Relation: Alternative Definition

Let A and B be sets.
Then R is a relation between A and B iff $R \subseteq A \times B$.
If $A = B$, then R is called a relation *on* A.
Q: Given A and B finite, how many relations exist between A and B?

Relations: More Examples

Let S be set of capitols of the world, and L the set of countries in the world.
The relation *CapitolOf* $\subseteq S \times L$ defined by *CapitolOf* $= \{\langle s, l\rangle | s$ is capitol in $l\}$ represent that *CapitolOf*(s, l) is true iff s is the capitol of l.

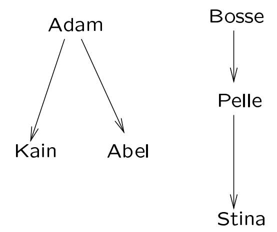
Relations *on* Sets: Examples (1)

Let M be set of human beings.
Let *FatherOf* $\subseteq M \times M$, such that *FatherOf*(f, b) iff f is the father of b.
Let *Siblings* $\subseteq M \times M$, and let *Siblings*(x, y) iff x and$y$ are siblings.

Relations *on* Sets: Examples (2)

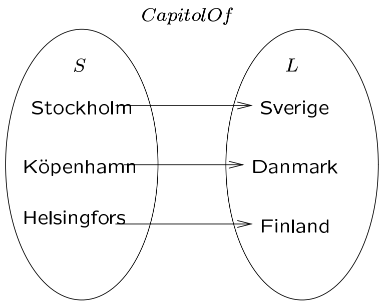
Let *Sister* $\subseteq M \times M$, and let *Sister*(x, y) iff x is y's sister.
Take *SameAge* binary relation on M $(\subseteq M \times M)$, let *SameAge*(x, y) iff x has the same age as y.

Drawing Relations *on* Sets



$FatherOf = \{\langle Pelle, Stina\rangle, \langle Adam, Kain\rangle, \langle Adam, Abel\rangle, \langle Bosse, Pelle\rangle\}$

Drawing Relations *between* Sets



Operations on Relations: Complement

Let R be a relation between A and B.
It is possible to find the *complement* to R as a set, i.e. R', as a relation.
What universe? $A \times B$ is understood, so $R' = (A \times B) - R$.
What does the complement do? *Negates* the proposition.
Everything not in R will be in R'.

Complement: Examples

Let $R(x, y)$ iff $x < y$, with $R \subseteq \mathrm{N} \times \mathrm{N}$.
Then, what is R'?
$R'(x, y)$ iff *not* $x < y$, i.e. iff $x \geq y$.
Let *Siblings(x,y)* $\subseteq M \times M$ such that *Siblings*(x, y) iff x and y are siblings.
Then we get that *Siblings*$'(x, y)$ iff x and y *not* are siblings.