Comments and questions (by Angelo Gilio) to the paper of Thomas Lukasiewicz: "Probabilistic Default Reasoning with Conditional Constraints".

1. This remark concerns the semantics of conditional assertions. As proposed (in 1936) by de Finetti, a conditional event $E|H$, with $H \neq \emptyset$ (notice that in this context to consider $E|H$, with $H = \emptyset$ has no sense), can be looked at as a three-valued entity: *true*, or *false*, or *indeterminate*, according to whether E and H are true, or E is true and H is false, or H is false. This semantics has been also adopted in (Dubois and Prade 1994), (Benferhat, Dubois and Prade 1997).

2. Then, an interesting (and in my opinion suitable, perhaps in next future), development of your paper could be the following one: instead of starting with a set of basic propositions $\Phi = \{B_1, B_2, \ldots, B_n\}$, you could start with a set of conditional assertions $\Phi = \{E_1|H_1, E_2|H_2, \ldots, E_n|H_n\}$. Then, a *possible world* would be a truth assignment $I : \Phi \longrightarrow \{\textbf{true, false, undetermined}\}$, and so on.

3. Notice that unconditional probabilities are not enough expressive for conditional assertions. Then, a probabilistic interpretation $\mathcal{P}$ should be defined as a (coherent conditional) probability assessment $\mathcal{P} = (p_1, \ldots, p_n)$ on $\{E_1|H_1, E_2|H_2, \ldots, E_n|H_n\}$. The truth of a probabilistic formula F in a probabilistic interpretation $\mathcal{P}$, denoted $\mathcal{P} \models F$, should be defined as:
   - $\mathcal{P} \models (\psi|\phi)[l, u]$ iff $P(\psi|\phi) \in [l, u]$.
   - ...

4. This remark concerns the concept of logical entailment. Given a set of probabilistic formulas $\mathcal{F}$ and a strict conditional constraint $(\psi|\phi)[l,u]$, to define the notion of *tight logical consequence* we don't need to assume $P(\phi) > 0$. In fact, in the framework of coherence, the condition $\mathcal{F} \models_{tight} (\psi|\phi)[l,u]$ can checked in all cases, included that one in which $\mathcal{F} \models_{tight} (\phi|\top)[0,0]$. In this way, also the "canonical" assumption $l = 1, u = 0$ when $\mathcal{F} \models_{tight} (\phi|\top)[0,0]$ could be avoided. Then, computing the 1-entailed tight intervals, in each one of the items (5), (7), (8), (10), (12) of Table 1, instead of $[1,0]$ one would obtain (more significantly) an interval $[l,u] \subseteq [0,1]$ and no problem of unsatisfiability would arise.

5. It seems that your proposal of weakening the notion of 1-entailment, with the aim of handling unsatisfiabilities, could be related in some sense to situations in which the antecedents of some conditional assertions have zero probability. If my intuition were true, this should be a "strong" motivation to develop a probabilistic default reasoning in the general context of coherence.

References

Dubois, D.; and Prade, H. 1994. Conditional objects as nonmonotonic consequence relationships. *IEEE Trans. Syst. Man Cybern.* 24: 1724-1740.

Benferhat, S.; Dubois, D.; and Prade, H. 1997. Nonmonotonic reasoning, conditional objects and possibility theory. *Artif. Intell.* 92(1-2): 259-276.