Code generation using pattern matching
 - For trees: Aho, Johnson 1976 (dynamic programming), Graham/Glanville 1978 (LR parsing), Fraser/Hanson/Proebsting 1992 (IBURG tool), ...
 - For DAGs: [Ertl 1999], [K., Bednarski 2006] (DP, ILP)

Macro Expansion of Quadruples

- Each quadruple is translated to a sequence of one or several target instructions that performs the same operation.

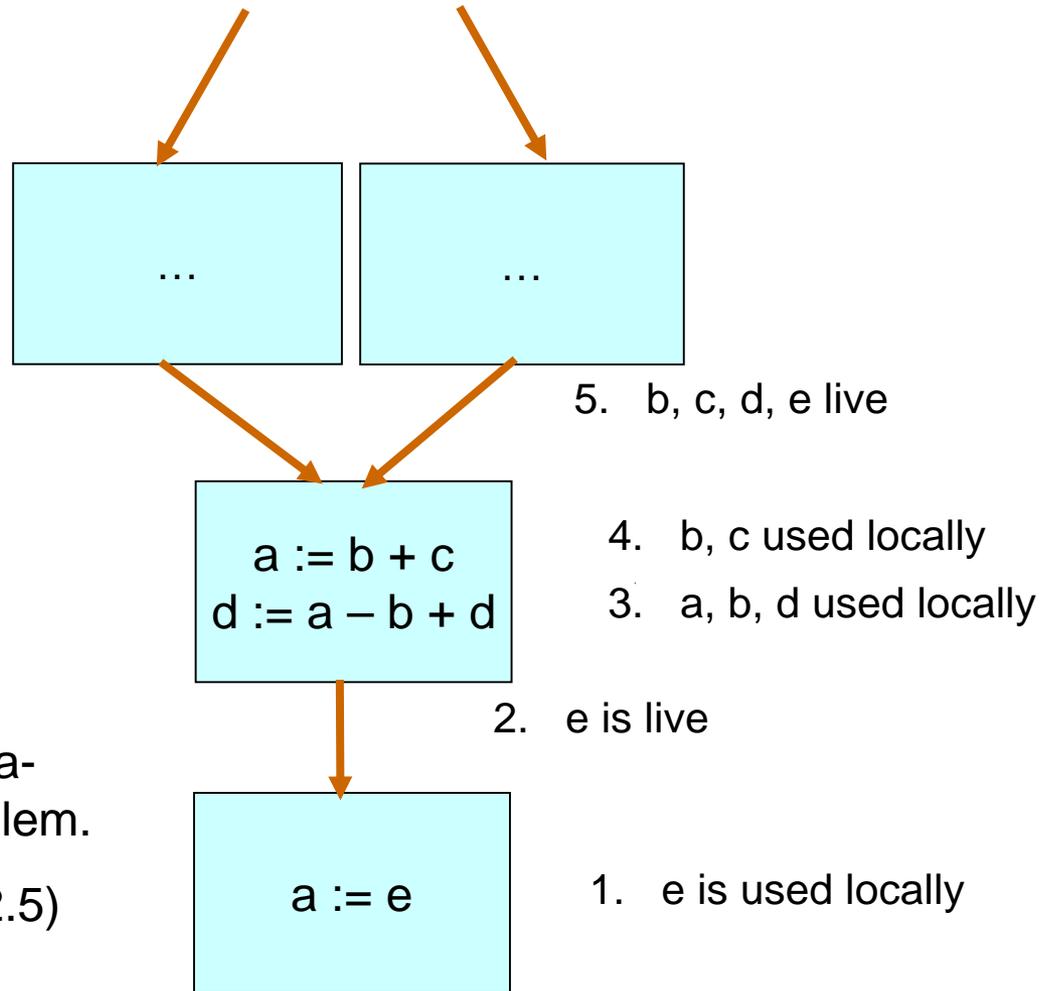
- ☺ very simple, quick to implement
- ☹ bad code quality
 - Cannot utilize powerful instructions/addressing modes that do the job of several quadruples in one step
 - Poor usage of registers

Simple Code Generation Algorithm (1)

- Input: Basic block graph (quadruples grouped in BB's)
- **Principle:**
Keep a (computed) value in a register as long as possible, and move it to memory only
 1. if the register is needed for another calculation
 2. at the end of a basic block
- A variable x is **used locally** after a point p if x 's value is used within the block after p before an assignment to x (if any) is made.
- All variables (except temporaries) are assumed to be **live** (may be used later before possibly being reassigned) after a basic block.

```
BB3:  
( ADD, a, b, x )  
...  
( MUL, x, y, t1 )  
...  
( ASGN, t1, 0, x )  
...
```

"is used locally" and "live"



"live variables"
is a backward data-
flow analysis problem.
(Textbook Sec 9.2.5)

Simple Code Generation Algorithm (2)

reg(R): current content (variable) stored in register R

adr(A): list of addresses ("home" memory location, register)
 where the *current* value of variable A resides

Generate code for a quadruple $Q = (op, B, C, A)$: (op a binary oper.)

- $(RB, RC, RA) \leftarrow \text{getreg}(Q)$; // selects registers for B, C, A – see later
- If $\text{reg}(RB) \neq B$
 generate **LOAD** B, RB; $\text{reg}(RB) \leftarrow B$; $\text{adr}(B) \leftarrow \text{adr}(B) \cup \{RB\}$
- If $\text{reg}(RC) \neq C$ // This step is not needed in the CISC case
 generate **LOAD** C, RC; $\text{reg}(RC) \leftarrow C$; $\text{adr}(C) \leftarrow \text{adr}(C) \cup \{RC\}$
- generate **OP** RB, RC, RA (where **OP** implements op)
 $\text{adr}(A) \leftarrow \{RA\}$; // old value of A in memory is now obsolete
 $\text{reg}(RA) \leftarrow A$;
- If B and/or C no longer used locally and are not live after the current basic block, free their registers RB, RC (update reg, adr)

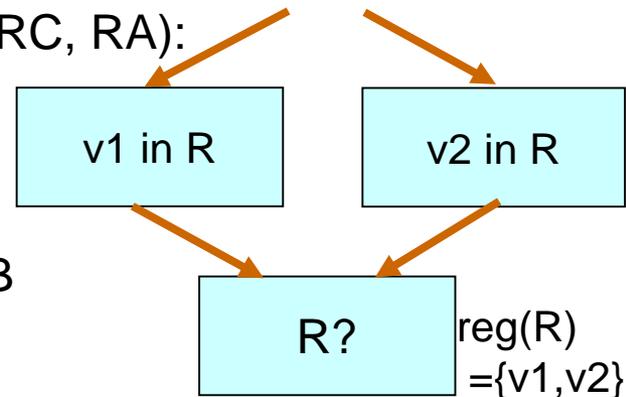
After all quadruples in the basic block have been processed,
 generate **STORES** for all non-temporary var's that only reside in a register

Simple Code Generation Algorithm RISC and CISC (3)

getreg (quadruple $Q = (op, B, C, A)$) determines (RB, RC, RA):

■ Determine RB:

- If $adr(B)$ contains a register RB and $reg(RB) == B$, then use RB.
- If $adr(B)$ contains a register RB with $reg(RB) != B$ or $adr(B)$ contains no register at all:
 - ▶ Use an empty register as RB if one is free.
 - ▶ If no register is free: Select any victim register R that does not hold C.
 - ▶ $V \leftarrow \{ v: R \text{ in } adr(V) \}$ (there *may* be several v's, due to control flow)
 - If for all v in V, $adr(v)$ contains some other place except R: OK, use R for RB.
 - If C in V, retry with another register R instead.
 - If A in V: OK, use R=RA for RB.
 - If all v in V not live after this basic block and not used locally after Q: OK, use R for RB
 - Otherwise: for each v in V,
 - » generate **STORE** R, v; // **spill** register R contents to memory
 - » $adr(v) = adr(v) - \{ R \} \cup \{ \&v \}$;



■ Determine RC: similarly // not needed for CISC

Prefer candidates R that require least spills

Simple Code Generation Algorithm (4)

- Determine RA: similarly, where...
 - Any R with $\text{reg}(R) = \{ A \}$ can be reused as RA
 - If B is not used after Q, and $\text{reg}(RB) = \{ B \}$, can use $RA=RB$.
 - (similarly for C and RC)

Example

Generate code for this basic block in pseudo-quadruple notation:

T1 := a + b;

T2 := c + d;

T3 := e + f;

T4 := T2 * T3;

g := T1 - T4;

Initially, no register is used.

Assume a, b, c, d, e, f, g are live after the basic block,
but the temporaries are not

Machine model: RISC machine model, only operations between registers,
but only 3 registers R0, R1, R2

Solution (NB – several possibilities, dep. on victim selection)

1. LOAD a, R0 // now $\text{adr}(a) = \{ \&a, R0 \}$, $\text{reg}(R0) = \{a\}$
2. LOAD b, R1
3. ADD R0, R1, R2 // now $\text{adr}(T1) = \{R2\}$, $\text{reg}(R2) = \{T1\}$
// reuse R0, R1 for c, d, as a, b still reside in memory
// use R0 for T2, as c still available in memory.
4. LOAD c, R0
5. LOAD d, R1
6. ADD R0, R1, R0 // now $\text{adr}(T2) = \{R0\}$, $\text{reg}(R0) = \{T2\}$
// reuse R1 for e, need a register for f – none free! Pick victim R0
7. STORE R0, 12(fp) // **spill** R0 to memory - a stack location for T2, e.g. at fp+12
8. LOAD e, R1
9. LOAD f, R0
10. ADD R1, R0, R1 // now $\text{adr}(T3) = \{R1\}$, $\text{reg}(R1) = \{T3\}$
11. LOAD T2, R0 // reload T2 to R0
12. MUL R0, R1, R0 // T4 in R0
13. SUB R2, R0, R2 // g in R2
14. STORE R2, g

```
T1 := a + b;  
T2 := c + d;  
T3 := e + f;  
T4 := T2 * T3;  
g := T1 - T4;
```

14 instructions,
including 9 memory accesses
(2 due to spilling)

Example – Slightly Reordered

Generate code for this basic block in pseudo-quadruple notation:



```
T2 := c + d;  
T3 := e + f;  
T4 := T2 * T3;  
T1 := a + b;  
g := T1 - T4;
```

Moving T1 := a + b; here does not modify the semantics of the code. (Why?)

Initially, no register is used.

Assume a, b, c, d, e, f, g are live after the basic block,
but the temporaries are not

Machine model: as above, but only 3 registers R0, R1, R2

Solution for Reordered Example

1. LOAD c, R0
2. LOAD d, R1
3. ADD R0, R1, R2 // now $\text{adr}(T2)=\{R2\}$, $\text{reg}(R2)=\{T2\}$
// reuse R0 for e, R1 for f:
4. LOAD e, R0
5. LOAD f, R1
6. ADD R0, R1, R0 // now $\text{adr}(T3) = \{R0\}$, $\text{reg}(R0)=\{T3\}$
// reuse R0 for T4:
7. MUL R0, R1, R0 // now $\text{adr}(T4)=\{R0\}$, $\text{reg}(R0)=\{T4\}$
// reuse R1 for a, R2 for b, R1 for T1:
8. LOAD a, R1
9. LOAD b, R2
10. ADD R1, R2, R1 // now $\text{adr}(T1) = \{R1\}$, $\text{reg}(R1)=\{T1\}$
// reuse R1 for g:
11. SUB R1, R0, R1 // g in R1
12. STORE R1, g

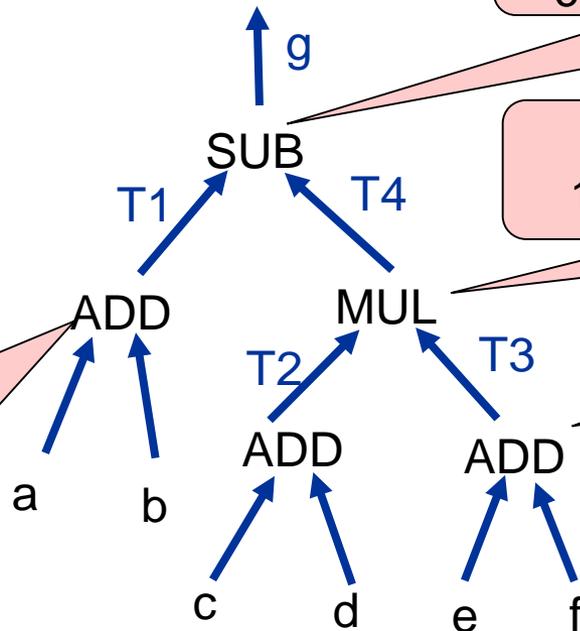
```
T2 := c + d;  
T3 := e + f;  
T4 := T2 * T3;  
T1 := a + b;  
g := T1 - T4;
```

12 instructions,
including 7 memory accesses
No spilling! Why?

Explanation

- Consider the **data flow graph** (here, an expression tree) of the example:

```
T1 := a + b;  
T2 := c + d;  
T3 := e + f;  
T4 := T2 * T3;  
g := T1 - T4;
```



Need 4 registers if subtree for T1 is computed first, 3 registers otherwise!

Need 3 registers to compute this subtree. 1 register will be occupied until last use of T4

Need 2 registers to compute this subtree

Need 1 register to hold value loaded for a leaf node

Need 2 registers to compute this subtree. 1 register will be occupied until last use of T1

Idea:

For each subtree $T(v)$ rooted at node v :
How many registers do we need (at least) to compute $T(v)$ without spilling?

Call this number $label(v)$ (a.k.a. "Ershov numbers")

If possible, at any v , code for "heavier" subtree of v (higher label) should come first.

Generating Code from Labeled Expression Trees

Labeling Algorithm [Ershov 1958] (textbook Section 8.10)

- Yields **space-optimal code** (proof: [Sethi, Ullman 1970])
(using a minimum #registers **without spilling**, or min. # stack locations) for expression **trees**.
 - (Time is fixed as no spill code is generated.)
- The problem is NP-complete for expression DAGs! [Sethi'75]
 - Solutions for DAGs: [K., Rauber '95], [K. '98], [Amaral et al.'00]
- If #machine registers exceeded: Spill code could be inserted afterwards for excess registers, but not necessarily (time-) optimal then...

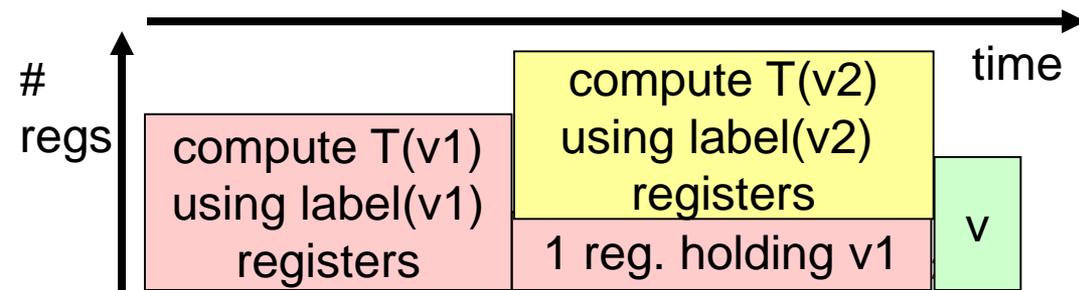
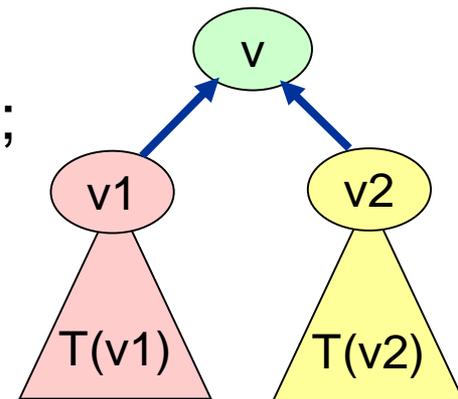
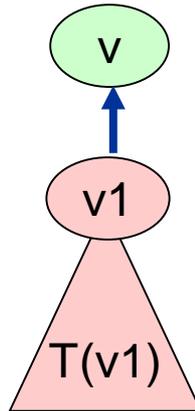
2 phases:

- **Labeling phase**
 - bottom-up traversal of the tree
 - computes $\text{label}(v)$ recursively for each node v
- **Code generation phase**
 - top-down traversal of the tree
 - recursively generating code for heavier subtree first

Labeling Phase for RISC Machine Model

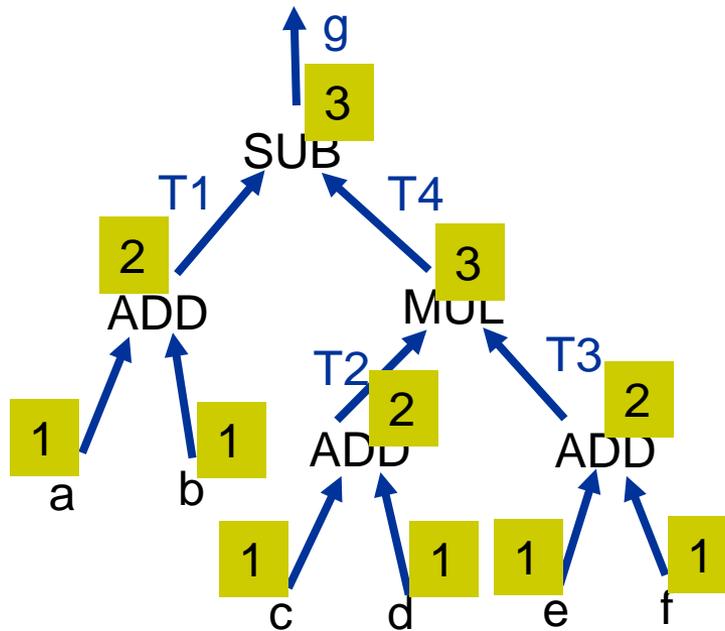
Bottom-up, calculate the register need for each subtree $T(v)$:

- If v is a **leaf node**,
 $\text{label}(v) \leftarrow 1$
- If v is a **unary operation** with operand node v_1 ,
 $\text{label}(v) \leftarrow \text{label}(v_1)$
- If v is a **binary operation** with operand nodes v_1, v_2 :
 $m \leftarrow \max(\text{label}(v_1), \text{label}(v_2))$;
 if $(\text{label}(v_1) = \text{label}(v_2))$ $\text{label}(v) \leftarrow m + 1$;
 else $\text{label}(v) \leftarrow m$;



Example – Labeling Phase

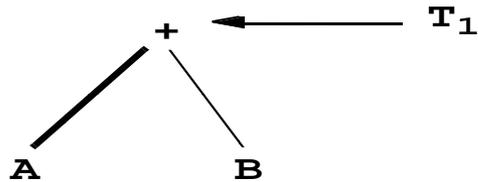
RISC Machine Model



Labeling Phase for CISC Machine Model

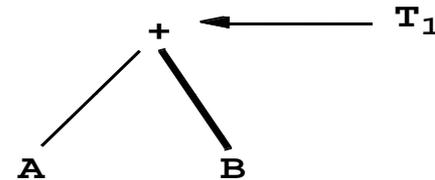
Bottom-up, calculate the register need for each subtree $T(v)$:

- If n is a left leaf
 \Rightarrow LABEL(n):= 1



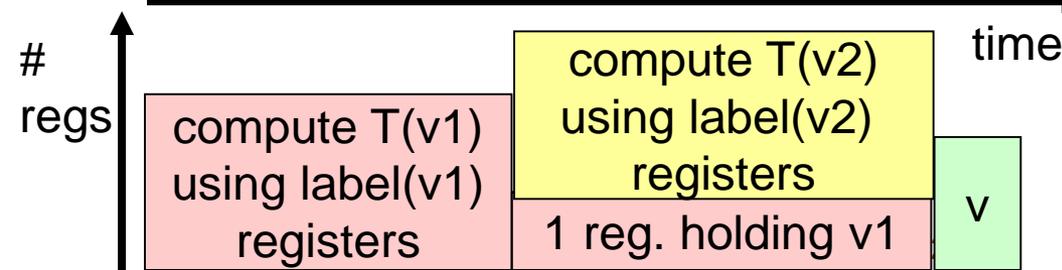
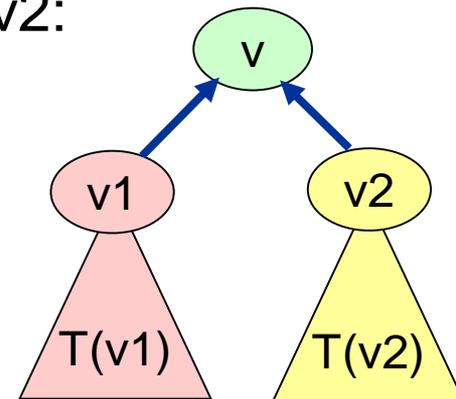
MOVE A,R0

- If n is a right leaf
 \Rightarrow LABEL(n):= 0



ADD B,R0

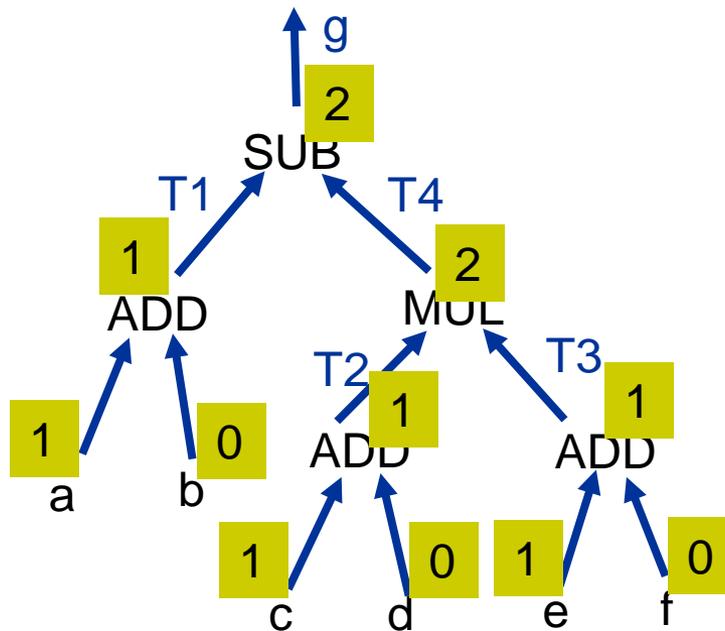
- If v is a **unary operation** with operand node v_1 ,
 $\text{label}(v) \leftarrow \text{label}(v_1)$
- If v is a **binary operation** with operand nodes v_1, v_2 :
 $m \leftarrow \max(\text{label}(v_1), \text{label}(v_2))$;
 if ($\text{label}(v_1) = \text{label}(v_2)$) $\text{label}(v) \leftarrow m + 1$;
 else $\text{label}(v) \leftarrow m$;



Example – Labeling Phase

CISC Machine Model

The CISC Machine model needs fewer registers than RISC since it can perform operations with an operand in memory



Code Generation Phase

RISC Machine Model

- Register stack **freeregs** of currently free registers, initially full
- Register assignment function **reg** from values to registers, initially empty

```
gencode( v ) { // generate space-opt. code for subtree T(v)
```

- If v is a leaf node:
 1. $R \leftarrow \text{freeregs.pop}()$; $\text{reg}(v) \leftarrow R$; // get a free register R for v
 2. `generate(LOAD v, R);`
- If v is a unary node with operand v1 in a register $\text{reg}(v1)=R1$:
 1. `generate(OP R1, R1);` $\text{reg}(v) \leftarrow R1$;
- If v is a binary node with operands v1, v2 in $\text{reg}(v1)=R1$, $\text{reg}(v2)=R2$:
 1. if ($\text{label}(v1) \geq \text{label}(v2)$) // code for T(v1) first:

```
gencode( v1 );
gencode( v2 );
```

else // code for T(v2) first:

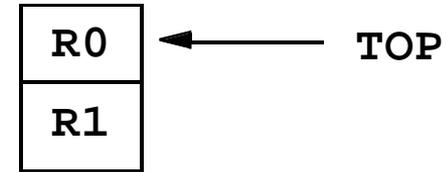
```
gencode( v2 );
gencode( v1 );
```
 2. `generate(OP R1, R2, R1);`
 3. `freeregs.push(R2);` // return register R2, keep R1 for v

```
}
```

Code Generation Phase – More Detailed for CISC Machine with memory-register ops

Data structures:

- **RSTACK**: the register stack, initialised with all available registers.
- **TSTACK**: Stack for temporary variables.



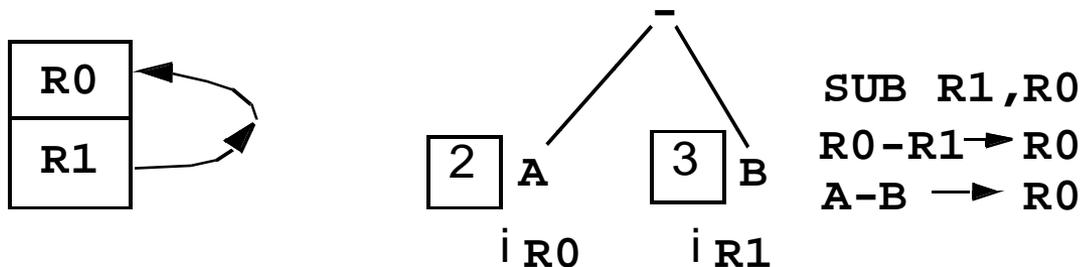
Procedures:

■ Gencode(n):

- Recursive procedure which generates code for sub-trees with root **n**.
- The result is placed in **RSTACK[TOP]**

■ Swap(RSTACK)

- swaps the top two elements at the top of the stack

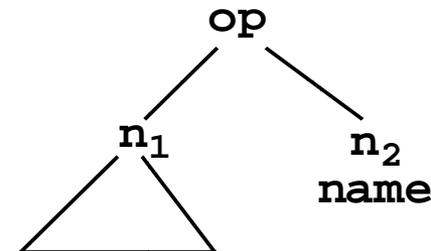


CISC Gencode(n) – 5 different cases (0 – 4) depending on the register needs for the sub-trees

- **Case 0:** $n = \text{left leaf} \Rightarrow$
Print('LOAD ', name, RSTACK[TOP]);



- **Case 1:** If n is a node with children n_1 and n_2 (left and right children, resp.) and $\text{LABEL}(n_2) = 0 \Rightarrow$
Gencode(n1);
Print(op, name, RSTACK[TOP]);



Gencode – case 2

■ Case 2:

If $1 \leq \text{LABEL}(n1) < \text{LABEL}(n2)$ and $\text{LABEL}(n1) < r$ where r is the number of registers in the machine.

Swap(RSTACK);

Gencode(n2);

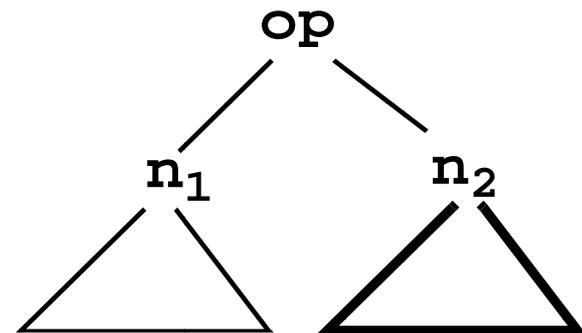
savereg := Pop(RSTACK);

Gencode(n1);

Print(op, savereg, RSTACK[TOP]);

Push(savereg, RSTACK);

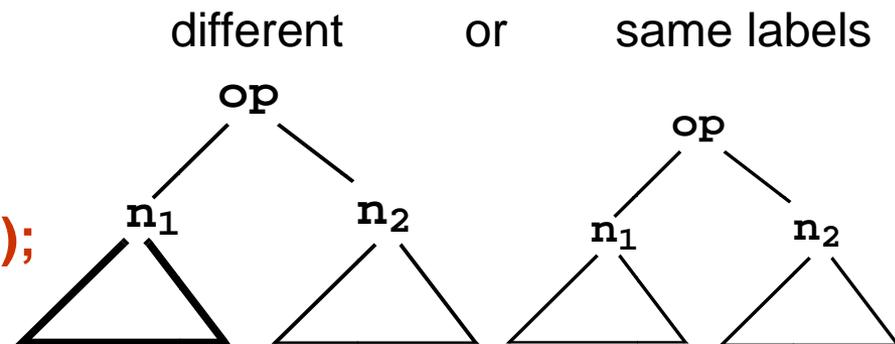
Swap(RSTACK);



Gencode – case 3 and case 4

- **Case 3:** If $1 \leq \text{LABEL}(n2) \leq \text{LABEL}(n1)$ and $\text{LABEL}(n2) < r$ where r is the number of registers in the machine.

```
Gencode(n1);  
savereg := Pop(RSTACK);  
Gencode(n2);  
Print(op, RSTACK[TOP], savereg);  
Push(savereg, RSTACK);
```



- **Case 4:** Both $n1$ and $n2$ have register needs $\geq r \Rightarrow$ store the result in the temporary stack **TSTACK**.

```
Gencode(n2); { recursive call }  
T := Pop(TSTACK);  
Print('STORE ', RSTACK[TOP], T);  
Gencode(n1);  
Print(op, T, RSTACK[TOP]);  
Push(T, TSTACK);
```

Remarks on the Labeling Algorithm

- Still one-to-one or one-to-many translation from quadruple operators to target instructions
- The code generated by `gencode()` is **contiguous** (a subtree's code is never interleaved with a sibling subtree's code).
 - E.g., code for a unary operation v immediately follows the code for its child $v1$.
 - Good for space usage, but sometimes bad for execution time on pipelined processors!
 - There are expression DAGs for which a non-contiguous code exists that uses fewer registers than any contiguous code for it. [K., Rauber 1995]
- The labeling algorithm can serve as a heuristic (but not as optimal algorithm) for DAGs if `gencode()` is called for common subexpressions only at the first time.

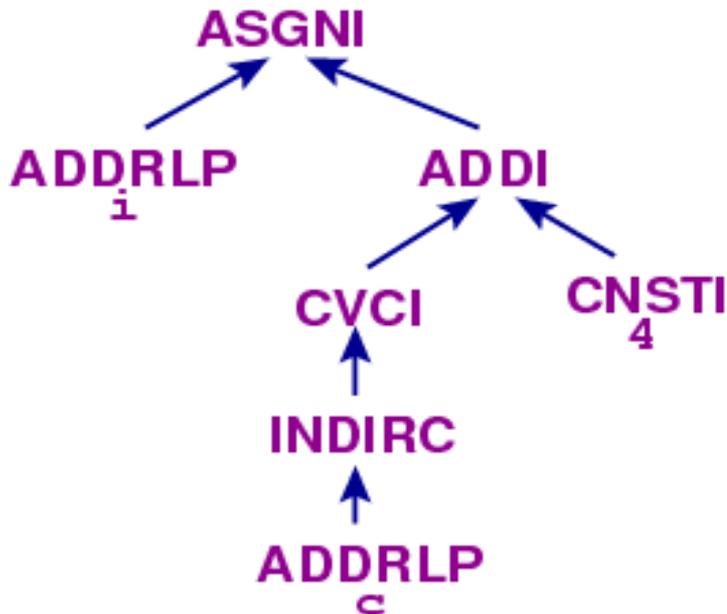
Exercise Ershov Labeling + Code gen

- Draw an AST tree for the expression: $(A+B)-(E-(C+D))$
- Perform the Ershov Labeling algorithm on the tree to compute the register needs labels for each node, assuming a CISC machine model (ops between regs and memory allowed)
- Generate code for the expression

Towards Code Generation by Pattern Matching

- Example: Data flow graph (expression tree) for $i = c + 4$
 - in LCC-IR (DAGs of quadruples) [Fraser,Hanson'95]
 - i, c : local variables

```
{ int i; char c; i=c+4; }
```



Intermediate code in quadruple form:

(Convention: last letter of opcode gives result type: **I**=int, **C**=char, **P**=pointer)

(**ADDRLP**, $i, 0, t1$) // $t1 \leftarrow fp+4$; addr i

(**ADDRLP**, $c, 0, t2$) // $t2 \leftarrow fp+12$; addr c

(**INDIRC**, $t2, 0, t3$) // $t3 \leftarrow M(t2)$; c value

(**CVCI**, $t3, 0, t4$) // convert char to int

(**CNSTI**, $4, 0, t5$) // create int-const 4

(**ADDI**, $t4, t5, t6$)

(**ASGNI**, $t6, 0, t1$) // $M(t1) \leftarrow t6$; store i

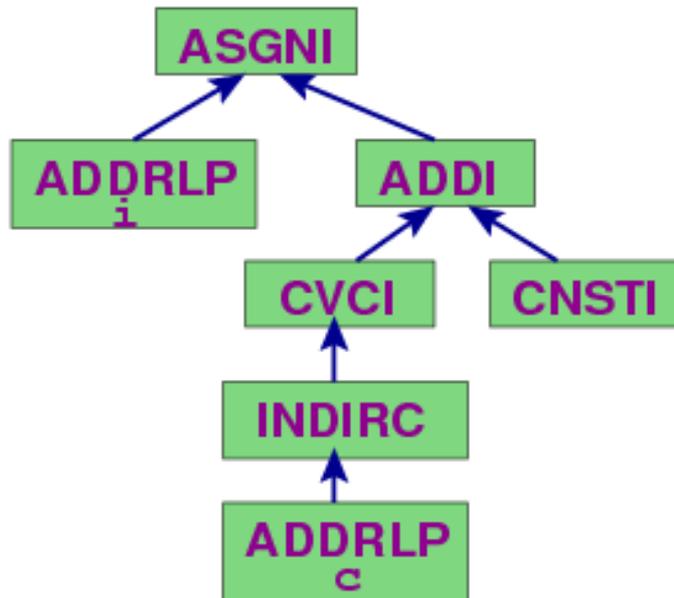
Pattern Matching Idea

- Given: Tree fragment of the intermediate code
- Given: Tree fragment describing a target machine instruction
- If the intermediate code tree fragment match the target machine instruction tree fragment, generate code with that instruction
- This method generates better code, since a single target machine instruction can match a whole tree fragment in the intermediate code

Recall: Macro Expansion

- For the example tree:
 - s1, s2, s3...: "symbolic" registers (allocated but not assigned yet)
 - Target processor has delayed load (1 delay slot)

```
{ int i; char c; i=c+4; }
```



naive instruction selection,
arranged by a postorder traversal:

```

addi fp, #4, s1      ! ADDR_LP (i)
addi fp, #8, s2      ! ADDR_LP (c)
load 0 (s2), s3      ! INDIRC
nop                  ! ", delay slot
                    ! CVCI
addi R0, #4, s4      ! CNSTI R0 assumed 0
addi s3, s4, s5      ! ADDI
store s4, 0 (s1)     ! ASGNI

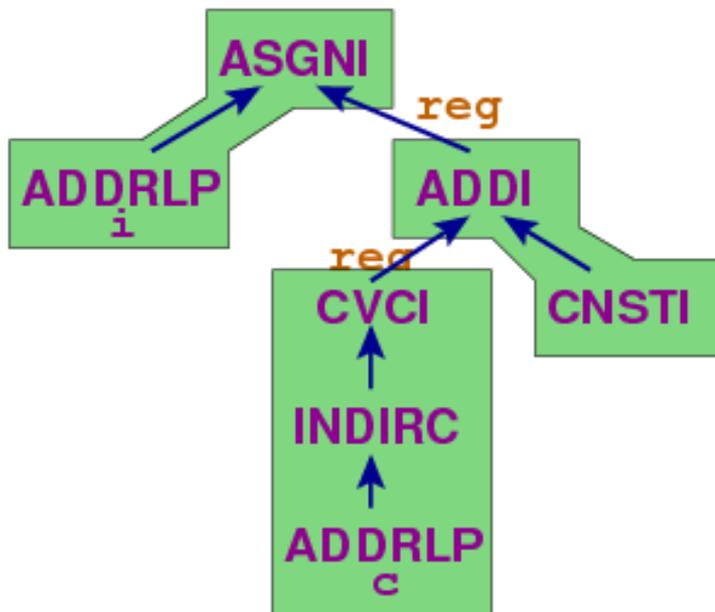
(needs 7cc)  cc - compute cycles
  
```

Using Tree Pattern Matching...

- Utilizing the available addressing modes of the target processor, 3 instructions and only 2 registers are sufficient to cover the entire tree:

```
{ int i; char c; i=c+4; }
```

pattern-matching instruction selection,
arranged by a postorder traversal:



```
load 8(fp), s3    ! ADDRFP+INDIRC+CVCI  
nop              ! ", delay slot
```

```
addi s3, #4, s4   ! CNSTI+ADDI
```

```
store s4, 4(fp)  ! ADDRPLP(i)+ASGNI
```

```
(needs 4cc)
```

Code Generation by Pattern Matching

- Powerful target instructions / addressing modes may cover the effect of several quadruples in one step.
- For each instruction and addressing mode, define a pattern that describes its behavior in terms of quadruples resp. data-flow graph nodes and edges (usually limited to tree fragment shapes: **tree pattern**).
- A pattern **matches** at a node v if pattern nodes, pattern operators and pattern edges coincide with a tree fragment rooted at v
- Each instruction (tree pattern) is associated with a **cost**, e.g. its time behavior or space requirements
- **Optimization problem:** Cover the entire data flow graph (expression tree) with matching tree patterns such that each node is covered *exactly once*, and the accumulated cost of all covering patterns is minimal.

Tree Grammar (Machine Grammar)

(E.g. to be used for code gen pattern matching by parsing)

The target processor is described by a **tree grammar** $G = (N, T, s, P)$

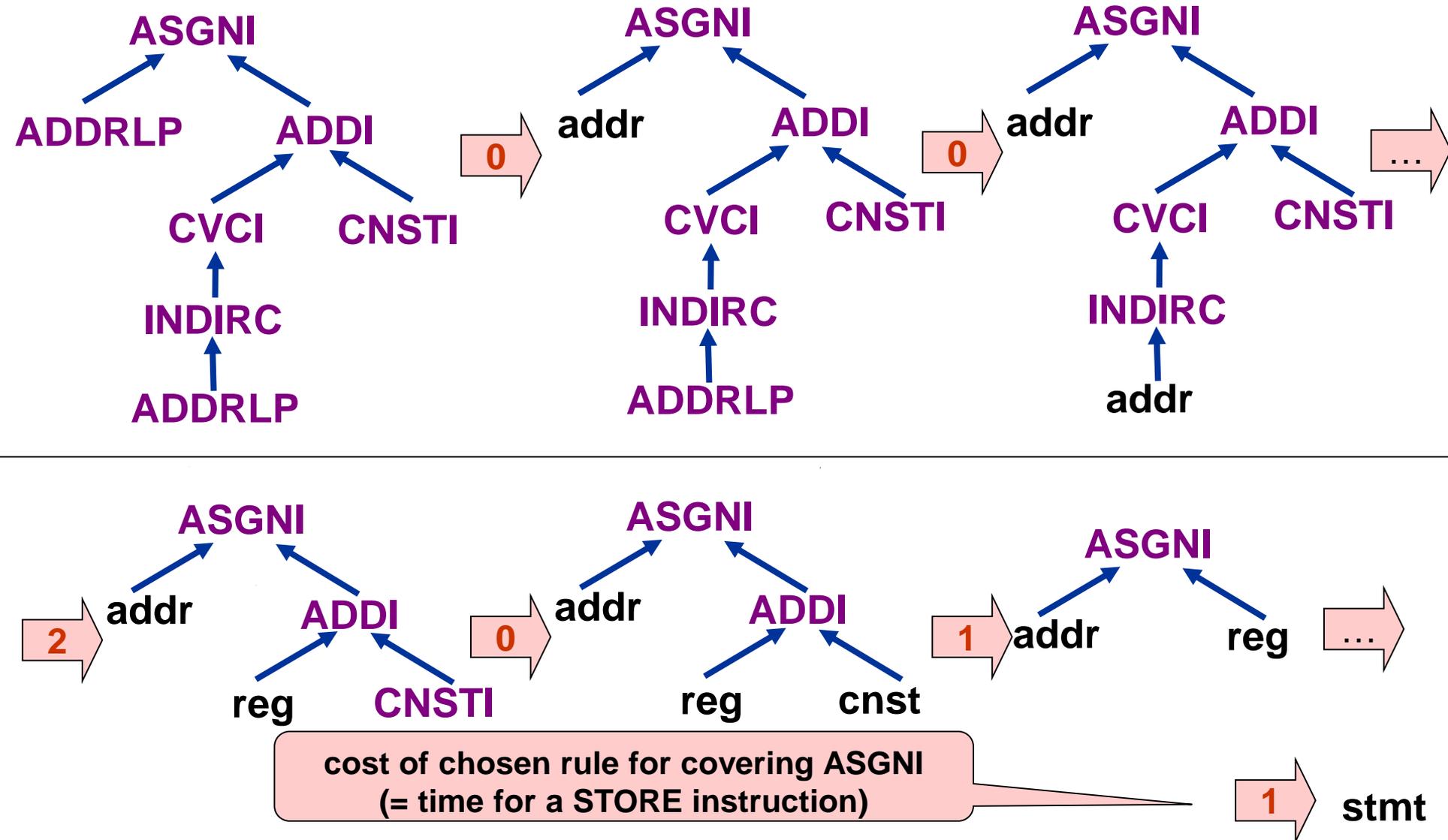
nonterminals $N = \{ \text{stmt, reg, con, addr, mem, ...} \}$ (start symbol is stmt)

terminals $T = \{ \text{CNSTI, ADDRPL, ...} \}$

production rules P :

	target instruction for pattern	cost
reg \rightarrow ADDI (reg, con)	addi %r, %c, %r	1
reg \rightarrow ADDI (reg, reg)	addi %r, %r, %r	1
stmt \rightarrow ASGNI (addr, reg)	store %r, %a	1
stmt \rightarrow ASGNI (reg, reg)	store %r, 0(%r)	1
reg \rightarrow ADDRPL	addi fp, #%d, %r	1
addr \rightarrow ADDRPL	%d(fp)	0
reg \rightarrow addr	addi %a, %r	1
reg \rightarrow INDIRC (addr)	load %a, %r; nop	2
reg \rightarrow INDIRC (reg)	load 0(%r), %r; nop	2
reg \rightarrow CVCI (INDIRC (addr))	load %a, %r; nop	2
reg \rightarrow CVCI (INDIRC (reg))	load 0(%r), %r; nop	2
con \rightarrow CNSTI	%d	0
reg \rightarrow con	addi R0, #%c, %r	1

Derivation Using an LR Parser:



Some Methods for Tree Pattern Matching

- Use a LR-parser for matching [Graham, Glanville 1978]
 - ☺ compact specification of the target machine using a context-free grammar ("machine grammar")
 - ☺ quick matching
 - ☹ not total-cost aware
(greedy local choices at reduce decisions → suboptimal)
- Combine tree pattern matching with dynamic programming for total cost minimization (→ More details in TDDC86 course)
[Aho, Ganapathi, Tjiang '89] [Fraser, Hanson, Proebsting'92]
- An LR parser is stronger than what is really necessary for matching tree patterns in a tree.
 - Right machine model is a **tree automaton**
= a finite automaton operating on input *trees*
rather than flat strings [Ferdinand, Seidl, Wilhelm '92]
- By Integer Linear Programming [Wilson et al.'94] [K., Bednarski '06]