Operations on Relations: Inverse

Since pairs are *ordered* we can flip them.
Let $R \subseteq A \times B$, and define
$R^{-1} = \{\langle y, x\rangle | \langle x, y\rangle \in R\}$,
i.e. we flip the arguments such that
$R^{-1}(y, x)$ iff $R(x, y)$.
Note that $R^{-1} \subseteq B \times A$,
and that $(R^{-1})^{-1} = R$.

Inverse: Example

Let $R = <^{-1}$, where R is a binary relation on $\mathbf{N}$.
Then $R(x, y)$ holds iff $y < x$, i.e. iff $x > y$.
Let *Siblings* be the same relation as before.
Since we always have
Siblings(x, y) iff *Siblings*(y, x) we get
Siblings^{-1} = *Siblings*!

Operations on Relations: Composition

If we have the relations $R \subseteq A \times B$ and
$S \subseteq B \times C$,
we can *compose* to one relation
$S \circ R$, given by
$S \circ R =$
$\{\langle x, z\rangle |$there is a y such that $\langle x, y\rangle \in$
R and $\langle y, z\rangle \in S\}$,
i.e. $(S \circ R)(x, z)$ says that $R(x, y)$ and $S(y, z)$ for
some y.

Composition: Example (1)

Take the relation *Father* defined as before.
Then we get that *Father* $\circ$ *Father*(x, y) iff
x is the grandfather of y.
What about *Siblings*?

Composition: Example (2)

Let A be a set and define
$id_A(x, y)$ iff $x = y$.
id_A is said to be the *identity relation on* A.
If $R \subseteq A \times B$ we get that
$R \circ id_A = R$, and that
$id_B \circ R = R$.

Operations on Relations: Union

If R and S are binary relations between A and
B, what does $R \cup S$ mean?
$(R \cup S)(x, y)$ iff $R(x, y)$ *or* $S(x, y)$.

Union: Example

(*Father* $\cup$ *Mother*)(x, y) says that
x is y's parent.
$(> \cup <)(x, y)$ says that $x \neq y$.

Operations on relations: Intersection

If R and S are binary relations between A and
B, what does $R \cap S$ mean?
$(R \cap S)(x, y)$ iff $R(x, y)$ *and* $S(x, y)$.