## Meeting 5: <br> Decision-making - the building blocks

## Example

Assume you are about to buy a car and you have a budget of SEK 260000.
You choose between a Skoda (SEK 260 000) and a Renault (SEK 150 000).
Both cars are of model 2020, and have about the same mileage. You are indifferent between them when it comes to exterior and interior looks, comfort, manoeuvring, equipment etc.

The relevant difference between them is that the Skoda is an electric car and the Renault is petrol-driven.

With today's prices of petrol and electricity the Skoda will cost you SEK 4 and the Renault will cost you SEK 9 per 10 km mixed driving.

You estimate to drive a total of 50000 km with the car.

- Which car would you choose based on today's "fuel" prices?
- How would future prices changes affect your choice?


## What characterises a decision problem?

You have a set of alternative acts (actions, decisions) to choose from.
An act can be beneficial for you depending on the state (of the world, of nature)
This depends on the outcome (consequence) of the combination of act and state.

You (normally) know

- which the different acts are
- which the different states are
- your relative preferences of the different consequences

You don't know

- the true state

You may

- be able to assign the probabilities of the different states
- have access to data to assist in your choice

Visualisation of a decision problem
Decision tree

| Actions | States | $c$ |
| :---: | :---: | :---: |
| $c\left(\boldsymbol{a}_{\mathbf{1}}, \boldsymbol{s}_{\mathbf{1}}\right)$ |  |  |
| $\boldsymbol{s}_{\mathbf{1}}$ | $c\left(\boldsymbol{a}_{\mathbf{1}}, \boldsymbol{s}_{\mathbf{2}}\right)$ |  |
| $c$ | $c\left(\boldsymbol{a}_{\mathbf{2}}, \boldsymbol{s}_{\mathbf{1}}\right)$ |  |
| $\boldsymbol{s}_{\mathbf{2}}$ |  |  |

Decision matrix

| Actions | States |  |
| :---: | :---: | :---: |
|  | $\boldsymbol{s}_{\mathbf{1}}$ | $\boldsymbol{s}_{\mathbf{2}}$ |
| $\boldsymbol{a}_{\mathbf{1}}$ | $c\left(\boldsymbol{a}_{\mathbf{1}}, \boldsymbol{s}_{1}\right)$ | $c\left(\boldsymbol{a}_{\mathbf{1}}, \boldsymbol{s}_{2}\right)$ |
| $\boldsymbol{a}_{\mathbf{2}}$ | $c\left(\boldsymbol{a}_{\mathbf{2}}, \boldsymbol{s}_{\mathbf{1}}\right)$ | $c\left(\boldsymbol{a}_{\mathbf{2}}, \boldsymbol{s}_{\mathbf{2}}\right)$ |
| $\boldsymbol{a}_{\mathbf{3}}$ | $c\left(\boldsymbol{a}_{\mathbf{3}}, \boldsymbol{s}_{1}\right)$ | $c\left(\boldsymbol{a}_{\mathbf{3}}, \boldsymbol{s}_{\mathbf{2}}\right)$ |

## Example

You are about to travel from city A to city B by car.
You can choose between taking route $1\left(a_{1}\right)$ or route $2\left(a_{2}\right)$.
If the traffic is heavy $\left(s_{1}\right)$, route 1 will take you approximately 80 minutes, while route 2 will take you approximately 60 minutes.
If the traffic is normal $\left(s_{2}\right)$, route 1 will take you approximately 50 minutes while route 2 will take you approximately 60 minutes.

Your aim is to get to city B as quickly as possible.

Heavy traffic


| Actions | States |  |
| :--- | :---: | :---: |
|  | Heavy <br> traffic | Normal <br> traffic |
| Route 1 | 80 min | 50 min |
| Route $\mathbf{2}$ | 60 min | 60 min |

## The classical description (not in the course book(s))

True state of world: $\boldsymbol{\theta}$ /nature

Data:

Decision procedure: $\delta$
Action (act): $\quad \delta(\boldsymbol{x}), a \quad$ The decision rule becomes an action when applied to given data $\boldsymbol{x}$

Loss function: $\quad L(\delta(\boldsymbol{x}), \boldsymbol{\theta}) \quad$ measures the loss from taking action $\delta(\boldsymbol{x})$ when $\boldsymbol{\theta}$ holds
Payoff function $\quad R(\delta(\boldsymbol{x}), \boldsymbol{\theta})$ measures the payoff from taking action $\delta(\boldsymbol{x})$ when $\boldsymbol{\theta}$ holds
Risk function:

Chance function

$$
D(\delta, \boldsymbol{\theta})=\int_{\boldsymbol{x}} L(\delta(\boldsymbol{x}), \boldsymbol{\theta}) f(\boldsymbol{x} \mid \boldsymbol{\theta}) d \boldsymbol{x}=E_{\widetilde{\boldsymbol{x}}}(L(\delta(\widetilde{\boldsymbol{x}}), \boldsymbol{\theta}))
$$

likelihood

$$
C(\delta, \boldsymbol{\theta})=\int_{\boldsymbol{x}} R(\delta(\boldsymbol{x}), \boldsymbol{\theta}) f(\boldsymbol{x} \mid \boldsymbol{\theta}) d \boldsymbol{x}=E_{\widetilde{\boldsymbol{x}}}(R(\delta(\widetilde{\boldsymbol{x}}), \boldsymbol{\theta}))
$$

Expected loss/payoff with respect to variation in $\boldsymbol{x}$
Functions of the state of world/nature and the decision rule (and not the action)

Decision-making under ignorance [non-probabilistic criteria] no assigned probabilities of the states

A decision rule $\delta^{*}$ is a maximin rule if $\quad C\left(\delta^{*},(\boldsymbol{\theta})\right)=\max _{\delta}\left\{\min _{\boldsymbol{\theta}} C(\delta, \boldsymbol{\theta})\right\}$
i.e. $\boldsymbol{\theta}$ is for each decision rule chosen to be the "worst" state $\Rightarrow$ lowest chance, and among these lowest chances the decision rule that gives the highest one of them is chosen.




$$
\text { A procedure } \delta^{*} \text { is a maximax rule if } \quad C\left(\delta^{*},(\boldsymbol{\theta})\right)=\max _{\delta}\left\{\max _{\boldsymbol{\theta}} C(\delta, \boldsymbol{\theta})\right\}
$$

i.e. $\boldsymbol{\theta}$ is for each decision rule chosen to be the "best" state $\Rightarrow$ highest chance, and among these highest chances the decision rule that gives the highest one of them is chosen.

The maximax rule is a typical optimistic rule


$$
\text { A procedure } \delta^{*} \text { is a minimax rule if } \quad D\left(\delta^{*},(\boldsymbol{\theta})\right)=\min _{\delta}\left\{\max _{\boldsymbol{\theta}} D(\delta, \boldsymbol{\theta})\right\}
$$

i.e. $\boldsymbol{\theta}$ is for each decision rule chosen to be the "worst" state $\Rightarrow$ highest risk, and among these highest risks the decision rule that gives the lowest one is chosen.

The minimax rule is a typical pessimistic rule


Maximin

$$
C\left(\delta^{*},(\boldsymbol{\theta})\right)=\max _{\delta}\left\{\min _{\boldsymbol{\theta}} C(\delta, \boldsymbol{\theta})\right\}
$$

Maximax

$$
C\left(\delta^{*},(\boldsymbol{\theta})\right)=\max _{\delta}\left\{\max _{\boldsymbol{\theta}} C(\delta, \boldsymbol{\theta})\right\}
$$

Minimax

$$
R\left(\delta^{*},(\boldsymbol{\theta})\right)=\min _{\delta}\left\{\max _{\boldsymbol{\theta}} R(\delta, \boldsymbol{\theta})\right\}
$$

Another decision rule under ignorance:
Lexical maximin (or minimax)
If the criteria maximin is used and it happens that two rules $\delta_{1}$ and $\delta_{2}$ both are solutions to $\max _{\delta}\left\{\min _{\boldsymbol{\theta}} C(\delta, \boldsymbol{\theta})\right\}$

- $\operatorname{try} \max _{\delta_{1}, \delta_{2}}\{$ next-to $\underset{\boldsymbol{\theta}}{ }-\min C(\delta, \boldsymbol{\theta})\}$
- 

...and if that cannot separate them

- $\operatorname{try} \max _{\delta_{1}, \delta_{2}}\{$ next-to-next-to-min $C(\delta, \boldsymbol{\theta})\}$
- etc.


## Example

Suppose you are about to make a decision on whether you should buy or rent a new laptop to have for two years $=24$ months.
$\rightarrow \delta_{1}=$ "Buy the laptop" $\delta_{2}=$ "Rent the laptop"


Now, assume $\theta$ is the mean time until the laptop breaks down for the first time.
Let $\theta$ assume three possible values: 6, 12 and 24 months.
The cost of the laptop is $\$ 500$ if you buy it and $\$ 30$ per month if you rent it.
If the laptop breaks down after 12 months (length of warranty) you'll have to replace it for the same cost as you bought it if you bought it. If you rented it you will get a new laptop for no cost provided you proceed with your contract.
Let $X$ be the time in months until the laptop breaks down and assume this variable is exponentially distributed with mean $\theta$.
$\rightarrow$ A cost function (negative payoff function) for an ownership of maximum 24 months may be defined as
$L\left(\delta_{1}(X), \theta\right)=500+500 \cdot \mathbf{1}_{\{X-12\}}$ and
$L\left(\delta_{2}(X), \theta\right)=30 \cdot 24=720$

$$
\text { where } \mathbf{1}_{\{y\}}= \begin{cases}0 & y<0 \\ 1 & y \geq 0\end{cases}
$$

As we will see later, this is not a proper loss function, but we use it like this here for purpose of illustration.

Then $\quad D\left(\delta_{1}, \theta\right)=E\left(500+500 \cdot \mathbf{1}_{\{x-12\}}\right)=500+500 \int_{12} 1 \cdot \frac{1}{\theta} e^{-\frac{x}{\theta}} d x=$

$$
=500 \cdot\left(1+e^{-12 / \theta}\right)
$$

$$
D\left(\delta_{2}, \theta\right)=720
$$

Now compare the risks for the three possible values of $\theta$.

| $\theta$ | $D\left(\delta_{1}, \theta\right)$ | $D\left(\delta_{2}, \theta\right)$ |
| :---: | :---: | :---: |
| 6 | $500 \cdot\left(1+e^{-12 / 6}\right)=568$ | 720 |
| 12 | $500 \cdot\left(1+e^{-12 / 12}\right)=684$ | 720 |
| 24 | $500 \cdot\left(1+e^{-12 / 24}\right)=803$ | 720 |

Clearly the risk for the first rule increases with $\theta$ while the risk for the second is constant. In searching for the minimax rule we therefore focus on the largest possible value of $\theta$ and there $\delta_{2}$ has the smallest risk.

Minimax decision rule $=\delta_{2}$ (rent the laptop)
For the maximin and maximax rules, let $C\left(\delta_{i}, \theta\right)=K-D\left(\delta_{i}, \theta\right)$

Details about the relation will come
$\Rightarrow$ Maximin decision rule $=\delta_{2}$ and Maximax decision rule $=\delta_{1}$

Note! In this description the rules are defined with expected payoffs and losses with respect to data (Chance and Risk functions). All rules may however be applied directly to payoff or loss tables for different states of the world/nature.

Example Route 1 and 2 revisited
The consequences are not expressed in terms of payoff or losses.

But the aim is to reach city B as quickly as possible.
$\Rightarrow$ Preference order is $50 \mathrm{~min}>60 \mathrm{~min}>80 \mathrm{~min}$

|  | States |  |
| :---: | :---: | :---: |
| Acts | Heavy <br> traffic | Normal <br> traffic |
| Route 1 | 80 min | 50 min |
| Route 2 | 60 min | 60 min |

" $>$ " means "preferred to"

## Maximin and Minimax:

Route 1: consequence with lowest preference is 80 min Route 2: consequence with lowest preference is 60 min . $60 \mathrm{~min}>80 \mathrm{~min} \Rightarrow$ The maximin (and minimax) rule is to choose Route 2.

Maximax:
Route 1: consequence with highest preference is 50 min .
Route 2: consequence with highest preference is 60 min .
$50 \mathrm{~min}>60 \mathrm{~min} \Rightarrow$ The maximax rule is to choose Route 1.

Decision-making under risk (Bayes decision rule(s) [probabilistic criteria]
To make formulas a bit simpler we use the notation
$p(\boldsymbol{\theta})=f^{\prime}(\boldsymbol{\theta}) \quad$ prior probability density (or mass) function for $\widetilde{\boldsymbol{\theta}}$
$f(\boldsymbol{x} \mid \boldsymbol{\theta}) \quad$ likelihood
$q(\boldsymbol{\theta} \mid \boldsymbol{x})=f^{\prime \prime}(\boldsymbol{\theta} \mid \boldsymbol{x})$ posterior probability density (or mass) function for $\widetilde{\boldsymbol{\theta}}$

Bayes risk of procedure $\delta: \quad B(\delta)=\int_{\boldsymbol{\theta} \in \boldsymbol{\Theta}} D(\delta, \boldsymbol{\theta}) p(\boldsymbol{\theta}) d \boldsymbol{\theta}$
i.e. the risk function averaged over the prior distribution.

A Bayes rule is a procedure that minimizes the Bayes risk

$$
\delta_{B}=\arg \min _{\delta}\left\{\int_{\boldsymbol{\theta} \in \boldsymbol{\Theta}} D(\delta, \boldsymbol{\theta}) p(\boldsymbol{\theta}) d \boldsymbol{\theta}\right\}
$$

Note! This is about the decision rule, not a specific action

However,

$$
\begin{aligned}
& \int_{\boldsymbol{\theta} \in \boldsymbol{\Theta}} D(\delta, \boldsymbol{\theta}) p(\boldsymbol{\theta}) d \boldsymbol{\theta}=\int_{\boldsymbol{\theta} \in \boldsymbol{\Theta}}\left(\int_{\boldsymbol{x}} L(\delta(\boldsymbol{x}), \boldsymbol{\theta}) f(\boldsymbol{x} \mid \boldsymbol{\theta}) d \boldsymbol{x}\right) p(\boldsymbol{\theta}) d \boldsymbol{\theta} \\
& =\int_{\boldsymbol{x}} \int_{\boldsymbol{\theta} \in \boldsymbol{\Theta}} L(\delta(\boldsymbol{x}), \boldsymbol{\theta}) f(\boldsymbol{x} \mid \boldsymbol{\theta}) p(\boldsymbol{\theta}) d \boldsymbol{\theta} d \boldsymbol{x}=\int_{\boldsymbol{x}} \int_{\boldsymbol{\theta} \in \boldsymbol{\Theta}} L(\delta(\boldsymbol{x}), \boldsymbol{\theta}) q(\boldsymbol{\theta} \mid \boldsymbol{x}) h(\boldsymbol{x}) d \boldsymbol{\theta}
\end{aligned}
$$

where $h(\boldsymbol{x})$ is the marginal likelihood

$$
q(\boldsymbol{\theta} \mid \boldsymbol{x})=\frac{f(\boldsymbol{x} \mid \boldsymbol{\theta}) p(\boldsymbol{\theta})}{h(\boldsymbol{x})}
$$

$\Rightarrow \int_{\boldsymbol{\theta} \in \boldsymbol{\Theta}} D(\delta, \boldsymbol{\theta}) p(\boldsymbol{\theta}) d \boldsymbol{\theta}=\int_{\boldsymbol{x}} h(\boldsymbol{x}) \int_{\boldsymbol{\theta} \in \boldsymbol{\Theta}} L(\delta(\boldsymbol{x}), \boldsymbol{\theta}) q(\boldsymbol{\theta} \mid \boldsymbol{x}) d \boldsymbol{\theta} d=\int_{\boldsymbol{x}} h(\boldsymbol{x}) E_{\widetilde{\boldsymbol{\theta}} \mid x}(L(\delta(\boldsymbol{x}), \widetilde{\boldsymbol{\theta}})) d \boldsymbol{x}$
$\Rightarrow \arg \min _{\delta}\left\{\int_{\boldsymbol{\theta} \in \boldsymbol{\Theta}} D(\delta, \boldsymbol{\theta}) p(\boldsymbol{\theta}) d \boldsymbol{\theta}\right\}=\arg \min _{\delta}\left\{E_{\widetilde{\boldsymbol{\theta}} \mid \boldsymbol{x}}(L(\delta(\boldsymbol{x}), \widetilde{\boldsymbol{\theta}}))\right\} \quad \begin{aligned} & \text { for any given } \\ & \text { value of } \boldsymbol{x}\end{aligned}$
Hence, a procedure that minimises the posterior expected loss is a Bayes decision rule (a probabilistic criterion).

Example: Rent or buy laptop cont.
Assume the three possible values of $\theta(6,12$ and 24) have the prior probabilities $0.2,0.3$ and 0.5 respectively.

Using the Bayes risks directly

$$
p(\theta)=\left\{\begin{array}{cc}
0.2 & \theta=6  \tag{pmf}\\
0.3 & \theta=12 \\
0.5 & \theta=24 \\
0 & \text { otherwise }
\end{array}\right.
$$

$B\left(\delta_{1}\right)=\left\langle\begin{array}{l}\theta \text { is discrete }- \\ \text { valued }\end{array}\right\rangle=\sum_{\theta \in \boldsymbol{\Theta}} D\left(\delta_{1}, \theta\right) p(\theta)=\sum_{\theta \in \boldsymbol{\Theta}} 500\left(1-e^{-12 / \theta}\right) p(\theta)$
$=500\left(1-e^{-12 / 6}\right) \cdot 0.2+500\left(1-e^{-12 / 12}\right) \cdot 0.3+500\left(1-e^{-12 / 24}\right) \cdot 0.5=280$
$B\left(\delta_{2}\right)=\sum_{\theta \in \boldsymbol{\Theta}} D\left(\delta_{2}, \theta\right) p(\theta)=\left\langle R_{D}\left(\delta_{2}, \theta\right) \equiv 720\right\rangle=720 \cdot 0.2+720 \cdot 0.3+720 \cdot 0.5=720$

Thus, the minimal Bayes risk is with procedure $\delta_{1}$ and therefore $\delta_{1}$ is the Bayes decision rule (among $\delta_{1}$ and $\delta_{2}$ ).

## Working with payoffs instead of losses

How do we define payoff and how do we define loss?

The payoff represents the net change in your total "wealth" as a function of your action and the actual state of the world/nature.

The payoff can then be seen as the consequence of your action with the actual state of the world/nature.

The term "wealth" is not necessarily to be interpreted in monetary units, but very often it should be possible - but perhaps difficult - to translate non-monetary consequences to cash equivalents

Defining payoff as the net change means that all costs involved are taken into consideration. Hence the payoff may be negative.

## Example

Suppose you are about to sell apples on an open market a Saturday in October.


You are choosing between selling one of two kinds of apples.

For the first kind - a lower quality apple - you can buy 100 kg apples for SEK 1800, and you deem a reasonable highest selling price to be SEK 30 per kg.

For the second kind - a higher quality apple - you can buy 80 kg for SEK 2000, and you deem a reasonable highest selling price to be 40 per kg.

Lower quality: 100 kg cost SEK 1800, selling price SEK 30 per kg Higher quality: 80 kg cost SEK 2000, selling price SEK 40 per kg

Assume the total demand for your apples that day is 50 kg (no matter what kind of apple).

Your payoff with the action to sell apples of the lower quality will be SEK $50 \times 30-1800=-300$.

Your payoff with the action to sell apples of the higher quality will be SEK $50 \times 40-2000=0$.

If the total demand is 100 kg ? If your action is to sell apples of the lower quality, your payoff will be SEK $100 \times 30-1800=1200$, and if your action is to sell apples of the higher quality, your payoff will be SEK $80 \times 40-2000=1200$.

The loss function is to be interpreted in terms of opportunity loss, i.e. for each state of the world it is the difference between the maximal payoff that can be obtained with that certain state and the payoff of the particular action - referred to as regret in the course book by Peterson.

This means,
$\delta_{B}=\arg \min _{\delta}\left\{E_{\widetilde{\boldsymbol{\theta}} \mid \boldsymbol{x}}(L(\delta(\boldsymbol{x}), \widetilde{\boldsymbol{\theta}}))\right\}=\arg \min _{\delta}\left\{E_{\widetilde{\boldsymbol{\theta}} \mid \boldsymbol{x}}\left(\begin{array}{c}{\underset{d}{\text { max }\{R(d(\boldsymbol{x}), \widetilde{\boldsymbol{\theta}})\}}}_{\text {not dending }}^{\text {on } \delta}\end{array}-R \boldsymbol{x}(\delta(\boldsymbol{x}), \widetilde{\boldsymbol{\theta}})\right\}\right.$
$=\arg \min _{\delta}\left\{-E_{\widetilde{\boldsymbol{\theta}} \mid \boldsymbol{x}}(R(\delta(\boldsymbol{x}), \widetilde{\boldsymbol{\theta}}))\right\}=\arg \max _{\delta}\left\{E_{\widetilde{\boldsymbol{\theta}} \mid \boldsymbol{x}}(R(\delta(\boldsymbol{x}), \widetilde{\boldsymbol{\theta}}))\right\}$
Hence, a procedure that maximises the posterior expected payoff is (also) a Bayes procedure.

Important, though: Payoff is not synonymous with utility
Note that implicit in the derivation above is:
$E_{\widetilde{\boldsymbol{\theta}} \mid \boldsymbol{x}}(L(\delta(\boldsymbol{x}), \widetilde{\boldsymbol{\theta}}))=E_{\widetilde{\boldsymbol{\theta}} \mid x}\left(\max _{d}\{R(d(\boldsymbol{x}), \widetilde{\boldsymbol{\theta}})\}\right)-E_{\widetilde{\boldsymbol{\theta}} \mid \boldsymbol{x}}(R(\delta(\boldsymbol{x}), \widetilde{\boldsymbol{\theta}}))=T-E_{\widetilde{\boldsymbol{\theta}} \mid \boldsymbol{x}}(R(\delta(\boldsymbol{x}), \widetilde{\boldsymbol{\theta}}))$
where $T$ is a quantity that does not depend on the action $\delta(\boldsymbol{x})$

The "complete" theoretical description hence takes its standpoint from decision procedures.

This covers both decision-making under ignorance (non-Bayesian) and decisionmaking under risk (Bayesian).

However, most accounts (including the course book(s)) of this course would focus on particular actions.

Thus we can say
Under ignorance (no probabilities and no data)...

- The maximin action is the action that has highest minimum payoff
- The minimax action is the action that has the lowest maximum loss
- The maximax action is the action that has the highest maximum payoff

Under risk (probabilities of states assigned, data may be available) ...

- A Bayes action is an action that maximises the expected payoff (prior or posterior) - equivalent to an action that minimises the expected loss (prior or posterior)

The relation between (opportunity) loss $(L)$ and payoff $(R)$ for an action $a^{*}$ is thus

$$
L\left(a^{*}, \boldsymbol{\theta}\right)=\max _{a}\{R(a, \boldsymbol{\theta})\}-R\left(a^{*}, \boldsymbol{\theta}\right)
$$

This implies that the loss can never be negative, while the payoff can be positive, negative or zero.

Applied to a payoff table with $m$ rows and $n$ column, where each row represents an action and each column a state of the world:
$L_{i j}=\max _{k}\left\{R_{k j}\right\}-R_{i j} \quad i=1, \ldots, m, j=1, \ldots, n$

|  | $\theta_{1}$ | $\theta_{2}$ | $\ldots$ |
| :---: | :---: | :---: | :---: |
| $a_{1}$ | $L_{11}, R_{11}$ | $L_{12}, R_{12}$ |  |
| $a_{2}$ | $L_{21}, R_{21}$ | $L_{22}, R_{22}$ |  |
| $\ldots$ |  |  |  |

where $L_{i j}$ and $R_{i j}$ stand for the loss and the payoff respectively when action $i$ is chosen with state $j$ of the world.

## (In)Admissibility of actions

An action $a_{1}$ is inadmissible if there exists another action $a_{2}$ such that
$R\left(a_{2}, \boldsymbol{\theta}\right) \geq R\left(a_{1}, \boldsymbol{\theta}\right)$ or $L\left(a_{2}, \boldsymbol{\theta}\right) \leq L\left(a_{1}, \boldsymbol{\theta}\right)$
for all states of the world/nature $\theta$ and
$R\left(a_{2}, \boldsymbol{\theta}\right)>R\left(a_{1}, \boldsymbol{\theta}\right)$ or $L\left(a_{2}, \boldsymbol{\theta}\right)<L\left(a_{1}, \boldsymbol{\theta}\right)$ (strict inequality) for at least one state $\theta$

The action $a_{2}$ is then said to dominate action $a_{1}$.

If an action is inadmissible, it should not be considered.

If an action is not dominated by any other action, it is admissible .

Example, apples cont.

We can form a payoff table as

|  | Demand is 50 kg | Demand is 100 kg |
| :---: | :---: | :---: |
| $a_{1}=$ Sell lower <br> quality apples | $R_{11}=-300$ | $R_{12}=1200$ |
| $a_{2}=$ Sell higher <br> quality apples | $R_{21}=0$ | $R_{22}=1200$ |

Since the decision to sell higher quality apples $\left(a_{2}\right)$ would give payoffs that do not fall short of the payoffs given by selling lower quality apples $\left(a_{1}\right)$, and the payoff with action $a_{2}$ when the demand is 50 kg is higher than the payoff with action $a_{1}$, the decision to sell lower quality apples is inadmissible.

> provided the only states of the world (possible demands) considered are 50 kg and 100 kg

## Example

Given the loss table below ...

| States: |  | I | II | III |
| :---: | :---: | :---: | :---: | :---: |
|  | 1 | 0 | 3 | 6 |
|  | 2 | 1 | 1 | 0 |
|  | 3 | 4 | 0 | 1 |


|  | $I$ | $I I$ | $I I I$ |
| :---: | :---: | :---: | :---: |
| 1 | 12 | 7 | 9 |
| 2 |  |  |  |
| 3 |  |  |  |

The relationship between Loss and Payoff for action $i$, state $j$, here $L_{i j}$ and $R_{i j}$ is

$$
L_{i j}=\max _{k} R_{k j}-R_{i j} \Leftrightarrow R_{i j}=\max _{k} R_{k j}-L_{i j}
$$

Loss table

| States: |  | $I$ | II | III |
| :---: | :---: | :---: | :---: | :---: |
| $$ | 1 | 0 | 3 | 6 |
|  | 2 | 1 | 1 | 0 |
|  | 3 | 4 | 0 | 1 |


|  | $I$ | $I I$ | III |
| :---: | :---: | :---: | :---: |
| 1 | 12 | 7 | 9 |
| 2 |  |  |  |
| 3 |  |  |  |

$$
\begin{aligned}
& L_{i j}=\max _{k} R_{k j}-R_{i j} \Leftrightarrow \\
& R_{i j}=\max _{k} R_{k j}-L_{i j}
\end{aligned}
$$

$$
\Rightarrow \max _{k} R_{k 1}-R_{11}=L_{11}=0 \Rightarrow \max _{k} R_{k 1}=R_{11}=12
$$

$$
\max _{k} R_{k 2}-R_{12}=L_{12}=3 \Rightarrow \max _{k} R_{k 2}=3+R_{12}=10
$$

$$
\max _{k} R_{k 3}-R_{13}=L_{13}=6 \Rightarrow \max _{k} R_{k 3}=6+R_{13}=15
$$

$\Rightarrow$ The payoff table is

|  | I | II | III |
| :---: | :---: | :---: | :---: |
| 1 | 12 | 7 | 9 |
| 2 | $12-1=11$ | $10-1=9$ | $15-0=15$ |
| 3 | $12-4=8$ | $10-0=10$ | $15-1=14$ |


|  | I | II | III |
| :---: | :---: | :---: | :---: |
| 1 | 12 | 7 | 9 |
| 2 | 11 | 9 | 15 |
| 3 | 8 | 10 | 14 |

Using the loss table

|  | $I$ | $I I$ | $I I I$ |
| :---: | :---: | :---: | :---: |
| 1 | 0 | 3 | 6 |
| 2 | 1 | 1 | 0 |
| 3 | 4 | 0 | 1 |

we see that no action gives a higher or equal loss compared to any of the other actions for all three states of the world. Hence, there is no inadmissible action.

Using the payoff table

|  | $I$ | $I I$ | $I I I$ |
| :---: | :---: | :---: | :---: |
| 1 | 12 | 7 | 9 |
| 2 | 11 | 9 | 15 |
| 3 | 8 | 10 | 14 |

we see that no action gives a lower or equal payoff compared to any of the other actions for all three states of the world .

