Take-Home Exercise 2(4)
Discrete Mathematics (HKGAB4) 1998:
Relations

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Relations

1. Let $A = \{1, 2\}$, $B = \{a, b, c\}$ and $C = \{d\}$. Write the following sets as lists.
   (a) $(A \times B) \times C$
   (b) $B \cup (C \times A)$
   (c) $(A \cup C) \times B$
   (d) $(C \times B) \circ (A \times C)$
2. Prove or disprove the following statements, for arbitrary sets A, B and C.
   (a) $A \times (B \cap C) = (A \times B) \cap (A \times C)$
   (b) $(A \times B) - (B \times A) = \emptyset$
3. Let $A = \{1, 2, 3\}$, $B = \{3, 4, 5\}$, and $R = \{\langle 1, 4\rangle, \langle 3, 3\rangle, \langle 3, 4\rangle, \langle 1, 3\rangle\}$ a relation from A to B. Write the following sets as lists.
   (a) $A \times B$
   (b) The domain of R
   (c) The range of R
   (d) R'
   (e) R^{-1}
   (f) The domain of $(R')^{-1}$
4. Let $A = \{1, 2, 3\}$ and $B = \{f, g, h\}$. Does the following statements hold, or not?
   (a) $\langle 1, g\rangle \in B \times A$
   (b) $\{\langle g, 2\rangle, \langle f, 2\rangle\} \subset B \times A$
5. Let $A = \{1, 2, 3, 4\}$ and R a relation on A, with $R = \{\langle 1, 2\rangle, \langle 2, 3\rangle, \langle 3, 4\rangle, \langle 4, 1\rangle\}$. What is $R \circ R$?
6. What properties do the following family relations have, of the properties we have discussed?
   (a) "Sister of"
   (b) "Cousin of"
   (c) "Mother of"

7. Let $A = \{a, b, c, d\}$, and let R_1, R_2, R_3 and R_4 be relations on A according to the following:

   (a) $R_1 = \{\langle a, b\rangle, \langle a, a\rangle, \langle b, b\rangle, \langle c, d\rangle, \langle b, a\rangle, \langle d, c\rangle, \langle c, c\rangle, \langle d, d\rangle\}$

   (b) $R_2 = \{\langle d, c\rangle, \langle c, b\rangle, \langle b, a\rangle, \langle a, d\rangle\}$

   (c) $R_3 = \{\langle a, b\rangle, \langle c, c\rangle, \langle a, c\rangle, \langle b, c\rangle, \langle d, d\rangle\}$

   (d) $R_4 = \emptyset$

   What properties does R_1, R_2, R_3 and R_4 have, of the ones we have discussed?

   Also decide whether the relation happens to be an equivalence relation, a (weak or strict) order, or a total order.

8. Invent a set A, and give the following:

   (a) An equivalence relation on A.

   (b) A weak order on A.

   (c) An asymmetric, connected relation on A.