# CSPs and complexity

An instance of the *constraint satisfaction problem* (CSP) consists of a set of variables, a set of values and a collection of constraints. Each constraint restricts the way in which (a finite subset of) the variables can be assigned values. The job is to determine if values can be assigned to the variables in a way so that all constraints are (simultaneously) satisfied.

The general CSP is NP-complete. However, when the CSP is restricted to a fixed *constraint language* (a set of allowed constraint-types) it can become easy to solve. It has been conjectured that this type of restricted CSPs are either NP-complete or in P. This is quite remarkable since it is known that if $P \neq NP$ an infinite set of intermediate complexity classes must exist. A large amount of research has been directed towards proving the conjecture, but as of yet no resolution has been found.

This project suggests that one work towards this kind of separation-result (that either it is polynomial-time, or it is NP-hard) for some relative of the CSP. Candidates could be the *surjective CSP* (in which a legal assignment of values to the variables must obey extra conditions) or one of several *optimisation versions* of the CSP (in which we can express that some assignments are preferred over others, and we want to find the best).

Depending on what problem(s) one decides to attack, a number of different ways of work might be useful: one would have to read some of the literature on the subject; to demonstrate that certain problems are polynomial-time solvable one might have to combine, extend or design entirely new algorithms; to find interesting examples and/or counterexamples one could possibly be helped by programming and computer experiments; etc.

For more information about the CSP, see [2, 4]; about surjective CSPs, see [1]; and about one of several optimisation versions of the CSP, see [3].

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## References


[1] Manuel Bodirsky, Jan Kára, and Barnaby Martin. The complexity of surjective homomorphism problems – a survey. *Discrete Applied Mathematics*, 160(12):1680–1690, 2012.

[2] Peter Jeavons, David Cohen, and Marc Gyssens. Closure properties of constraints. *Journal of the ACM*, 44(4):527–548, 1997.

[3] Peter Jonsson and Gustav Nordh. Introduction to the maximum solution problem. In Nadia Creignou, Phokion G. Kolaitis, and Heribert Vollmer, editors, *Complexity of Constraints*, volume 5250 of *Lecture Notes in Computer Science*, pages 255–282. Springer Berlin Heidelberg, 2008.

[4] Wikipedia. Constraint satisfaction problem — Wikipedia, the free encyclopedia, 2013. [Online].