Domain and Range

Let R be a relation such that $R \subseteq A \times B$ (R is defined "from A to B").
R's *domain* is the set $\{a \mid \text{there is a } b \in B \text{ such that } R(a,b)\}$,
and R's *range* is the set $\{b \mid \text{there is a } a \in A \text{ such that } R(a,b)\}$.

Domain and Range: Example (1)

Let M be the set of all human beings, and *Mother*(x,y) iff x is y's mother.
The domain of *Mother* is the set of all mothers, and the range is the set of all humans (i.e. M).

Domain and Range: Example (2)

Let S be the set of all Swedish cities, and H all swedish universities, and $R \subseteq H \times S$ such that $R(h,s)$ iff h is a university in the city s.
R's domain is H, and it's range is the set of all cities with a university.

Reflexivity

A relation R on a set A is *reflexive* iff $R(x,x)$ for every $x \in A$.

Reflexivity: Examples

- $=$ on $\mathbb{N}$ (actually on anything)
- *SameAge*(x,y) on the set of humans
- $\leq$ on $\mathbb{N}$
- $\subseteq$ on sets

Reflexivity: Counter Examples

- *Father*(x,y) on the set of humans
- *Siblings*(x,y) on the set of humans
- *ProvidesFor*(x,y) on the set of humans
- $\subset$ on sets

Symmetry

A relation R on a set A is *symmetric* if, whenever $R(x,y)$ holds, $R(y,x)$ holds too.

Symmetry: Examples

- *Siblings*(x,y) on the set of humans
- *Cousins*(x,y) on the set of humans
- $=$ on $\mathbb{N}$ (or, on anything)

Symmetry: Counter Examples

- $\leq$ on $\mathbb{N}$
- *Sister*(x, y) on the set of humans
- $\subseteq$ on sets

Antisymmetry

A relation R on a set A is *antisymmetric* iff whenever $R(x, y)$ and $R(y, x)$ hold , $x = y$ holds too.

Antisymmetry: Examples

- $\leq$ on $\mathbb{N}$
- $\subseteq$ on sets
- *Father*(x, y) on humans

Antisymmetry: Counter Example

- *Siblings*(x, y) on humans

Transitivity

A relation R on a set A is *transitive* iff when $R(x, y)$ and $R(y, z)$ holds, $R(x, z)$ holds.

Transitivity: Examples

- $<$ on $\mathbb{N}$
- *OlderThan*(x, y) on the set of humans
- $\subseteq$ on sets
- $\emptyset$ on any set A

Transitivity: Counter Examples

- $R(x, y)$ on $\mathbb{N}$ iff $y - x < 10$
- *Father*(x, y) on the set of humans

Negated Properties

- Irreflexivity
- Asymmetry
- Intransitivity

There are even more properties in the book!

What Happens if We Invert, R^{-1}?

- Reflexivity is preserved.
- Symmetry? All arrows go in both direction: preserved.

Complement?

- R symmetric yields R' symmetric

Complete table can be found on page 45.

Equivalence Relations

A relation R is an *equivalence relation* iff R is reflexive, transitive och symmetric.

Equivalence Relations: Example

- $=$ on $\mathbb{N}$ (or, on anything)
- "Has the same shoe size" on humans
- "Has the same [whatever]" on [anything]