Proposal for a bachelor's or master's thesis

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1 Intervals of Strong Partial Clones

In the last 15 years the *algebraic approach* has proved to be a successful approach to study the computational complexity of many hard decision problems [1]. Informally this approach makes it possible to relate problem instances to a particular kind of algebra, and, depending on the structure of this algebra, one can for example say whether a set of instances results in *easy problems* (solvable in polynomial time) or results in *hard problems* (NP-complete or even harder). When considering the Boolean constraint satisfaction problem (SAT) the algebraic approach can even be used to obtain a dichotomy theorem which separates all problems solvable in polynomial time to those problems which are NP-complete.

Unfortunately even such powerful dichotomy theorems are not close to resolving the $\mathrm{P} \stackrel{?}{=} \mathrm{NP}$ question. This question is one of the most well-studied problems in theoretical computer science and was first posed by Kurt Gödel in 1956 in a personal correspondence with John von Neumann [3]. Not only is this problem notoriously difficult, but it is also one of the remaining Millennium problems, and has a monetary reward of one million dollars. Thus, solving $\mathrm{P} \stackrel{?}{=} \mathrm{NP}$ not only implies infinite fame and glory, but allows one to live a luxurious life while tackling the five remaining Millennium problems.

In this thesis proposal we do not aim to solve $\mathrm{P} \stackrel{?}{=} \mathrm{NP}$, although this would be most welcome, but to tackle the related question of comparing NP-complete problems with respect to worst-case running times. In particular we are interested in comparing the complexity of SAT problems parameterized by constraint languages. To do this we exploit a connection between relations implementable by conjunctive queries over a constraint languages, and algebras known as *strong partial clones* [2]. Although there exists quite a lot of strong partial clones, in fact so many that they are impossible to enumerate, it is still possible to describe parts of this structure when considering restricted *intervals*.

The aim of this proposal is thus to describe the structures of certain intervals of strong partial clones. Although the question is quite complicated to formulate, some of these intervals can be enumerated by computer programs using straightforward techniques. The thesis would therefore consist of writing such a program as efficiently as possible, and, depending on the scope of the thesis (master's or bachelor's) try to prove that these intervals can be completely enumerated in the sense that no strong partial clones in the interval are omitted. Even a partial classification of some of these intervals could lead to a better understanding of the complexity differences between NP-complete SAT problems.

Some programming proficiency is required. Tools for concurrent programming (OpenMP and MPI) might also be useful since this would allow efficient computations on any of the university's supercomputers.

2 Contact

Sounds interesting? Do not hesitate to contact the author!

References

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2. P. Jonsson, V. Lagerkvist, G. Nordh, and B. Zanuttini. Complexity of SAT problems, clone theory and the exponential time hypothesis. In *In Proceedings of the 24th Annual ACM-SIAM Symposium on Discrete Algorithms (SODA-2013)*, pages 1264–1277, 2013.
3. R. J. Lipton. The Gödel letter. http://rjlipton.wordpress.com/the-gdel-letter/, 2009.