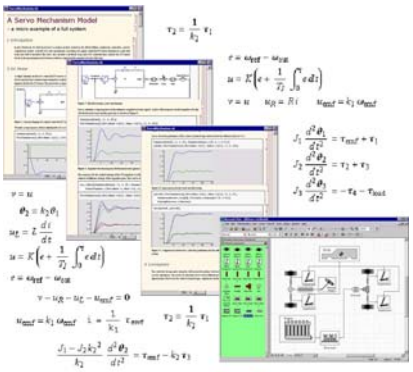
Principles of Object-Oriented Modeling and Simulation with Modelica



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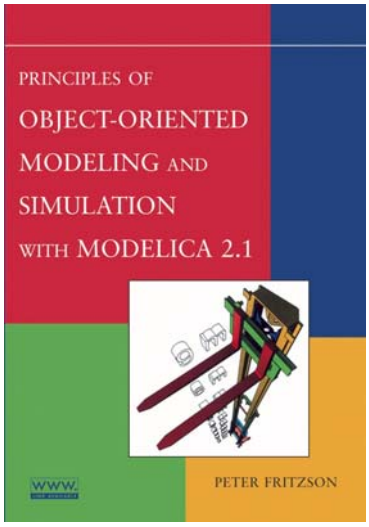
pelab

MODELICA

Course Based on Recent Book, 2004



Peter Fritzson

Principles of Object Oriented Modeling and Simulation with Modelica 2.1

Wiley-IEEE Press

940 pages

MODELICA pelab

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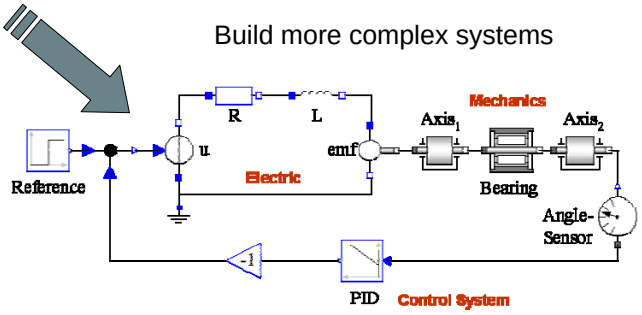
Content

- Modelica – The Next Generation Modeling Language
- OpenModelica Environment
- The Modelica Language – Modelica Classes and Inheritance
  - This section including hands-on exercises on textual modeling using the **OpenModelica** environment
- MathModelica Environment
- Components, Connectors and Connections – Modelica Libraries and Graphical Modeling
- Graphical Modeling Exercises
  - This section including hands-on exercises on graphical modeling using the **MathModelica** environment

Why Modeling & Simulation ?

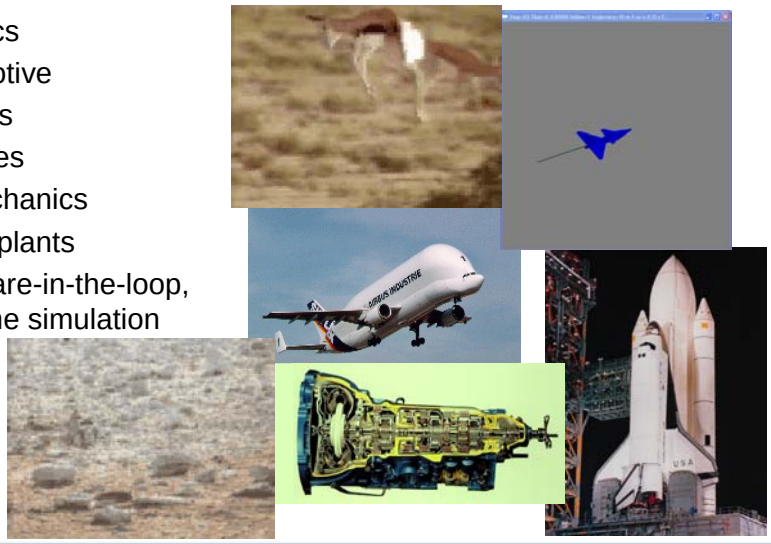
- Increase understanding of complex systems
- Design and optimization
- Virtual prototyping
- Verification

Build more complex systems



Examples of Complex Systems

- Robotics
- Automotive
- Aircrafts
- Satellites
- Biomechanics
- Power plants
- Hardware-in-the-loop, real-time simulation



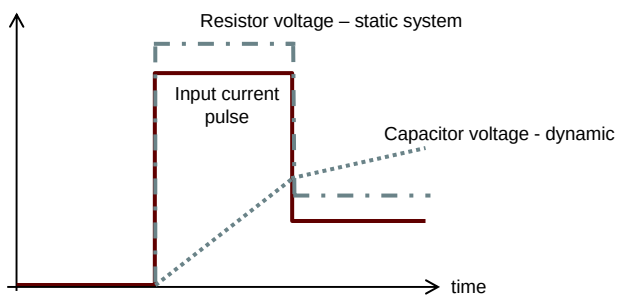
Kinds of Mathematical Models

- Dynamic *vs.* Static models
- Continuous-time *vs.* Discrete-time dynamic models
- Quantitative *vs.* Qualitative models

Dynamic vs. Static Models

A **dynamic** model includes *time* in the model

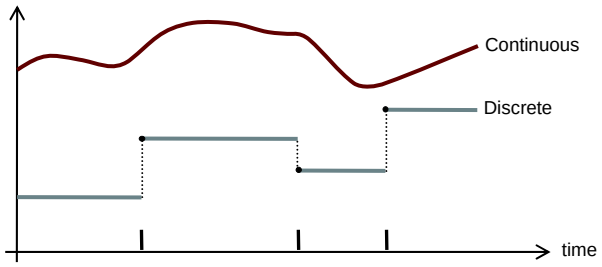
A **static** model can be defined *without* involving *time*



Continuous-Time vs. Discrete-Time Dynamic Models

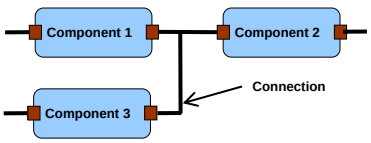
Continuous-time models may evolve their variable values *continuously* during a time period

Discrete-time variables change values a *finite* number of times during a time period

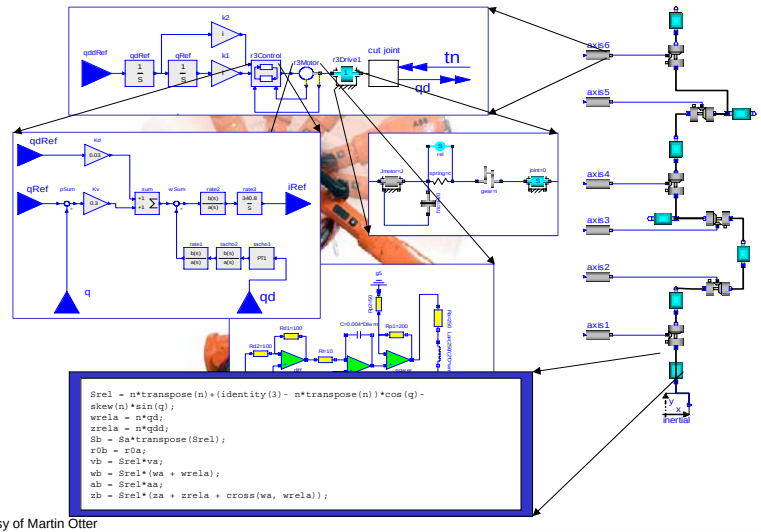


Principles of Graphical Equation-Based Modeling

- Each icon represents a physical component i.e. Resistor, mechanical Gear Box, Pump
- Composition lines represent the actual physical connections i.e. electrical line, mechanical connection, heat flow
- Variables at the interfaces describe interaction with other component
- Physical behavior of a component is described by equations
- Hierarchical decomposition of components

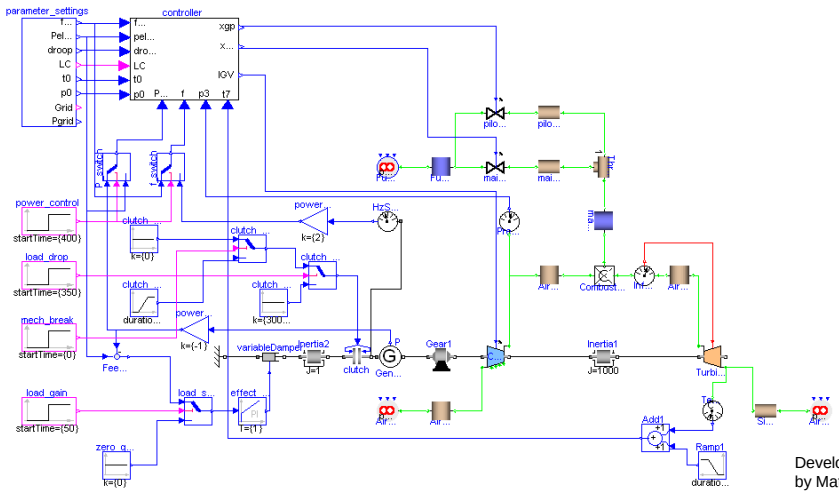


Application Example – Industry Robot



Courtesy of Martin Otter

GTX Gas Turbine Power Cutoff Mechanism



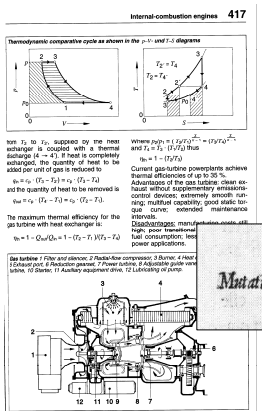
Developed by MathCore for Siemens

Courtesy of Siemens Industrial Turbomachinery AB

Modelica – The Next Generation Modeling Language

Stored Knowledge

Model knowledge is stored in books and human minds which computers cannot access


Internal-combustion engines 417

Thermodynamic comparative cycle as shown in the p-V- und T-S diagrams

from T_2 to T_3, supplied by the heat exchanger is coupled with a thermal discharge (4 → 4'). If heat is completely exchanged, the quantity of heat to be added per unit of gas is reduced to

$$q_{zu} = c_p \cdot (T_3 - T_2) = c_p \cdot (T_3 - T_4)$$

and the quantity of heat to be removed is

$$q_{ab} = c_p \cdot (T_4 - T_1) = c_p \cdot (T_2 - T_1).$$

The maximum thermal efficiency for the gas turbine with heat exchanger is:

$$\eta_{th} = 1 - Q_{ab}/Q_{zu} = 1 - (T_2 - T_1)/(T_3 - T_4)$$

Where $p_2/p_1 = (V_2/V_1)^{\frac{\kappa}{\kappa-1}} = (T_2/T_4)^{\frac{\kappa}{\kappa-1}}$ and $T_4 = T_3 \cdot (T_1/T_2)$ thus

$$\eta_{th} = 1 - (T_1/T_2)$$

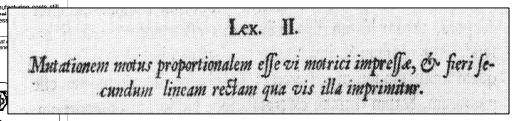
Current gas-turbine powerplants achieve thermal efficiencies of up to 35 %.

Advantages of the gas turbine: clean exhaust without supplementary emissions-control devices; extremely smooth running; multifuel capability; good static torque curve; extended maintenance intervals.

Disadvantages: manufacturing costs still high, power dependent on ambient; fuel consumption; less suitable for power applications.

Gas turbine: 1 Filter and silencer, 2 Radial-flow compressor, 3 Burner, 4 Heat exchanger, 5 Exhaust port, 6 Reduction gearset, 7 Power turbine, 8 Adjustable guide vanes, 9 Gas turbine, 10 Starter, 11 Auxiliary equipment drive, 12 Lubricating oil pump

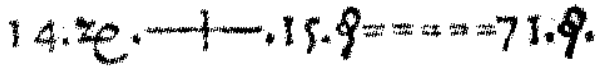
"The change of motion is proportional to the motive force impressed"
– Newton


Lex. II.

Mutationem motus proportionalem esse vi motrici impressæ, & fieri secundum lineam rectam qua vis illa imprimitur.

The Form – Equations

- Equations were used in the third millennium B.C.
- Equality sign was introduced by Robert Recorde in 1557



Newton still wrote text (Principia, vol. 1, 1686)
"The change of motion is proportional to the motive force impressed"

CSSL (1967) introduced a special form of "equation":

```
variable = expression
v = INTEG(F)/m
```

Programming languages usually do not allow equations!

Modelica – The Next Generation Modeling Language

Declarative language

Equations and mathematical functions allow acausal modeling, high level specification, increased correctness

Multi-domain modeling

Combine electrical, mechanical, thermodynamic, hydraulic, biological, control, event, real-time, etc...

Everything is a class

Strongly typed object-oriented language with a general class concept, Java & MATLAB-like syntax

Visual component programming

Hierarchical system architecture capabilities

Efficient, non-proprietary

Efficiency comparable to C; advanced equation compilation, e.g. 300 000 equations, ~150 000 lines on standard PC

Object Oriented Mathematical Modeling with Modelica

- The static *declarative structure* of a mathematical model is emphasized
- OO is primarily used as a *structuring concept*
- OO *is not* viewed as dynamic object creation and sending messages
- *Dynamic model* properties are expressed in a *declarative way* through equations.
- Acausal classes supports *better reuse of modeling and design knowledge* than traditional classes

Modelica Acausal Modeling

- What is *acausal* modeling/design?
- Why does it increase *reuse*?

  The acausality makes Modelica library classes *more reusable* than traditional classes containing assignment statements where the input-output causality is fixed.

- Example: a resistor *equation*:

```
R*i = v;
```

- can be used in three ways:

```
i := v/R;
v := R*i;
R := v/i;
```


Brief Modelica History

- First Modelica design group meeting in fall 1996
  - International group of people with expert knowledge in both language design and physical modeling
  - Industry and academia
- Modelica Versions
  - 1.0 released September 1997
  - 2.0 released March 2002
  - Latest version, 2.2 released March 2005
  - Next version, 3.0 to be released fall 2007
- Modelica Association established 2000
  - Open, non-profit organization

Modelica Conferences

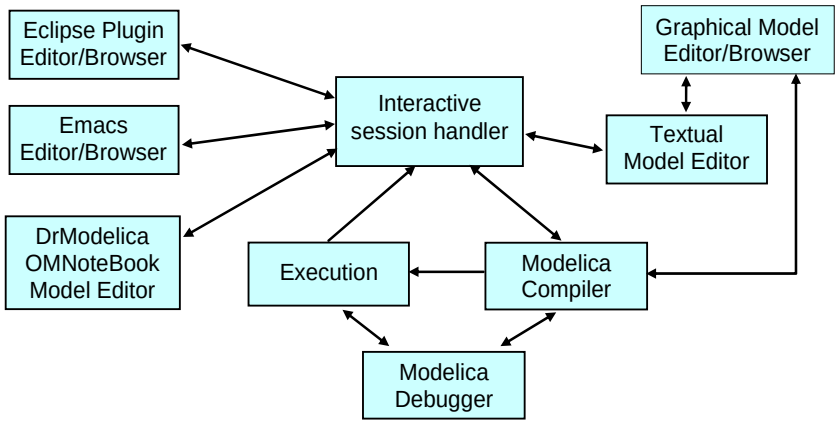
- The 1st International Modelica conference October, 2000
- The 2nd International Modelica conference March 18-19, 2002
- The 3rd International Modelica conference November 5-6, 2003 in Linköping, Sweden
- The 4th International Modelica conference March 6-7, 2005 in Hamburg, Germany
- The 5th International Modelica conference September 4-5, 2006 in Vienna, Austria
- Coming: 6th International Modelica conference March 3-4, 2008 in Bielefeld, Germany

OpenModelica Environment

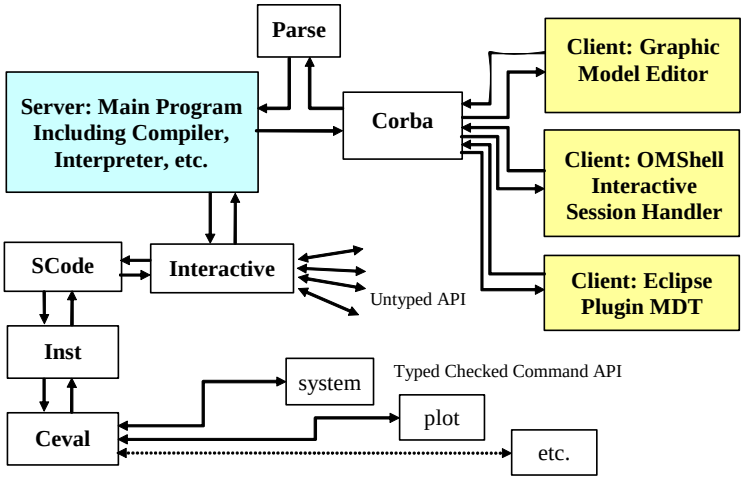
OpenModelica

- Goal: comprehensive modeling and simulation environment for research, teaching, and industrial usage
- Free, open-source for both academic and commercial use
- Available under Berkley New BSD open source license
- The OpenModelica compiler (OMC) implemented in MetaModelica, a slightly extended Modelica
- Invitation for open-source cooperation around OpenModelica, tools, and applications

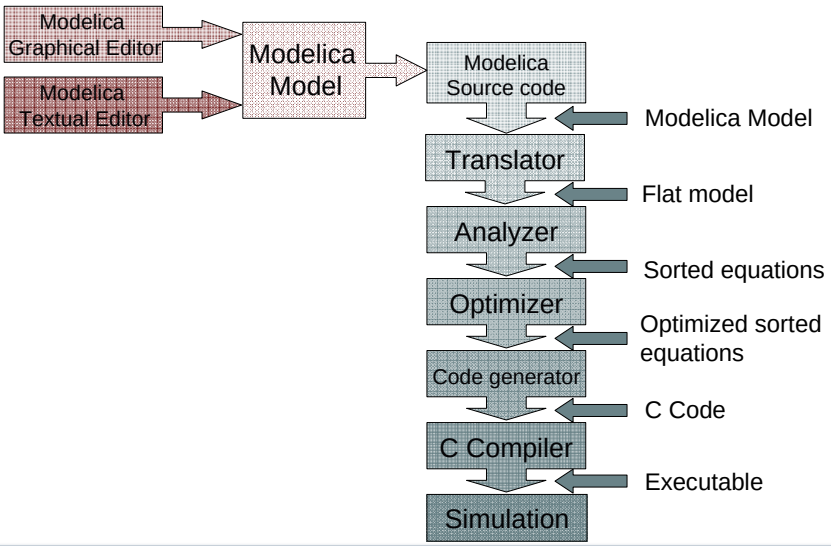
OpenModelica Environment Architecture



OpenModelica Client-Server Architecture



The Compiler (OMC) Translates Model to Simulation Code



Released in OpenModelica 1.4.3

- OpenModelica compiler/interpreter – OMC
- Interactive session handler – OMShell
- OpenModelica Notebook with DrModelica – OMNotebook
- OpenModelica Eclipse plugin MDT
- Available: MathModelica Lite graphic editor (not part of OpenModelica)

Platforms

- All OpenModelica GUI tools (OMShell, OMNotebook, ...) are developed on the Qt4 GUI library, portable between Windows, Linux, Mac
- Both compilers (OMC, MMC) are portable between the three platforms

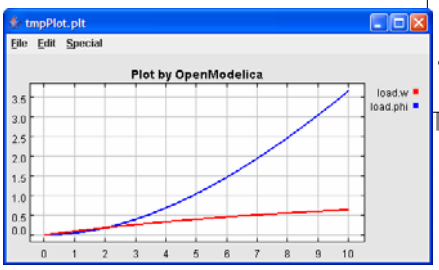
- Windows – currently main development and release platform
- Linux – available
- Mac – recently available

Interactive Session Handler – on dcmotor Example

(Session handler called OMShell – OpenModelica Shell)

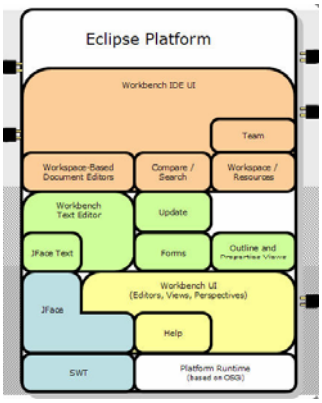
```
>>simulate(dcmotor,startTime=0.0,stopTime=10.0)
>>plot({load.w,load.phi})
```

```
model dcmotor
  Modelica.Electrical.Analog.Basic.Resistor r1(R=10);
  Modelica.Electrical.Analog.Basic.Inductor i1;
  Modelica.Electrical.Analog.Basic.EMF emf1;
  Modelica.Mechanics.Rotational.Inertia load;
  Modelica.Electrical.Analog.Basic.Ground g;
  Modelica.Electrical.Analog.Sources.ConstantVoltage v;
equation
  connect(v.p,r1.p);
  connect(v.n,g.p);
  connect(r1.n,i1.p);
  connect(i1.n,emf1.p);
  connect(emf1.n,g.p);
  connect(emf1.flange_b,load.flange_a);
end dcmotor;
```

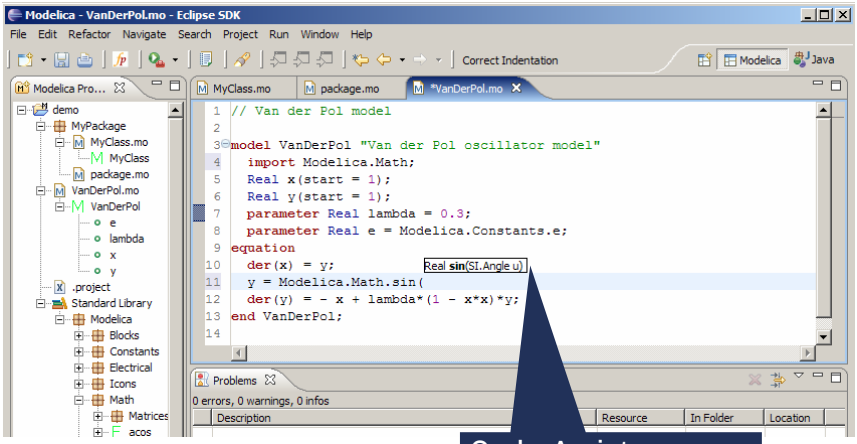


OpenModelica MDT – Eclipse Plugin

- Browsing of packages, classes, functions
- Automatic building of executables; separate compilation
- Syntax highlighting
- Code completion,
  Code query support for developers
- Automatic Indentation
- Debugger
  (Prel. version for algorithmic subset)



OpenModelica Eclipse Plugin – Usage Example



Code Assistance on function calling.

The Modelica Language – Modelica Classes and Inheritance

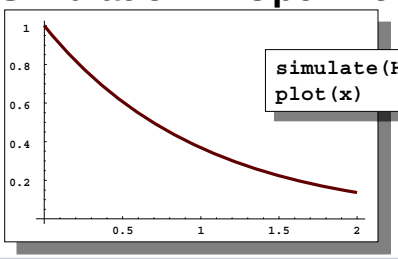
Simplest Model – Hello World!

A Modelica "Hello World" model

Equation: x' = - x
Initial condition: x(0) = 1

```
class HelloWorld "A simple equation"
  Real x(start=1);
equation
  der(x)= -x;
end HelloWorld;
```

Simulation in OpenModelica environment




```
simulate(HelloWorld, stopTime = 2)
plot(x)
```

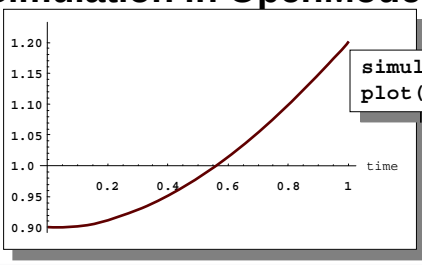

Another Example

Include algebraic equation

Algebraic equations contain
no derivatives

```
class DAEexample
  Real x(start=0.9);
  Real y;
equation
  der(y)+(1+0.5*sin(y))*der(x)
    = sin(time);
  x - y = exp(-0.9*x)*cos(y);
end DAEexample;
```

Simulation in OpenModelica environment

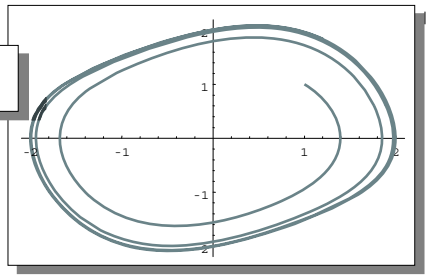



```
simulate(DAEexample, stopTime = 1)
plot(x)
```


Example class: Van der Pol Oscillator

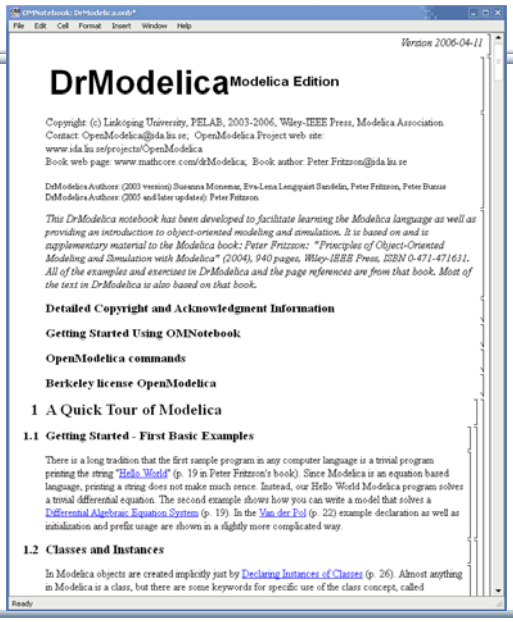
```
class VanDerPol  "Van der Pol oscillator model"
  Real x(start = 1)  "Descriptive string for x"; // x starts at 1
  Real y(start = 1)  "y coordinate";             // y starts at 1
  parameter Real lambda = 0.3;
equation
  der(x) = y;                          // This is the 1st diff equation //
  der(y) = -x + lambda*(1 - x*x)*y;  /* This is the 2nd diff equation */
end VanDerPol;
```

```
simulate(VanDerPol,stopTime = 25)
plotParametric(x,y)
```

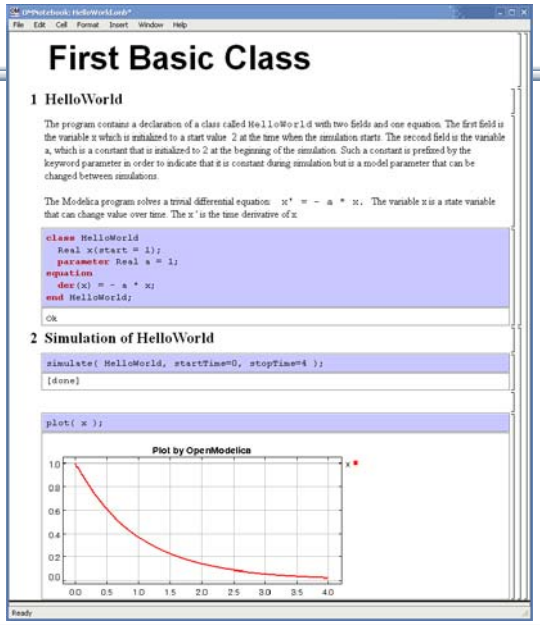


OpenModelica OMNotebook Electronic Notebook with DrModelica

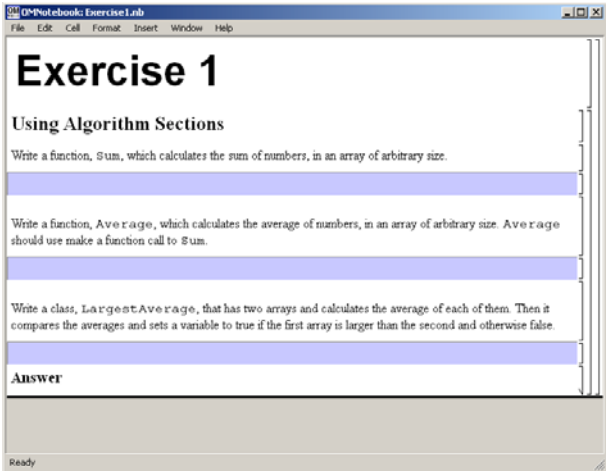
- Primarily for teaching
- Interactive electronic book
- Platform independent



Cells with both Text and Graphics



Exercises and Answers in OMNotebook DrModelica



Some OMNotebook Commands
(see also OpenModelica Users Guide)

- Shift-return (evaluated a cell)
- File Menu (open, close, etc.)
- Text Cursor (vertical), Cell cursor (horisontal)
- Cell types: text cells & executable code cells
- Copy, paste, group cells
- Copy, pase, group text
- Command Completion (shift-tab)

Small Exercises (1.1 and 1.2)

- Locate the VanDerPol model in DrModelica (link from Section 2.1), using OMNotebook!
- **Exercise 1.1**: Simulate and plot VanDerPol. Do a slight change in the model, re-simulate and re-plot.
- Open the Exercises Modelica Tutorial Introduction found in the Tutorial directory.
- **Exercise 1.2**. Simulate and plot the HelloWorld example. Do a slight change in the model, re-simulate and re-plot. Try command-completion, val( ), etc.

```
class HelloWorld "A simple equation"
  Real x(start=1);
equation
  der(x)= -x;
end HelloWorld;
```

```
simulate(HelloWorld, stopTime = 2)
plot(x)
```


OMNotebook Cell Editing Exercises (optional)

- Select and copy a cell (tree to the right)
- Position the horizontal cell cursor: first click between cells; then click once more on top of the horizontal line
- Paste the cell

- Note: You can find most Users Guide examples in the UsersGuideExamples.onb in the testmodels directory

Modelica Language Continued
Variables and Constants

Built-in primitive data types

Boolean	**true** or **false**
Integer	Integer value, e.g. **42** or **–3**
Real	Floating point value, e.g. **2.4e-6**
String	String, e.g. **"Hello world"**
Enumeration	Enumeration literal e.g. **ShirtSize.Medium**

Variables and Constants cont'

- Names indicate meaning of constant
- Easier to maintain code
- Parameters are constant during simulation
- Two types of constants in Modelica
  - `constant`
  - `parameter`

```
constant  Real    PI=3.141592653589793;
constant  String  redcolor = "red";
constant  Integer one = 1;
parameter Real    mass = 22.5;
```


Comments in Modelica

1) Declaration comments, e.g. `Real x "state variable";`

```
class VanDerPol  "Van der Pol oscillator model"
  Real x(start = 1)  "Descriptive string for x"; // x starts at 1
  Real y(start = 1)  "y coordinate";             // y starts at 1
  parameter Real lambda = 0.3;
equation
  der(x) = y;                          // This is the 1st diff equation //
  der(y) = -x + lambda*(1 - x*x)*y;  /* This is the 2nd diff equation */
end VanDerPol;
```

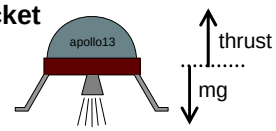
2) Source code comments, disregarded by compiler

2a) C style, e.g. `/* This is a C style comment */`

2b) C++ style, e.g. `// Comment to the end of the line…`

A Simple Rocket Model

Rocket



$$acceleration = \frac{thrust - mass \cdot gravity}{mass}$$

$$mass' = -massLossRate \cdot \mathrm{abs}(thrust)$$

$$altitude' = velocity$$

$$velocity' = acceleration$$

```
class Rocket "rocket class"
  parameter String name;
  Real mass(start=1038.358);
  Real altitude(start= 59404);
  Real velocity(start= -2003);
  Real acceleration;
  Real thrust;  // Thrust force on rocket
  Real gravity; // Gravity forcefield
  parameter Real massLossRate=0.000277;
equation
  (thrust-mass*gravity)/mass = acceleration;
  der(mass)  = -massLossRate * abs(thrust);
  der(altitude) = velocity;
  der(velocity) = acceleration;
end Rocket;
```

new model

parameters (changeable before the simulation)

floating point type

differentiation with regards to time

declaration comment

start value

name + default value

mathematical equation (acausal)

Celestial Body Class

A class declaration creates a *type name* in Modelica

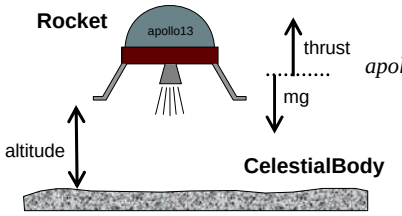
```
class CelestialBody
  constant  Real    g = 6.672e-11;
  parameter Real    radius;
  parameter String  name;
  parameter Real    mass;
end CelestialBody;
```

An *instance* of the class can be declared by *prefixing* the type name to a variable name

```
...
CelestialBody moon;
...
```

The declaration states that **`moon`** is a variable containing an object of type **`CelestialBody`**

Moon Landing



$$apollo.gravity = \frac{moon.g \cdot moon.mass}{(apollo.altitude + moon.radius)^2}$$

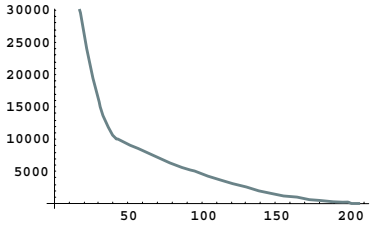
only access inside the class

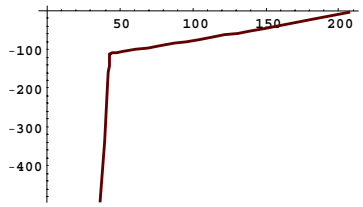
access by dot notation outside the class

```
class MoonLanding
  parameter Real force1 = 36350;
  parameter Real force2 = 1308;
  protected
  parameter Real thrustEndTime = 210;
  parameter Real thrustDecreaseTime = 43.2;
  public
  Rocket          apollo(name="apollo13");
  CelestialBody     moon(name="moon",mass=7.382e22,radius=1.738e6);
equation
  apollo.thrust = if (time < thrustDecreaseTime) then force1
                  else if (time < thrustEndTime) then force2
                  else 0;
  apollo.gravity=moon.g*moon.mass/(apollo.altitude+moon.radius)^2;
end MoonLanding;
```


Simulation of Moon Landing

```
simulate(MoonLanding, stopTime=230)
plot(apollo.altitude, xrange={0,208})
plot(apollo.velocity, xrange={0,208})
```





It starts at an altitude of 59404 (not shown in the diagram) at time zero, gradually reducing it until touchdown at the lunar surface when the altitude is zero

The rocket initially has a high negative velocity when approaching the lunar surface. This is reduced to zero at touchdown, giving a smooth landing

Restricted Class Keywords

- The `class` keyword can be replaced by other keywords, e.g.: `model`, `record`, `block`, `connector`, `function`, ...
- Classes declared with such keywords have restrictions
- Restrictions apply to the contents of restricted classes

- Example: A `model` is a class that cannot be used as a connector class
- Example: A `record` is a class that only contains data, with no equations
- Example: A `block` is a class with fixed input-output causality

```
model CelestialBody
  constant  Real    g = 6.672e-11;
  parameter Real    radius;
  parameter String  name;
  parameter Real    mass;
end CelestialBody;
```


Modelica Functions

- Modelica Functions can be viewed as a special kind of restricted class with some extensions
- A function can be called with arguments, and is instantiated dynamically when called
- More on functions and algorithms later

```
function sum
  input  Real arg1;
  input  Real arg2;
  output Real result;
algorithm
  result := arg1+arg2;
end sum;
```


Inheritance

parent class to Color

restricted kind of class without equations

child class or subclass

keyword denoting inheritance

```
record ColorData
  parameter Real red = 0.2;
  parameter Real blue = 0.6;
  Real   green;
end ColorData;

class Color
  extends ColorData;
equation
  red + blue + green = 1;
end Color;
```

```
class ExpandedColor
  parameter Real red=0.2;
  parameter Real blue=0.6;
  Real green;
equation
  red + blue + green = 1;
end ExpandedColor;
```

Data and behavior: field declarations, equations, and certain other contents are copied into the subclass

Inheriting definitions

```
record ColorData
  parameter Real red = 0.2;
  parameter Real blue = 0.6;
  Real   green;
end ColorData;

class ErrorColor
  extends ColorData;
  parameter Real blue = 0.6;
  parameter Real red = 0.3;
equation
  red + blue + green = 1;
end ErrorColor;
```

Legal!
Identical to the inherited field blue

Inheriting multiple identical definitions results in only one definition

Illegal!
Same name, but different value

Inheriting multiple different definitions of the same item is an error

Inheritance of Equations

```
class Color
  parameter Real red=0.2;
  parameter Real blue=0.6;
  Real green;
equation
  red + blue + green = 1;
end Color;
```

```
class Color2  // OK!
  extends Color;
equation
  red + blue + green = 1;
end Color2;
```

`Color` is identical to `Color2`

Same equation twice leaves one copy when inheriting

```
class Color3  // Error!
   extends Color;
equation
  red + blue + green = 1.0;
  // also inherited: red + blue + green = 1;
end Color3;
```

`Color3` is overdetermined

Different equations means two equations!

Multiple Inheritance

Multiple Inheritance is fine – inheriting both geometry and color

```
class Color
  parameter Real red=0.2;
  parameter Real blue=0.6;
  Real green;
equation
  red + blue + green = 1;
end Color;
```

```
class Point
  Real x;
  Real y,z;
end Point;
```

```
class ColoredPoint
  extends Point;
  extends Color;
end ColoredPoint;
```

multiple inheritance

Equivalent to

```
class ColoredPointWithoutInheritance
  Real x;
  Real y, z;
  parameter Real red = 0.2;
  parameter Real blue = 0.6;
  Real green;
equation
  red + blue + green = 1;
end ColoredPointWithoutInheritance;
```


Multiple Inheritance cont'

Only one copy of multiply inherited class `Point` is kept

```
class Point
  Real x;
  Real y;
end Point;
```

```
class VerticalLine
  extends Point;
  Real vlength;
end VerticalLine;
```

Diamond Inheritance

```
class HorizontalLine
  extends Point;
  Real hlength;
end HorizontalLine;
```

```
class Rectangle
  extends VerticalLine;
  extends HorizontalLine;
end Rectangle;
```


Simple Class Definition – Shorthand Case of Inheritance

- Example:

```
class SameColor = Color;
```

Equivalent to:

inheritance

```
class SameColor
  extends Color;
end SameColor;
```

- Often used for introducing new names of types:

```
type Resistor = Real;
```

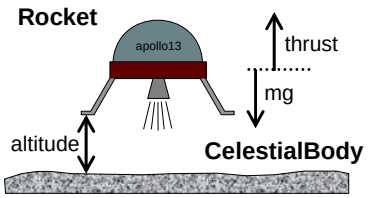
```
connector MyPin = Pin;
```


Inheritance Through Modification

- Modification is a concise way of combining inheritance with declaration of classes or instances
- A modifier modifies a declaration equation in the inherited class
- Example: The class `Real` is inherited, modified with a different `start` value equation, and instantiated as an `altitude` variable:

```
...
 Real altitude(start= 59404);
...
```


The Moon Landing Example Using Inheritance




```
model Rocket "generic rocket class"
  extends Body;
  parameter Real massLossRate=0.000277;
  Real altitude(start= 59404);
  Real velocity(start= -2003);
  Real acceleration;
  Real thrust;
  Real gravity;
equation
  thrust-mass*gravity= mass*acceleration;
  der(mass)= -massLossRate*abs(thrust);
  der(altitude)= velocity;
  der(velocity)= acceleration;
end Rocket;
```

```
model Body "generic body"
  Real   mass;
  String name;
end Body;
```

```
model CelestialBody
  extends Body;
  constant  Real g = 6.672e-11;
  parameter Real radius;
end CelestialBody;
```


The Moon Landing Example using Inheritance cont'

inherited parameters

```
model MoonLanding
 parameter Real force1 = 36350;
 parameter Real force2 = 1308;
 parameter Real thrustEndTime = 210;
 parameter Real thrustDecreaseTime = 43.2;
 Rocket          apollo(name="apollo13", mass(start=1038.358) );
 CelestialBody   moon(mass=7.382e22,radius=1.738e6,name="moon");
equation
 apollo.thrust = if (time<thrustDecreaseTime) then force1
                 else if (time<thrustEndTime) then force2
                 else 0;
 apollo.gravity =moon.g*moon.mass/(apollo.altitude+moon.radius)^2;
end Landing;
```


Inheritance of Protected Elements

If an `extends`-clause is preceded by the `protected` keyword, all inherited elements from the superclass become protected elements of the subclass

```
class Color
  Real red;
  Real blue;
  Real green;
equation
  red + blue + green = 1;
end Color;
```

```
class Point
  Real x;
  Real y,z;
end Point;
```

```
class ColoredPoint
  protected
    extends Color;
  public
    extends Point;
end ColoredPoint;
```

Equivalent to

```
class ColoredPointWithoutInheritance
  Real x;
  Real y,z;
  protected Real red;
  protected Real blue;
  protected Real green;
equation
  red + blue + green = 1;
end ColoredPointWithoutInheritance;
```

The inherited fields from `Point` keep their protection status since that `extends`-clause is preceded by `public`

A protected element cannot be accessed via dot notation!

Do Exercise 1.4 on Model with Simple Systems of Equations

$$\dot{x} = 2 * x * y - 3 * x$$
$$\dot{y} = 5 * y - 7 * x * y$$
$$x(0) = 2$$
$$y(0) = 3$$

Algorithms and Functions

(Only If there is enough time, otherwise skip to page 71, Components, Connectors, Graphical modeling

Algorithm Sections

Whereas equations are very well suited for physical modeling, there are situations where computations are more conveniently expressed as algorithms, i.e., sequences of instructions, also called statements

```
algorithm
  ...
  <statements>
  ...
<some keyword>
```

Algorithm sections can be embedded among equation sections

```
equation
  x = y*2;
  z = w;
algorithm
  x1 := z+x;
  x2 := y-5;
  x1 := x2+y;
equation
  u = x1+x2;
  ...
```


Iteration Using for-statements in Algorithm Sections

```
for <iteration-variable> in <iteration-set-expression> loop
    <statement1>
    <statement2>
    ...
end for
```

The general structure of a `for`-statement with a single iterator

```
class SumZ
  parameter Integer n = 5;
  Real[n]  z(start = {10,20,30,40,50});
  Real sum;
algorithm
  sum := 0;
  for i in 1:n loop      // 1:5 is {1,2,3,4,5}
    sum := sum + z[i];
  end for;
end SumZ;
```

A simple `for`-loop summing the five elements of the vector `z`, within the class `SumZ`

Examples of `for`-loop headers with different range expressions

```
for k in 1:10+2 loop            // k takes the values 1,2,3,...,12
for i in {1,3,6,7} loop         // i takes the values 1, 3, 6, 7
for r in 1.0 : 1.5 : 5.5 loop   // r takes the values 1.0, 2.5, 4.0, 5.5
```


Iterations Using while-statements in Algorithm Sections

```
while <conditions> loop
  <statements>
end while;
```

The general structure of a `while`-loop with a single iterator.

```
class SumSeries
  parameter Real eps = 1.E-6;
  Integer i;
  Real sum;
  Real delta;
algorithm
  i := 1;
  delta := exp(-0.01*i);
  while delta>=eps loop
    sum := sum + delta;
    i := i+1;
    delta := exp(-0.01*i);
  end while;
end SumSeries;
```

The example class `SumSeries` shows the `while`-loop construct used for summing a series of exponential terms until the loop condition is violated , i.e., the terms become smaller than eps.

if-statements

```
if <condition> then
  <statements>
elseif <condition> then
  <statements>
else
  <statementss>
end if
```

The general structure of `if`-statements.
The `elseif`-part is optional and can occur zero or more times whereas the optional `else`-part can occur at most once

The `if`-statements used in the class `SumVector` perform a combined summation and computation on a vector `v`.

```
class SumVector
  Real sum;
  parameter Real v[5] = {100,200,-300,400,500};
  parameter Integer n = size(v,1);
algorithm
  sum := 0;
  for i in 1:n loop
    if v[i]>0 then
      sum := sum + v[i];
    elseif v[i] > -1 then
      sum := sum + v[i] -1;
    else
      sum := sum - v[i];
    end if;
  end for;
end SumVector;
```


Function Declaration

The structure of a typical function declaration is as follows:

```
function <functionname>
  input TypeI1 in1;
  input TypeI2 in2;
  input TypeI3 in3;
  ...
  output TypeO1 out1;
  output TypeO2 out2;
  ...
protected
  <local variables>
  ...
algorithm
  ...
  <statements>
  ...
end <functionname>;
```

All internal parts of a function are optional, the following is also a legal function:

```
function <functionname>
end <functionname>;
```

Modelica functions are *declarative mathematical functions*:

- Always return the same result(s) given the same input argument values

Function Call

Two basic forms of arguments in Modelica function calls:

- *Positional* association of actual arguments to formal parameters
- *Named* association of actual arguments to formal parameters

Example function called on next page:

```
function PolynomialEvaluator
  input  Real A[:];    // array, size defined
                       // at function call time
  input  Real x := 1.0 // default value 1.0 for x
  output Real sum;
protected
  Real   xpower;          // local variable xpower
algorithm
  sum := 0;
  xpower := 1;
  for i in 1:size(A,1) loop
    sum := sum + A[i]*xpower;
    xpower := xpower*x;
  end for;
end PolynomialEvaluator;
```

The function `PolynomialEvaluator` computes the value of a polynomial given two arguments:
a coefficient vector `A` and a value of `x`.

Positional and Named Argument Association

Using *positional* association, in the call below the actual argument `{1,2,3,4}` becomes the value of the coefficient vector `A`, and `21` becomes the value of the formal parameter `x`.

```
...
algorithm
  ...
  p:= polynomialEvaluator({1,2,3,4},21)
```

The same call to the function `polynomialEvaluator` can instead be made using *named* association of actual parameters to formal parameters.

```
...
algorithm
  ...
  p:= polynomialEvaluator(A={1,2,3,4},x=21)
```


External Functions

It is possible to call functions defined outside the Modelica language, implemented in C or FORTRAN 77

```
function polynomialMultiply
  input  Real a[:], b[:];
  output Real c[:] := zeros(size(a,1)+size(b, 1) - 1);
external
end polynomialMultiply;
```

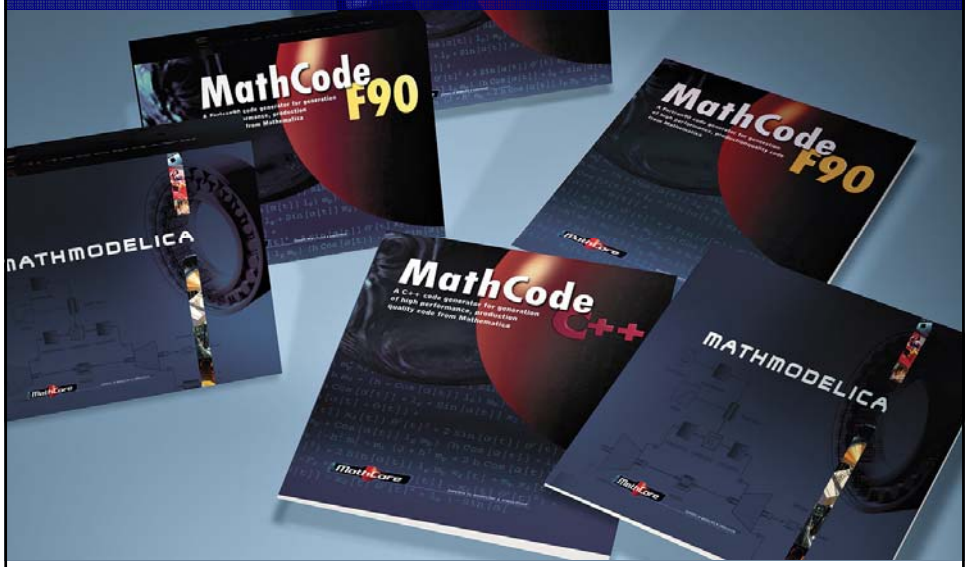
The body of an external function is marked with the keyword **`external`**

If no language is specified, the implementation language for the external function is assumed to be C. The external function `polynomialMultiply` can also be specified, e.g. via a mapping to a FORTRAN 77 function:

```
function polynomialMultiply
  input  Real a[:], b[:];
  output Real c[:] := zeros(size(a,1)+size(b, 1) - 1);
external "FORTRAN 77"
end polynomialMultiply;
```


If enough time, Do Exercise 1.5 on Functions and algorithms

MathModelica Environment



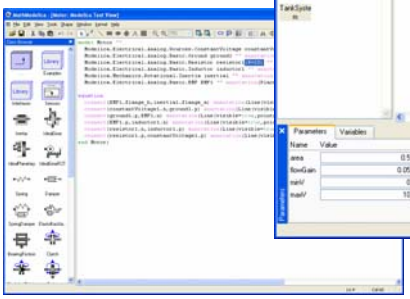
MathModelica Editions

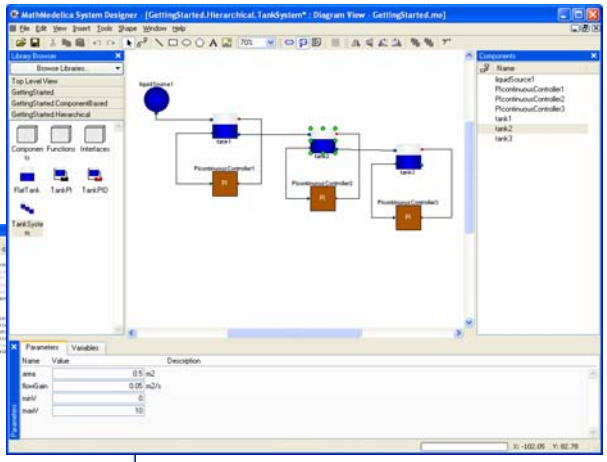
	MathModelica Lite	MathModelica System Designer	MathModelica System Designer Professional
Libraries (kernel)	✓ partial	✓	✓
Model Editor	✓ partial	✓	✓
Simulation Center	⊘	✓	✓
Mathematica connection	⊘	⊘	✓
MathCore Premier Service	⊘	✓	✓

MathModelica® Lite™

Ideal for small student projects. It is free for academic and personal use, but not available for industry.

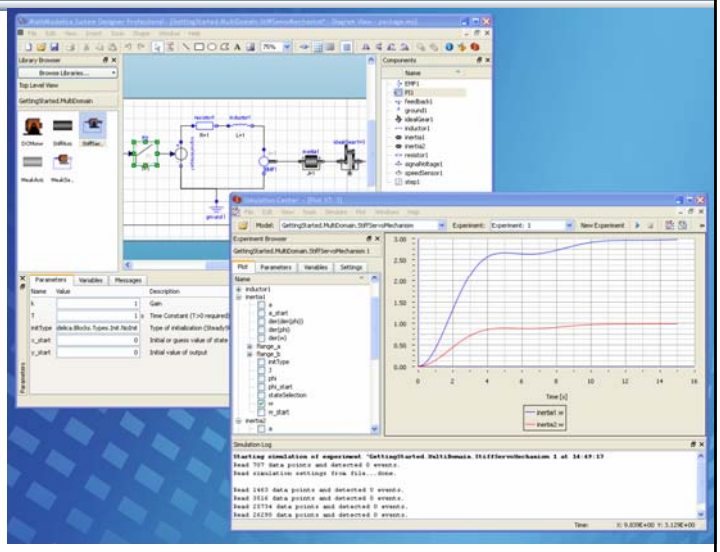
Runs with, but is not part of OpenModelica





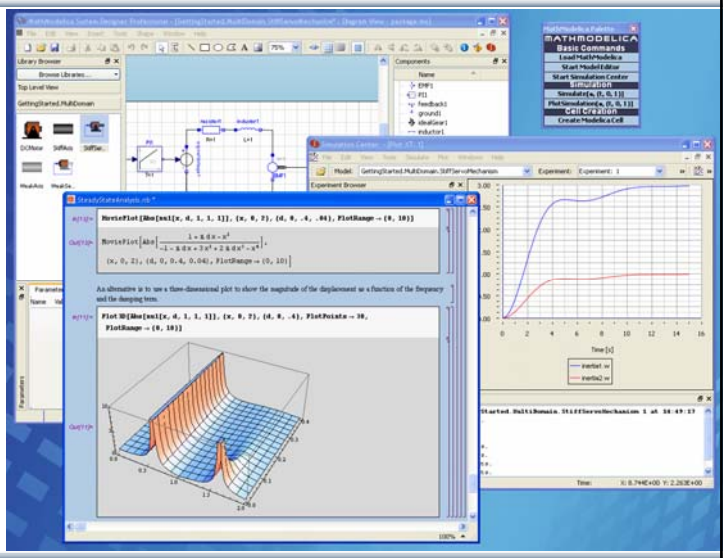
MathModelica® System Designer™

Suited for **modeling and simulation projects** in industry and academia.



MathModelica® System Designer Professional™

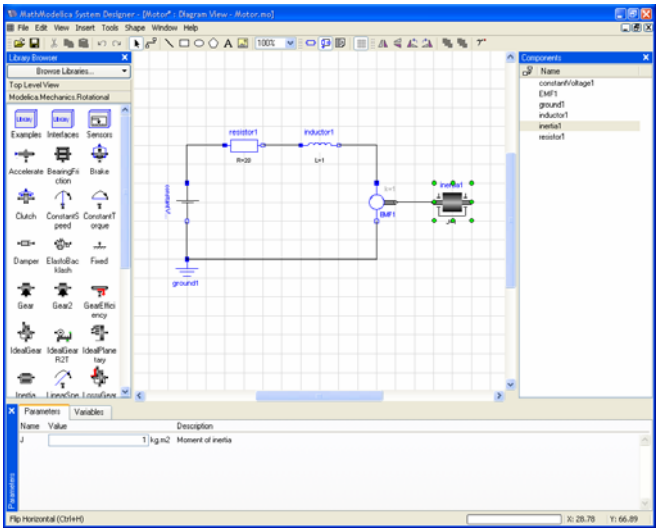
Targeted at **research in industry and academia**, offering unparalleled possibilities for analyzing results.



Components, Connectors and Connections – Modelica Libraries and Graphical Modeling

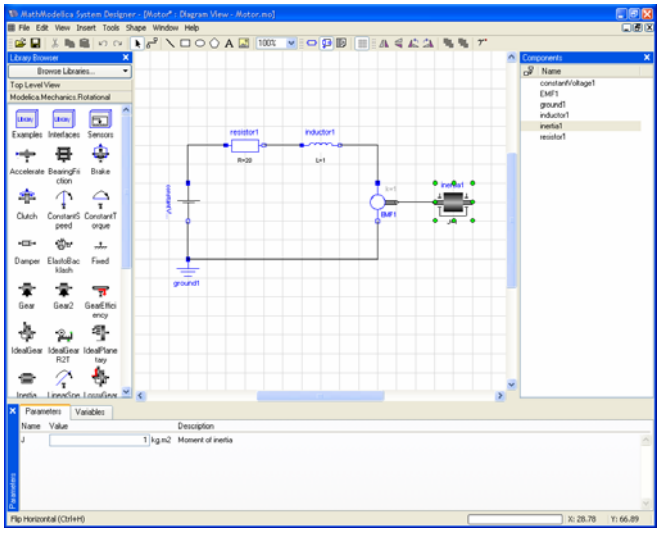
Live example of electric circuit

Graphical Modeling Using Drag and Drop Composition



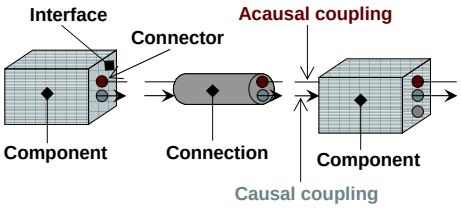
Courtesy MathCore Engineering AB

Completed DCMotor using Graphical Composition



Courtesy MathCore Engineering AB

Software Component Model



A component class should be defined *independently of the environment,* very essential for *reusability*

A component may internally consist of other components, i.e. *hierarchical* modeling

Complex systems usually consist of large numbers of *connected* components

Connectors and Connector Classes

Connectors are instances of *connector classes*

electrical connector

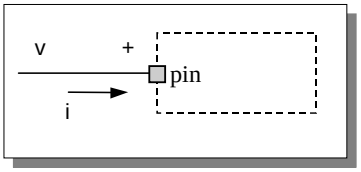
connector class

keyword `flow` indicates that currents of connected pins sum to zero.

an instance `pin` of class `Pin`

```
connector Pin
  Voltage      v;
  flow Current i;
end Pin;

Pin pin;
```



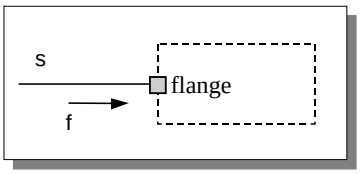
mechanical connector

connector class

an instance `flange` of class `Flange`

```
connector Flange
  Position     s;
  flow Force   f;
end Flange;

Flange flange;
```



The `flow` prefix

Two kinds of variables in connectors:

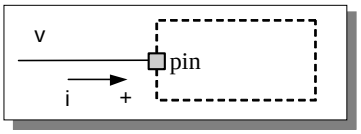
- *Non-flow variables potential* or energy level
- *Flow variables* represent some kind of flow

Coupling

- *Equality coupling*, for non-`flow` variables
- *Sum-to-zero coupling*, for `flow` variables

The value of a `flow` variable is *positive* when the current or the flow is *into* the component

positive flow direction:



Physical Connector Classes Based on Energy Flow

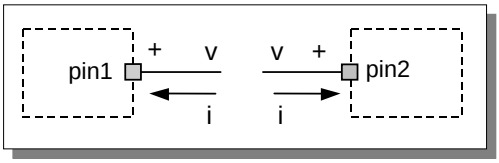
Domain Type	Potential	Flow	Carrier	Modelica Library
Electrical	Voltage	Current	Charge	`Electrical.Analog`
Translational	Position	Force	Linear momentum	`Mechanical.Translational`
Rotational	Angle	Torque	Angular momentum	`Mechanical.Rotational`
Magnetic	Magnetic potential	Magnetic flux rate	Magnetic flux	
Hydraulic	Pressure	Volume flow	Volume	`HyLibLight`
Heat	Temperature	Heat flow	Heat	`HeatFlow1D`
Chemical	Chemical potential	Particle flow	Particles	Under construction
Pneumatic	Pressure	Mass flow	Air	`PneuLibLight`

`connect`-equations

Connections between connectors are realized as *equations* in Modelica

```
connect(connector1,connector2)
```

The two arguments of a `connect`-equation must be references to *connectors*, either to be declared directly *within* the *same class* or be *members* of one of the declared variables in that class




```
Pin pin1,pin2;
//A connect equation
//in Modelica:
connect(pin1,pin2);
```

Corresponds to

```
pin1.v = pin2.v;
pin1.i + pin2.i =0;
```


Connection Equations

```
Pin pin1,pin2;
//A connect equation
//in Modelica
connect(pin1,pin2);
```

Corresponds to

```
pin1.v = pin2.v;
pin1.i + pin2.i =0;
```

Multiple connections are possible:

```
connect(pin1,pin2); connect(pin1,pin3); ... connect(pin1,pinN);
```

Each primitive connection set of nonflow variables is used to generate equations of the form:

$$v_1 = v_2 = v_3 = \ldots v_n$$

Each primitive connection set of flow variables is used to generate *sum-to-zero* equations of the form:

$$i_1 + i_2 + \ldots (-i_k) + \ldots i_n = 0$$

Acausal, Causal, and Composite Connections

Two *basic* and one *composite* kind of connection in Modelica

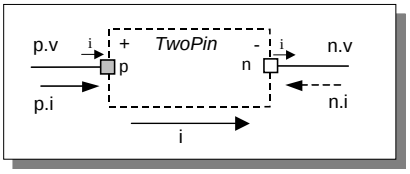
- *Acausal connections*
- *Causal connections*, also called *signal* connections
- *Composite connections*, also called structured connections, composed of basic or composite connections

connector class
fixed causality

```
connector OutPort
  output Real signal;
end OutPort
```


Common Component Structure

The base class `TwoPin` has two connectors `p` and `n` for positive and negative pins respectively



partial class (cannot be instantiated)

positive pin

negative pin

```
partial model TwoPin
  Voltage     v
  Current     i
  Pin p;
  Pin n;
equation
  v = p.v - n.v;
  0 = p.i + n.i;
  i = p.i;
end TwoPin;
// TwoPin is same as OnePort in
// Modelica.Electrical.Analog.Interfaces
```

```
connector Pin
  Voltage        v;
  flow Current   i;
end Pin;
```

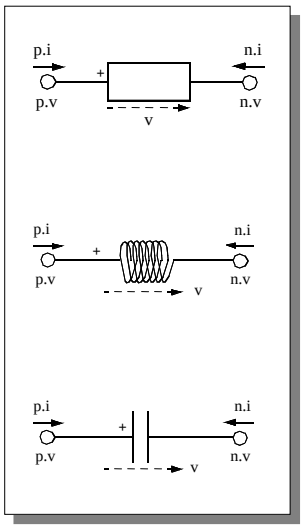
electrical connector class

Electrical Components

```
model Resistor "Ideal electrical resistor"
  extends TwoPin;
  parameter Real R;
equation
  R*i = v;
end Resistor;
```

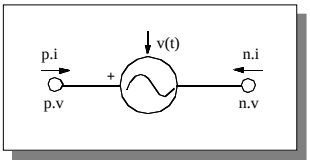
```
model Inductor "Ideal electrical inductor"
  extends TwoPin;
  parameter Real L "Inductance";
equation
  L*der(i) = v;
end Inductor;
```

```
model Capacitor "Ideal electrical capacitor"
  extends TwoPin;
  parameter Real C ;
equation
  i=C*der(v);
end Capacitor;
```

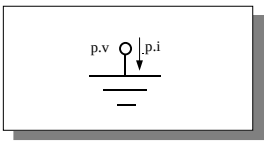


Electrical Components cont'

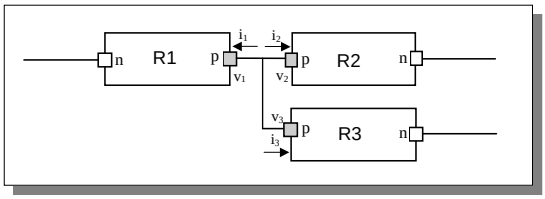
```
model Source
   extends TwoPin;
   parameter Real A,w;
equation
   v = A*sin(w*time);
end Resistor;
```




```
model Ground
   Pin p;
equation
   p.v = 0;
end Ground;
```



Resistor Circuit




```
model ResistorCircuit
  Resistor R1(R=100);
  Resistor R2(R=200);
  Resistor R3(R=300);
equation
  connect(R1.p, R2.p);
  connect(R1.p, R3.p);
end ResistorCircuit;
```

Corresponds to

```
R1.p.v = R2.p.v;
R1.p.v = R3.p.v;
R1.p.i + R2.p.i + R3.p.i = 0;
```


Modelica Standard Library Used for Graphical Modeling

Modelica Standard Library (called `Modelica`) is a standardized predefined package developed by Modelica Association

It can be used freely for both commercial and noncommercial purposes under the conditions of *The Modelica License*.

Modelica libraries are available online including documentation and source code from http://www.modelica.org/library/library.html.

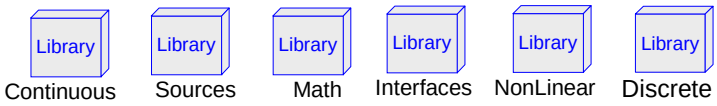
Modelica Standard Library cont'

Modelica Standard Library contains components from various application areas, with the following sublibraries:

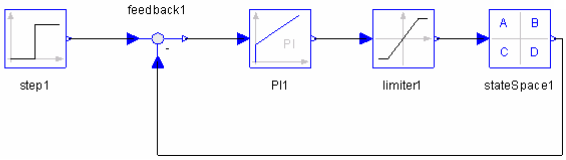
- Blocks — Library for basic input/output control blocks
- Constants — Mathematical constants and constants of nature
- Electrical — Library for electrical models
- Icons — Icon definitions
- Math — Mathematical functions
- Mechanics — Library for mechanical systems
- Media — Media Media models for liquids and gases
- SIunits — Type definitions based on SI units according to ISO 31-1992
- Stategraph — Hierarchical state machines (analogous to Statecharts)
- Thermal — Components for thermal systems
- Utility — Utilities Utility functions especially for scripting

Modelica.Blocks

This library contains input/output blocks to build up block diagrams.



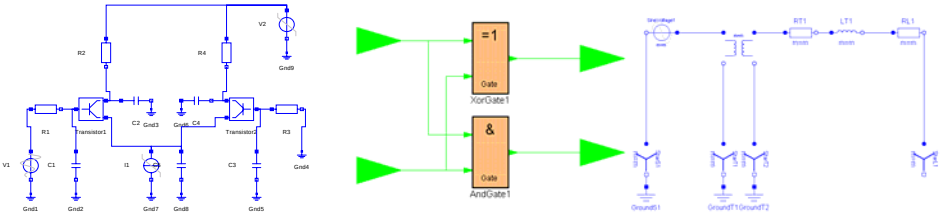
Example:



Modelica.Electrical

Electrical components for building analog, digital, and multiphase circuits

Examples:

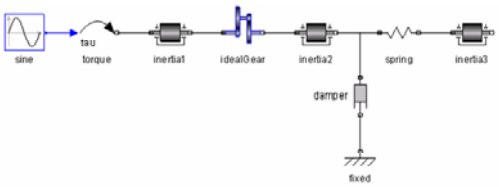


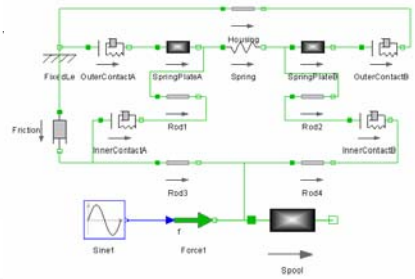
Modelica.Mechanics

Package containing components for mechanical systems

Subpackages:

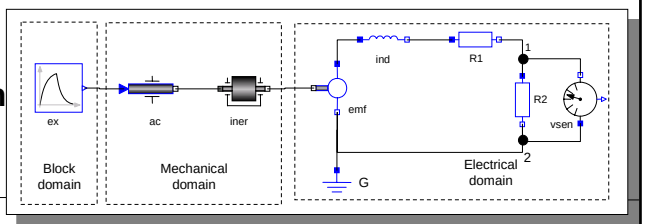
- Rotational — 1-dimensional rotational mechanical components
- Translational — 1-dimensional translational mechanical components
- MultiBody — 3-dimensional mechanical components





Example of Connecting Components from Multiple Domains

- **Block domain**
- **Mechanical domain**
- **Electrical domain**

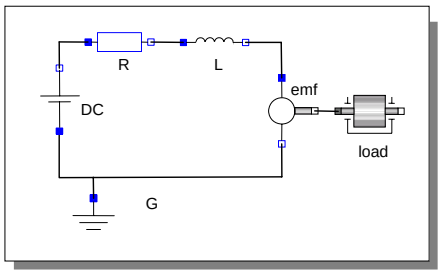



```
model Generator
  Modelica.Mechanics.Rotational.Accelerate ac;
  Modelica.Mechanics.Rotational.Inertia iner;
  Modelica.Electrical.Analog.Basic.EMF emf(k=-1);
  Modelica.Electrical.Analog.Basic.Inductor ind(L=0.1);
  Modelica.Electrical.Analog.Basic.Resistor R1,R2;
  Modelica.Electrical.Analog.Basic.Ground G;
  Modelica.Electrical.Analog.Sensors.VoltageSensor vsens;
  Modelica.Blocks.Sources.Exponentials ex(riseTime={2},riseTimeConst={1});
equation
  connect(ac.flange_b, iner.flange_a); connect(iner.flange_b, emf.flange_b);
  connect(emf.p, ind.p); connect(ind.n, R1.p); connect(emf.n, G.p);
  connect(emf.n, R2.n); connect(R1.n, R2.p); connect(R2.p, vsens.n);
  connect(R2.n, vsens.p); connect(ex.outPort, ac.inPort);
end Generator;
```


Simple Modelica DCMotor Model Multi-Domain (Electro-Mechanical)

A DC motor can be thought of as an electrical circuit which also contains an electromechanical component.

```
model DCMotor
   Resistor R(R=100);
   Inductor L(L=100);
   VsourceDC DC(f=10);
   Ground G;
   EMF emf(k=10,J=10, b=2);
   Inertia load;
equation
   connect(DC.p,R.n);
   connect(R.p,L.n);
   connect(L.p, emf.n);
   connect(emf.p, DC.n);
   connect(DC.n,G.p);
   connect(emf.flange,load.flange);
end DCMotor;
```

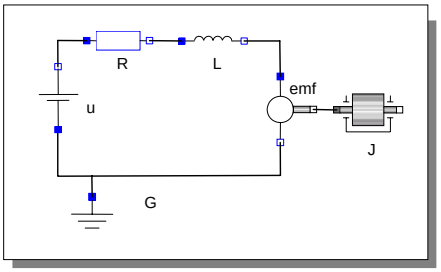


Graphical Modeling Exercises

Exercise 2.1

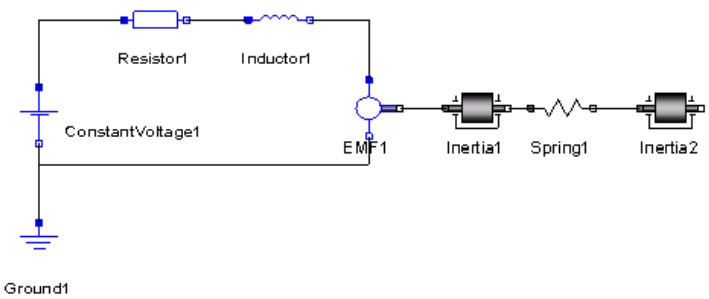
- Draw the `DCMotor` model using the graphic connection editor using models from the following Modelica libraries:
  `Mechanics.Rotational`,
  `Electrical.Analog.Basic`,
  `Electrical.Analog.Sources`

- Simulate it for 15s and plot the variables for the outgoing rotational speed on the inertia axis and the voltage on the voltage source (denoted u in the figure) in the same plot.



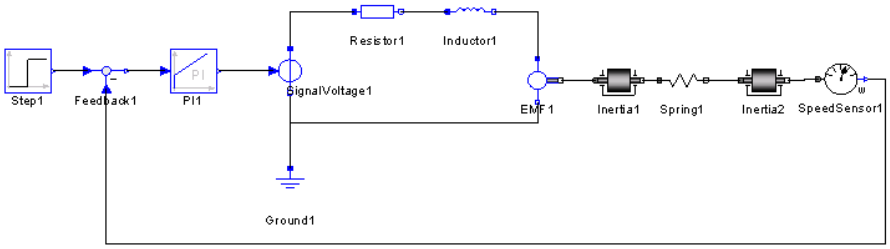
Optional Exercise 2.2

- If there is enough time: Add a torsional spring to the outgoing shaft and another inertia element. Simulate again and see the results. Adjust some parameters to make a rather stiff spring.



Optional Exercise 2.3

- If there is enough time: Add a PI controller to the system and try to control the rotational speed of the outgoing shaft. Verify the result using a step signal for input. Tune the PI controller by changing its parameters in MathModelica.

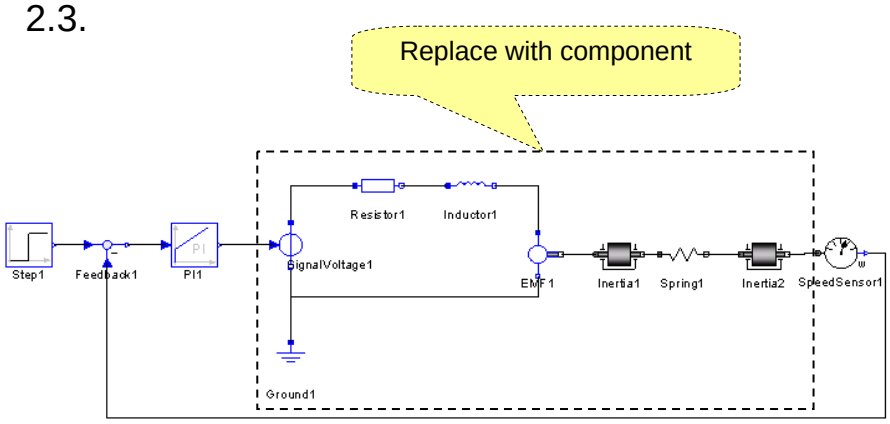


Live example

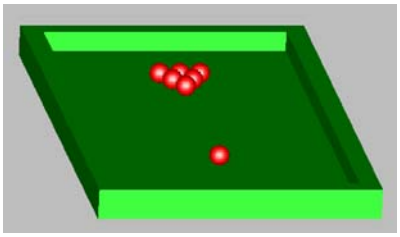
- Building a component with icon

Optional Exercise 2.4

- Make a component of the model in Exercise 2.2, and use it when building the model in exercise 2.3.



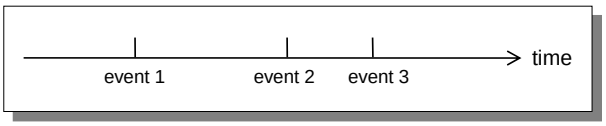
Optional Extra Material Discrete Events and Hybrid Systems



Picture: Courtesy Hilding Elmqvist

Events

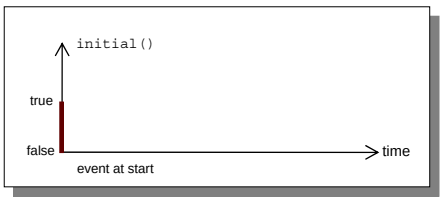
Events are ordered in time and form an event history



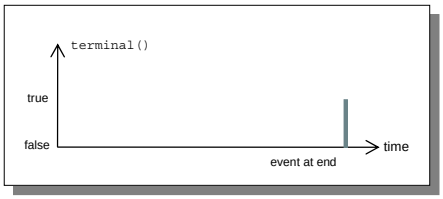
- A *point* in time that is instantaneous, i.e., has zero duration
- An *event condition* that switches from false to true in order for the event to take place
- A set of *variables* that are associated with the event, i.e. are referenced or explicitly changed by equations associated with the event
- Some *behavior* associated with the event, expressed as *conditional equations* that become active or are deactivated at the event. *Instantaneous equations* is a special case of conditional equations that are only active *at* events.

`initial` and `terminal` events

Initialization actions are triggered by `initial()`

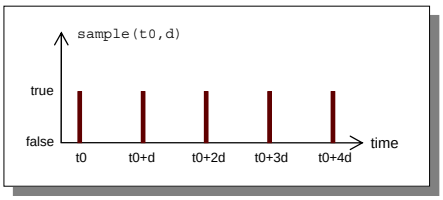


Actions at the end of a simulation are triggered by `terminal()`



Generating Repeated Events

The call `sample(t0,d)` returns true and triggers events at times `t0+i*d`, where `i=0,1,…`




```
class SamplingClock
  parameter Modelica.SIunits.Time  first,interval;
  Boolean clock;
equation
  clock = sample(first,interval);
  when clock then
   ...
  end when;
end SamplingClock;
```


Expressing Event Behavior in Modelica

if-equations, if-statements, and if-expressions express different behavior in different operating regions

```
if <condition> then
  <equations>
elseif <condition> then
  <equations>
else
  <equations>
end if;
```

```
model Diode "Ideal diode"
  extends TwoPin;
  Real s;
  Boolean off;
equation
  off = s < 0;
  if off then
    v=s
  else
    v=0;
  end if;
  i = if off then 0 else s;
end Diode;
```

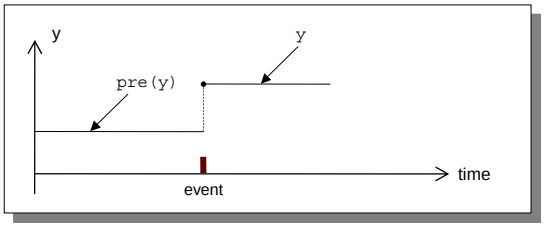
when-equations become active at events

```
when <conditions> then
  <equations>
end when;
```

```
equation
  when x > y.start then
    ...
```


Obtaining Predecessor Values of a Variable Using `pre()`

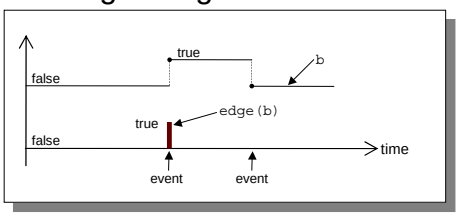
At an event, `pre(y)` gives the previous value of y immediately before the event, except for event iteration of multiple events at the same point in time when the value is from the previous iteration



- The variable *y* has one of the basic types `Boolean`, `Integer`, `Real`, `String`, or `enumeration`, a subtype of those, or an array type of one of those basic types or subtypes
- The variable y is a discrete-time variable
- The `pre` operator can *not* be used within a function

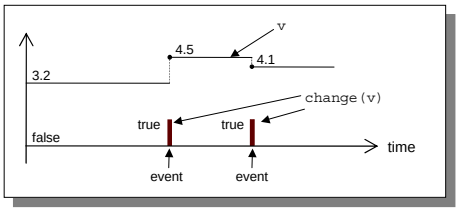
Detecting Changes of Boolean Variables Using `edge()` and `change()`

Detecting changes of boolean variables using `edge()`



The expression `edge(b)` is true at events when b switches from false to true

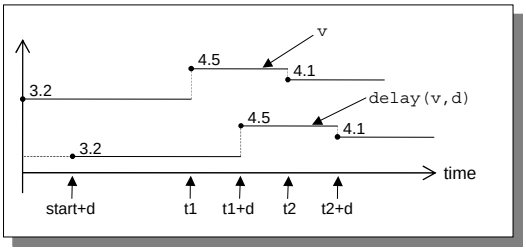
Detecting changes of discrete-time variables using `change()`



The expression `change(v)` is true at instants when v changes value

Creating Time-Delayed Expressions

Creating time-delayed expressions using `delay()`



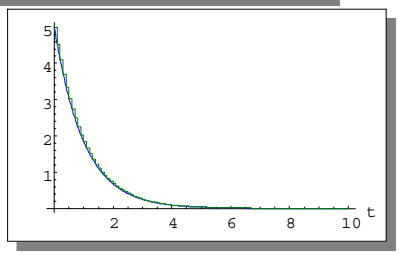
In the expression `delay(v,d)` *v* is delayed by a delay time *d*

A Sampler Model

```
model Sampler
  parameter Real sample_interval = 0.1;
  Real x(start=5);
  Real y;
equation
  der(x) = -x;
  when sample(0, sample_interval) then
    y = x;
  end when;
end Sampler;
```

```
simulate(Sampler, startTime = 0, stopTime = 10)

plot({x,y})
```

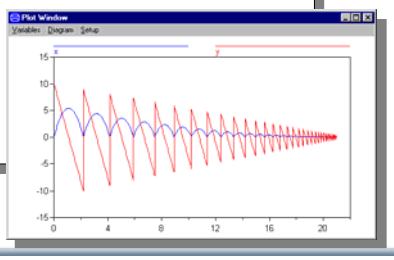


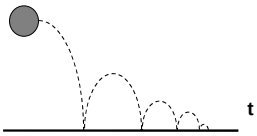
Discontinuous Changes to Variables at Events via When-Equations/Statements

The value of a *discrete-time* variable can be changed by placing the variable on the left-hand side in an equation within a `when`-equation, or on the left-hand side of an assignment statement in a `when`-statement

The value of a *continuous-time* state variable can be instantaneously changed by a `reinit`-equation within a `when`-equation

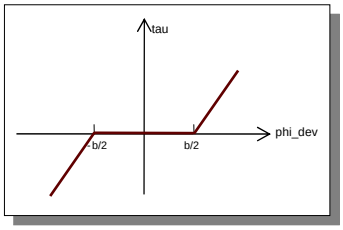
```
model BouncingBall "the bouncing ball model"
  parameter Real g=9.18;   //gravitational acc.
  parameter Real c=0.90;   //elasticity constant
  Real x(start=0),y(start=10);
equation
  der(x) = y;
  der(y)=-g;
  when x<0 then
    reinit(y, -c*y);
  end when;
end BouncingBall;
```



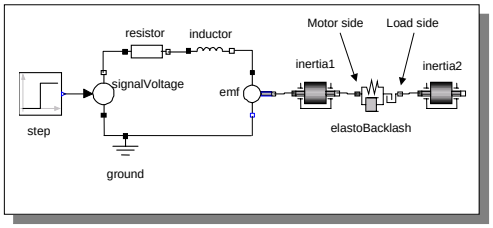


A Mode Switching Model Example

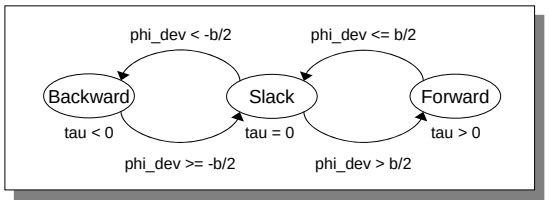
Elastic transmission with slack



DC motor transmission with elastic backlash



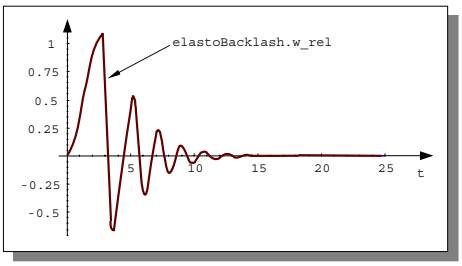
A finite state automaton `SimpleElastoBacklash` model



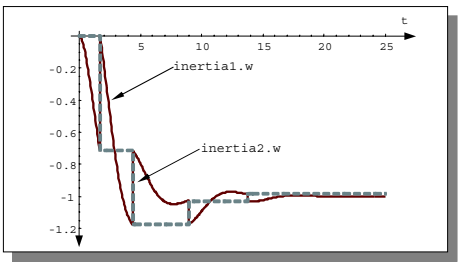
A Mode Switching Model Example cont'

```
partial model SimpleElastoBacklash
  Boolean backward, slack, forward;  // Mode variables
  parameter Real  b                  "Size of backlash region";
  parameter Real  c = 1.e5           "Spring constant (c>0), N.m/rad";
  Flange_a        flange_a           "(left) driving flange - connector";
  Flange_b        flange_b           "(right) driven flange - connector";
  parameter Real  phi_rel0 = 0       "Angle when spring exerts no torque";
  Real         phi_rel               "Relative rotation angle betw. flanges";
  Real         phi_dev               "Angle deviation from zero-torque pos";
  Real         tau                   "Torque between flanges";
equation
  phi_rel    = flange_b.phi - flange_a.phi;
  phi_dev    = phi_rel - phi_rel0;
  backward   = phi_rel < -b/2;                  // Backward angle gives torque tau<0
  forward    = phi_rel > b/2;                   // Forward angle gives torque tau>0
  slack      = not (backward or forward);       // Slack angle gives no torque
  tau = if forward then                         // Forward angle gives
           c*(phi_dev - b/2)                    // positive driving torque
        else (if backward then                  // Backward angle gives
                c*(phi_dev + b/2)               // negative braking torque
              else                              // Slack gives
                0);                             // zero torque
end SimpleElastoBacklash
```


A Mode Switching Model Example cont'



Relative rotational speed between the flanges of the `Elastobacklash` transmission



We define a model with less mass in `inertia2(J=1)`, no damping `d=0`, and weaker string constant `c=1e-5`, to show even more dramatic backlash phenomena

The figure depicts the rotational speeds for the two flanges of the transmission with elastic backlash

Optional Exercise 1.5

- Locate the BouncingBall model in one of the hybrid modeling sections of DrModelica (the When-Equations link in Section 2.9), run it, change it slightly, and re-run it.