Master's Thesis Proposal

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Graceful Graphs

A graph $G = (V, E)$ is said to be labeled by f if each vertex $v \in V$ is assigned a non-negative integer value $f(v)$ and each edge $e = uv \in E$ is assigned the value $|f(u) - f(v)|$. The labeling is graceful if $f : V \to \{0, 1, 2, \ldots, |E|\}$ is an injection and if all edges of G have distinct labels from $\{1, 2, \ldots, |E|\}$. A graph is said to be graceful if it admits a graceful labelling. There is a famous and well investigated conjecture (the Graceful Tree Conjecture) stating that every tree is graceful. For a through survey of graceful labelings, see [1].

The computational complexity of deciding whether a graph is graceful is not known. In fact, it is a rather well known example of the rare natural problems in NP which are not known to be NP-complete nor in P [2, 3]. The goal of the project is to determine the complexity of the graceful graph problem.

There are essentially two known types of necessary conditions for a graph to be graceful:
1. The graph needs to have the right parity.
2. The graph needs to have the right density (not too many/few edges).
One possible starting point for the project could be to try to strengthen the known necessary conditions, cf. [4]. The ultimate (and maybe unrealistic) goal in this direction would be to come up with conditions that can be conjectured to be sufficient for a graph to be graceful.

References

[1] J.A. Gallian. A dynamic survey of graph labeling. *The Electronic Journal of Combinatorics*, 2009.

[2] D.S. Johnson. The NP-completeness column: An ongoing guide. *J. Algorithms*, 4(1):87–100, 1983.

[3] D.S. Johnson. The NP-completeness column. *ACM Transactions on Algorithms*, 1(1):160–176, 2005.

[4] T.A. Redl. Graceful graphs and graceful labelings: two mathematical programming formulations and some other new results. *Congr. Numer.*, 164:17–31, 2003.